Design Criteria and Performance Analysis for Distributed Space-Time Coding

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Abstract

In this paper, the design of distributed space-time codes for wireless relay networks is considered. Distributed space-time coding (DSTC) can be achieved through node cooperation to emulate multiple antennas transmitter. First, the decode-and-forward protocol, in which each relay node decodes the symbols received from the source node before retransmission, is considered. A space-time code designed to achieve full diversity and maximum coding gain over multiple-input multiple-output (MIMO) channels is proved to achieve full diversity but not necessarily maximizing the coding gain if used with the decode-and-forward protocol. Next, the amplify-and-forward protocol is considered; each relay node can only perform simple operations such as linear transformation of the received signal and then amplify the signal before retransmission. A space-time code designed to achieve full diversity and maximum coding gain over MIMO channels is proved to achieve full diversity and maximum coding gain if used with the amplify-and-forward protocol.

Next, the design of DSTC that can mitigate the relay nodes synchronization errors is considered. Most of the previous works on cooperative transmission assume perfect synchronization between the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays are identical. Perfect synchronization is difficult to achieve among randomly located relay nodes. To simplify the synchronization in the network, a diagonal structure is imposed on the space-time code used. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay node to transmit at any time slot. Hence, it is not necessary to synchronize simultaneous "in-phase" transmissions of randomly located relay nodes, which greatly simplifies the synchronization among the relay nodes. The code design criterion for distributed space-time codes based on the diagonal structure is derived. The work shows that the code design criterion is to maximize the minimum product distance.

Keywords: Coding gain, distributed space-time coding, space-time coding, spatial diversity, wireless relay networks.

I. INTRODUCTION

Recently, there has been much interest in modulation techniques to achieve transmit diversity motivated by the increased capacity of multiple-input multiple-output (MIMO) channels [1]. To achieve transmit diversity the transmitter needs to be equipped with more than one antenna. The antennas should be well separated to have uncorrelated fading among the different antennas; hence, higher diversity orders and higher coding gains are achievable. It is affordable to equip base stations with more than one antenna, but it is difficult to equip the small mobile units with more than one antenna with uncorrelated fading. In such a case, transmit diversity can only be achieved through user cooperation leading to what is known as cooperative diversity [2], [3].

Designing protocols that allow several single-antenna terminals to cooperate via forwarding each others' data can increase the system reliability by achieving spatial diversity. Another benefit results from boosting the system throughput by employing cooperation between nodes in the network. In [2] and [3], various node cooperation protocols were proposed and outage probability analyses for these protocols were provided. The concepts of decode-and-forward and amplify-and-forward have been introduced in these works. Symbol error rate performance analyses for the single-node and multi-node decode-and-forward cooperation protocols were provided in [4], [5]. Performance analyses for the single node and multi-node amplify-and-forward cooperation protocols can be found in [6], [7]. In [8], the authors emphasized the importance of studying distributed multistage relaying, in which each stage acts as a virtual antenna, and they envisioned that this is a promising direction to achieve the very high data rate requirements of future wireless systems.

The main problem with the multi-node decode-and-forward protocol [5] and the multi-node amplify-and-forward protocol [6], [9] is the loss in the data rate as the number of relay nodes increases. The use of orthogonal subchannels for the relay node transmissions, either through TDMA or FDMA, results in a high loss of the system spectral efficiency. This leads to the use of what is known as distributed space-time coding, where relay nodes are allowed to simultaneously transmit over the same channel by emulating a space-time code. The term *distributed* comes from the fact that the virtual multi-antenna transmitter is distributed between randomly located relay nodes. The space-time code design criteria for virtual antenna arrays was considered in [10], where the design of space-time trellis codes (STTCs) was considered. It was proposed in [2] to use relay nodes to form a virtual multi-antenna transmitter to achieve diversity. In addition, an outage analysis was presented for the system.

Several works have considered the application of the existing space-time codes in a distributed fashion

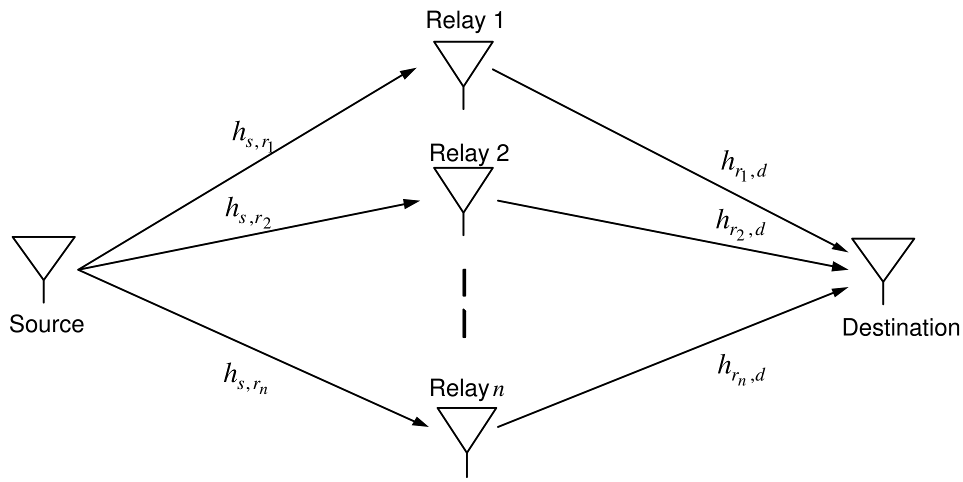

Fig. 1. Simplified system model for the two-hop distributed space-time codes.

for the wireless relay network [11], [12], [13], [14]. All of these works have considered a two-hop relay network where a direct link between the source and the destination nodes does not exist, as shown in Fig. 1. In [11], space-time block codes were used in a completely distributed fashion. Each relay node transmits a randomly selected column from the space-time code matrix. This system achieves a diversity of order one, as the signal-to-noise (SNR) tends to infinity, limited by the probability of having all of the relay nodes selecting to transmit the same column of the space-time code matrix. In [12], distributed space-time coding based on the Alamouti scheme and amplify-and-forward cooperation protocol was analyzed. An expression for the average symbol error rate (SER) was derived. In [13], a performance analysis of the gain of using cooperation among nodes was considered assuming that the number of relays available for cooperation is a Poisson random variable. The authors compared the performance of different distributed space-time codes designed for the MIMO channels under this assumption. In [14], the performance the linear dispersion (LD) space-time codes of [15] was analyzed when used for distributed space-time coding in wireless relay networks. These works did not account for the code design criteria for the space-time codes when employed in a distributed fashion. An answer to the question of whether or not a space-time code, which achieves full diversity and maximum coding gain over MIMO channels, can also achieve full diversity and maximum coding gain if used in a distributed fashion.

In this paper, the two-hop relay network model depicted in Fig. 1, where the system lacks a direct link from the source to destination node, is considered. The results apply to that system model and other relaying network models might have different results. Every node is assumed to be equipped with only one antenna. In the analysis, the wireless channel between any two nodes is assumed to be a Rayleigh flat fading channel. First, distributed space-time coding in conjunction with the decode-and-forward cooperation protocol is considered. In this scheme, the relay node forwards the source symbols if it has decoded correctly. Hence, not all of the relays assigned to help the source will forward the source information. A space-time code designed to achieve full diversity and maximum coding gain over MIMO channels will achieve full diversity if used with the decode-and-forward cooperation protocol. However, it will not necessarily maximize the coding gain.

The design of distributed space-time codes used in conjunction with the amplify-and-forward cooperation protocol is considered. In this scheme, the relay nodes do not decode the received signals from the source node, but they can perform simple operations to the received signal such as linear transformation. Each relay node amplifies the received signal after processing and retransmits it to the destination. In the amplify-and-forward cooperation protocol all of the assigned relay nodes for helping the source will always forward the source information. A space-time code designed to achieve full diversity and maximum coding gain over MIMO channels is proved to achieve full diversity and maximum coding gain when used with the amplify-and-forward protocol [16].

Most of the previous works on cooperative transmission assume perfect synchronization among the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays are identical. Perfect synchronization is difficult to be achieved among randomly located relay nodes. Synchronization mismatches can result in inter-symbol interference, which can highly degrade system performance. However, if the receiver is able to estimate the synchronization mismatches, it can apply a maximum likelihood (ML) detector and the system will incur a lower performance degradation. This comes at the expense of increased receiver complexity to estimate the synchronization mismatches and increased overhead in the system in terms of the required training symbols.

To simplify the synchronization in the network, a diagonal structure is imposed on the distributed space-time code. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay to transmit at any time slot (assuming TDMA). Hence, it is not necessary to synchronize simultaneous in-phase transmissions of randomly located relay nodes, which greatly simplifies the synchronization among the relay nodes. Fig. 2 shows the time frame structure for the conventional distributed space-time codes and the diagonal distributed space-time codes (DDSTCs). From Fig 2 it is clear that perfect propagation delay synchronization and carrier frequency synchronization are not needed since only one relay is transmitting at any given time slot. The code design criterion for the distributed space-time code based on diagonal structure is derived. It turns out that the code design

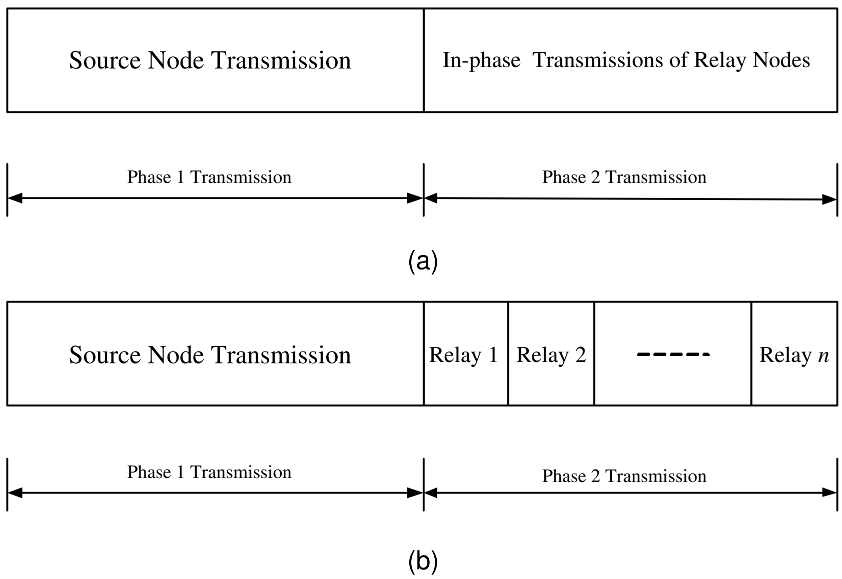

Fig. 2. Time frame structure for (a) Conventional distributed space-time codes (b) DDSTCs.

criterion is to maximize the minimum product distance of the code, which was used perviously to design the diagonal algebraic space-time (DAST) codes [17] and to design full-rate full-diversity space frequency (SF) codes [18].

The rest of the paper is organized as follows. In Section II, the distributed space-time coding with decode-and-forward protocol system model is described and the performance analysis of the system is provided. In Section III, the distributed space-time coding with amplify-and-forward protocol system model is described and the performance analysis of the system is provided. In Section IV, the code design criterion for distributed space-time code based on diagonal structure is derived. In Section V, simulation results are presented. Finally, Section VI concludes the paper.

II. DSTC WITH THE DECODE-AND-FORWARD PROTOCOL

In this section, the system model for DSTC with decode-and-forward cooperation protocol is presented, and a system performance analysis is provided. The notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{C})$ is used to denote that the random vector $\mathbf{x}$ is a circularly symmetric complex Gaussian random vector with mean $\mathbf{m}$ and covariance matrix $\mathbf{C}$.

A. DSTC with the Decode-and-Forward Protocol System Model

The source node is assumed to have n relay nodes assigned for cooperation. The system has two phases given as follows. In phase 1, the source transmits data to the relay nodes with power P_1. The received signal at the k-th relay is modeled as

$$\mathbf{y}_{s,r_k} = \sqrt{P_1} h_{s,r_k} \mathbf{s} + \mathbf{v}_{s,r_k}, \quad k = 1, 2, \cdots, n, \tag{1}$$

where $\mathbf{s}$ is an $L \times 1$ transmitted data vector with a power constraint $||\mathbf{s}||_F^2 \leq L$, where $||\cdot||_F^2$ denotes the Frobenius norm[1] and $h_{s,r_k} \sim \mathcal{CN}(0, \delta_{s,r_k}^2)$ denotes the channel gain between the source node and the k-th relay node. The channel gains from the source node to the relay nodes are assumed to be independent. All channel gains are fixed during the transmission of one data packet and can vary from one packet to another, i.e., a block flat-fading channel model is assumed. In (1), $\mathbf{v}_{s,r_i} \sim \mathcal{CN}(\mathbf{0}, N_o \mathbf{I}_n)$ denotes additive white Gaussian noise (AWGN), where $\mathbf{I}_n$ denotes the $n \times n$ identity matrix.

The n relay nodes try to decode the received signals from the source node. Each relay node is assumed to be capable of deciding whether or not it has decoded correctly. If a relay node decodes correctly, it will forward the source data in the second phase of the cooperation protocol; otherwise, it remains idle. This can be achieved through the use of cyclic redundancy check (CRC) codes [19]. Alternatively, this performance can be approached by setting a SNR threshold at the relay nodes, and the relay will only forward the source data if the received SNR is larger than that threshold [5]. For the analysis in this section, the relay nodes are assumed to be synchronized either by a centralized or a distributed algorithm.

In phase 2, the relay nodes that have decoded correctly re-encode the data vector $\mathbf{s}$ with a pre-assigned code structure. In the subsequent development no specific code design will be assumed, instead a generic space-time (ST) code structure is considered. The ST code is distributed among the relays such that each relay will emulate a single antenna in a multiple-antenna transmitter. Hence, each relay will generate a column in the corresponding ST code matrix. Let $\mathbf{X}_r$ denote the $K \times n$ space-time code matrix with $K \geq n$. Column k of $\mathbf{X}_r$ represents the code transmitted from the k-th relay node. The signal received at the destination is given by

$$\mathbf{y_d} = \sqrt{P_2} \mathbf{X}_r \mathbf{D_I} \mathbf{h}_d + \mathbf{v}_d, \tag{2}$$

where

$$\mathbf{h}_d = [h_{r_1,d}, h_{r_2,d}, \cdots, h_{r_n,d}]^T$$

[1]The Frobenius norm of the $m \times n$ matrix $\mathbf{A}$ is defined as $||\mathbf{A}||_F^2 = \sum_{i=1}^m \sum_{j=1}^n |\mathbf{A}(i,j)|^2 = TR(\mathbf{A}\mathbf{A}^{\mathcal{H}}) = TR(\mathbf{A}^{\mathcal{H}}\mathbf{A})$, where $TR(\cdot)$ is the trace of a matrix and $\mathbf{A}^{\mathcal{H}}$ is the Hermitian transpose of $\mathbf{A}$.

is an $n \times 1$ channel gains vector from the n relays to the destination, $h_{r_k,d} \sim \mathcal{CN}(0, \delta^2_{r_k,d})$, and P_2 is the relay node power where equal power allocation among the relay nodes is assumed. The channel gains from the relay nodes to the destination node are assumed to be statistically independent as the relays are spatially separated. The $K \times 1$ vector $\mathbf{v}_d \sim \mathcal{CN}(\mathbf{0}, N_o\mathbf{I}_K)$ denotes AWGN at the destination node. The matrix $\mathbf{D_I}$ is the state matrix, which will be defined later.

The state of the k-th relay, i.e., whether it has decoded correctly or not, is denoted by the random variable I_k ($1 \leq k \leq n$), which takes values 1 or 0 if the relay decodes correctly or erroneously, respectively. Let

$$\mathbf{I} = [I_1, I_2, \cdots, I_n]^T$$

denote the state vector of the relay nodes and $n_{\mathbf{I}}$ denote the number of relay nodes that have decoded correctly corresponding to a certain realization $\mathbf{I}$. The random variables I_k's are statistically independent as the state of each relay depends only on its channel conditions to the source node, which are independent from other relays. The matrix

$$\mathbf{D_I} = \text{diag}(I_1, I_2, \cdots, I_n)$$

in (2) is defined as the state matrix of the relay nodes. An energy constraint is imposed on the generated ST code such that $||\mathbf{X}_r||_F^2 \leq L$, and this guarantees that the transmitted power per source symbol is less than or equal to $P_1 + P_2$.

B. DSTC with the Decode-and-Forward Protocol Performance Analysis

In this section, the pairwise error probability (PEP) performance analysis for the cooperation scheme described in Section II-A is provided. The diversity and coding gain achieved by the protocol are then analyzed.

The random variable I_k can be easily seen to be a Bernoulli random variable. Therefore, the probability distribution of I_k is given by

$$I_k = \begin{cases} 0 & \text{with probability} = 1 - (1 - SER_k)^L \\ 1 & \text{with probability} = (1 - SER_k)^L, \end{cases} \tag{3}$$

where SER_k is the un-coded SER at the k-th relay node and is modulation dependent. For M-ary quadrature amplitude modulation (M-QAM, $M = 2^p$ with p even), the exact expression can be shown to be upper bounded by [20]

$$SER_k \leq \frac{2N_o g}{bP_1\delta^2_{s,r_k}}, \tag{4}$$

where $b = 3/(M-1)$ and $g = \frac{4R}{\pi}\int_0^{\pi/2}\sin^2\theta d\theta - \frac{4R^2}{\pi}\int_0^{\pi/4}\sin^2\theta d\theta$, in which $R = 1 - \frac{1}{\sqrt{M}}$.

The destination is assumed to have perfect channel state information (CSI) as well as the relay nodes state vector. The destination applies a maximum likelihood (ML) receiver, which will be a minimum distance rule. The conditional pairwise error probability (PEP) is given by

$$\begin{aligned} &\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I},\ \mathbf{h}_d\right) \\ &\quad = \Pr\left(\|\mathbf{y_d} - \sqrt{P_2}\mathbf{X}_1\mathbf{D_I}\mathbf{h}_d\|_F^2 > \|\mathbf{y_d} - \sqrt{P_2}\mathbf{X}_2\mathbf{D_I}\mathbf{h}_d\|_F^2 | \mathbf{I},\ \mathbf{h}_d,\ \mathbf{X}_1 \text{was transmitted}\right), \end{aligned} \tag{5}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are two possible transmitted codewords. The conditional PEP can be expressed as quadratic form of a complex Gaussian random vector as

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I},\ \mathbf{h}_d\right) = \Pr\left(q < 0 | \mathbf{I},\ \mathbf{h}_d\right), \tag{6}$$

where

$$q = \begin{bmatrix} \mathbf{w}_1^{\mathcal{H}} & \mathbf{w}_2^{\mathcal{H}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_n \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix},$$

$\mathbf{w}_1 = \sqrt{P_2}\left(\mathbf{X}_1 - \mathbf{X}_2\right)\mathbf{D_I}\mathbf{h}_d + \mathbf{v}_d$, $\mathbf{w}_2 = \mathbf{v}_d$. The random vectors $\mathbf{h}_d$ and $\mathbf{I}$ are mutually independent as they arise from independent processes. First, the conditional PEP was averaged over the channel realizations $\mathbf{h}_d$. By defining the signal matrix

$$\mathbf{C_I} = \left(\mathbf{X}_1 - \mathbf{X}_2\right)\mathbf{D_I}\text{diag}(\delta_{r_1,d}^2, \delta_{r_2,d}^2, \cdots, \delta_{r_n,d}^2)\mathbf{D_I}\left(\mathbf{X}_1 - \mathbf{X}_2\right)^{\mathcal{H}}, \tag{7}$$

the conditional PEP in (6) can be tightly upper bounded by [21]

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I}\right) \le \frac{\begin{pmatrix} 2\Delta(\mathbf{I}) - 1 \\ \Delta(\mathbf{I}) - 1 \end{pmatrix} N_0^{\Delta(\mathbf{I})}}{P_2^{\Delta(\mathbf{I})}\prod_{i=1}^{\Delta(\mathbf{I})}\lambda_i^{\mathbf{I}}}, \tag{8}$$

where $\Delta(\mathbf{I})$ is the number of nonzero eigenvalues of the signal matrix and $\lambda_i^{\mathbf{I}}$'s are the nonzero eigenvalues of the signal matrix corresponding to the state vector $\mathbf{I}$. The nonzero eigenvalues of the signal matrix are the same as the nonzero eigenvalues of the matrix [22]

$$\mathbf{\Gamma}\left(\mathbf{X}_1, \mathbf{X}_2\right) = \text{diag}(\delta_{r_1,d}, \delta_{r_2,d}, \cdots, \delta_{r_n,d})\mathbf{D_I}\mathbf{\Phi}\left(\mathbf{X}_1, \mathbf{X}_2\right)\mathbf{D_I}\text{diag}(\delta_{r_1,d}, \delta_{r_2,d}, \cdots, \delta_{r_n,d}),$$

where

$$\mathbf{\Phi}\left(\mathbf{X}_1, \mathbf{X}_2\right) = \left(\mathbf{X}_1 - \mathbf{X}_2\right)^{\mathcal{H}}\left(\mathbf{X}_1 - \mathbf{X}_2\right).$$

The employed space-time code is assumed to achieve full diversity and maximum coding gain over MIMO channels, which means that the matrix $\mathbf{\Phi}\left(\mathbf{X}_1, \mathbf{X}_2\right)$ is full rank of order n for any pair of distinct codewords $\mathbf{X}_1$ and $\mathbf{X}_2$. Achieving maximum coding gain means that the minimum of the products

$\prod_{i=1}^{n} \lambda_i$, where the λ_i's are the eigenvalues of the matrix $\mathbf{\Phi}\left(\mathbf{X}_1, \mathbf{X}_2\right)$, is maximized over all the pairs of distinct codewords [23].

Clearly, if the matrix $\mathbf{\Phi}\left(\mathbf{X}_1, \mathbf{X}_2\right)$ has a rank of order n then the matrix $\mathbf{\Gamma}\left(\mathbf{X}_1, \mathbf{X}_2\right)$ will have a rank of order $n_{\mathbf{I}}$, which is the number of relays that have decoded correctly. Equation (8) can now be rewritten as

$$\Pr\left(\mathbf{X}_1 \rightarrow \mathbf{X}_2 | \mathbf{I}\right) \leq \frac{\begin{pmatrix} 2n_{\mathbf{I}} - 1 \\ n_{\mathbf{I}} - 1 \end{pmatrix} N_0^{n_{\mathbf{I}}}}{P_2^{n_{\mathbf{I}}} \prod_{i=1}^{n_{\mathbf{I}}} \lambda_i^{\mathbf{I}}}. \tag{9}$$

Second, the conditional PEP was averaged over the relays' state vector $\mathbf{I}$. The dependence of the expression in (9) on $\mathbf{I}$ appears through the set of nonzero eigenvalues $\left\{\lambda_i^{\mathbf{I}}\right\}_{i=1}^{n_{\mathbf{I}}}$, which depends on the number of relays that have decoded correctly and their realizations. The state vector $\mathbf{I}$ of the relay nodes determines which columns from the ST code matrix are replaced with zeros and thus affect the resulting eigenvalues.

The probability of having a certain realization of $\mathbf{I}$ is given by

$$\Pr(\mathbf{I}) = \left(\prod_{k \in CR(\mathbf{I})} (1 - SER_k)^L \right) \left(\prod_{k \in ER(\mathbf{I})} \left(1 - (1 - SER_k)^L\right) \right), \tag{10}$$

where $CR(\mathbf{I})$ is the set of relays that have decoded correctly and $ER(\mathbf{I})$ is the set of relays that have decoded erroneously corresponding to the $\mathbf{I}$ realization. For simplicity of presentation symmetry is assumed between all relays, that is $\delta_{s,r_k}^2 = \delta_{s,r}^2$ and $\delta_{r_k,d}^2 = \delta_{r,d}^2$ for all k. Averaging over all realizations of the states of the relays, gives the PEP at high SNR as

$$PEP = \Pr(\mathbf{X}_1 \rightarrow \mathbf{X}_2) \leq \sum_{k=0}^{n} \left((1 - SER)^L\right)^k \left(1 - (1 - SER)^L\right)^{n-k} \sum_{\mathbf{I}:\, n_{\mathbf{I}} = k} \frac{\begin{pmatrix} 2k - 1 \\ k - 1 \end{pmatrix} N_0^k}{P_2^k \prod_{i=1}^{k} \lambda_i^{\mathbf{I}}}, \tag{11}$$

where SER is now the symbol error rate at any relay node due to the symmetry assumption.

The diversity order of a system determines the average rate with which the error probability decays at high enough SNR. In order to compute the diversity order of the system, the PEP in (11) is rewritten in terms of the SNR defined as $SNR = P/N_o$, where $P = P_1 + P_2$ is the transmitted power per source symbol. Let $P_1 = \alpha P$ and $P_2 = (1 - \alpha)P$, where $\alpha \in (0, 1)$. Substituting these definitions along with the SER expressions at the relay nodes from (4) into (11) and considering high SNR, the PEP can be upper bounded as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq SNR^{-n} \sum_{k=0}^{n} \left(\frac{2Lg}{b\alpha\delta_{s,r}^2} \right)^{n-k} \sum_{\mathbf{I}:\, n_{\mathbf{I}}=k} \frac{\begin{pmatrix} 2k-1 \\ k-1 \end{pmatrix}}{(1-\alpha)^k \prod_{i=1}^{k} \lambda_i^{\mathbf{I}}}, \tag{12}$$

where at high SNR $1-(1-SER)^L \approx L \cdot SER$ and upper bounding $1 - L \cdot SER$ by 1. The diversity gain is defined as $d = \lim_{SNR\to\infty} -\frac{\log(PEP)}{\log(SNR)}$. Applying this definition to the PEP in (12), when the number of cooperating nodes is n, gives

$$d_{DF} = \lim_{SNR\to\infty} -\frac{\log(PEP)}{\log(SNR)} = n. \tag{13}$$

Hence, any code that is designed to achieve full diversity over MIMO channels will achieve full diversity in the distributed relay network if it is used in conjunction with the decode-and-forward protocol. Some of these codes can be found in [15], [17], [23], [24], [25].

If full diversity is achieved, the coding gain is

$$C_{DF} = \left(\sum_{k=0}^{n} \left(\frac{2ng}{b\alpha\delta_{s,r}^2} \right)^{n-k} \sum_{\mathbf{I}:\, n_{\mathbf{I}}=k} \frac{\begin{pmatrix} 2k-1 \\ k-1 \end{pmatrix}}{(1-\alpha)^k \prod_{i=1}^{k} \lambda_i^{\mathbf{I}}} \right)^{-\frac{1}{n}}, \tag{14}$$

which is a term that does not depend on the SNR. To minimize the PEP bound the coding gain of the distributed space-time code needs to be maximized. This is different from the determinant criterion in the case of MIMO channels [23]. Hence, a space-time code designed to achieve full diversity and maximum coding gain over MIMO channels will achieve full diversity but not necessarily maximizing the coding gain if used in a distributed fashion with the decode-and-forward protocol. Intuitively, the difference is due to the fact that in the case of distributed space-time codes with decode-and-forward protocol, not all of the relays will always transmit their corresponding code matrix columns. The design criterion used in the case of distributed space-time codes makes sure that the coding gain is significant over all sets of possible relays that have decoded correctly. Although it is difficult to design codes to maximize the coding gain as given by (14), this expression gives insight on how to design good codes. The code design should take into consideration the fact that not all of the relays will always transmit in the second phase.

III. DSTC with the Amplify-and-Forward Protocol

In this section, the distributed space-time coding based on the amplify-and-forward protocol is introduced. In this case, the relay nodes do not perform any hard-decision operation on the received data vectors. The system model is presented and a performance analysis is provided.

A. DSTC with the Amplify-and-Forward Protocol System Model

The system has two phases as follows. In phase 1, if n relays are assigned for cooperation, the source transmits data to the relays with power P_1 and the signal received at the k-th relay is as modeled in (1) with $L = n$. For simplicity of presentation, symmetry of the relay nodes is assumed, i.e., $h_{s,r_k} \sim \mathcal{CN}(0, \delta_{s,r}^2),\ \forall k$ and $h_{r_k,d} \sim \mathcal{CN}(0, \delta_{r,d}^2),\ \forall k$. In the amplify-and-forward protocol, relay nodes do not decode the received signals. Instead, the relays can only amplify the received signal and perform simple operations such as permutations of the received symbols or other forms of *unitary* linear transformations. Let $\mathbf{A}_k$ denote the $n \times n$ unitary transformation matrix at the k-th relay node. Each relay will normalize the received signal by the factor $\sqrt{\frac{P_2/n}{P_1\delta_{s,r}^2+N_0}}$ to satisfy a long term-power constraint. It can be easily shown that this normalization will give a transmitted power per symbol of $P = P_1 + P_2$.

The $n \times 1$ received data vector from the relay nodes at the destination node can be modeled as

$$\mathbf{y_d} = \sqrt{\frac{P_2/n}{P_1\delta_{s,r}^2 + N_0}}\tilde{\mathbf{X}}_r\mathbf{h}_d + \mathbf{v}_d, \tag{15}$$

where $\mathbf{h}_d = [h_{r_1,d}, h_{r_2,d}, \cdots, h_{r_n,d}]^T$ is an $n \times 1$ vector channel gains from the n relays to the destination where $h_{r_i,d} \sim \mathcal{CN}(0, \delta_{r,d}^2)$, $\tilde{\mathbf{X}}_r$ is the $n \times n$ code matrix given by

$$\tilde{\mathbf{X}}_r = [h_{s,r_1}\mathbf{A}_1\mathbf{s}, h_{s,r_2}\mathbf{A}_2\mathbf{s}, \cdots, h_{s,r_n}\mathbf{A}_n\mathbf{s}],$$

and $\mathbf{v}_d$ denotes additive white Gaussian noise. Each element of $\mathbf{v}_d$ given the channel coefficients has the distribution of $\mathcal{CN}\left(0, N_0\left(1 + \frac{P_2/n}{P_1\delta_{s,r}^2+N_o}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)\right)$, and $\mathbf{v}_d$ accounts for both the noise propagated from the relay nodes as well as the noise generated at the destination. It can be easily shown that restricting the linear transformations at the relay nodes to be unitary causes the elements of the vector $\mathbf{v}_d$, given the channel coefficients, to be mutually independent.

Now, the received vector in (15) can be rewritten as

$$\mathbf{y_d} = \sqrt{\frac{P_2P_1/n}{P_1\delta_{s,r}^2 + N_0}}\mathbf{X}_r\mathbf{h} + \mathbf{v}_d, \tag{16}$$

where

$$\mathbf{h} = [\ h_{s,r_1}h_{r_1,d}, h_{s,r_2}h_{r_2,d}, \cdots, h_{s,r_n}h_{r_n,d}]^T$$

and

$$\mathbf{X}_r = [\mathbf{A}_1\mathbf{s}, \mathbf{A}_2\mathbf{s}, \cdots, \mathbf{A}_n\mathbf{s}]$$

plays the role of the space-time codeword.

B. DSTC with the Amplify-and-Forward Protocol Performance Analysis

In this section, a pairwise error probability analysis is made to derive the code design criteria. With the ML decoder, the PEP of mistaking $\mathbf{X}_1$ by $\mathbf{X}_2$ can be upper bounded by the following Chernoff bound

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\exp\left(-\frac{P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\mathbf{h}^{\mathcal{H}}(\mathbf{X}_1-\mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1-\mathbf{X}_2)\mathbf{h}\right)\right\}, \tag{17}$$

where the expectation is over the channel coefficients. Taking the expectation in (17) over the source-to-relay channel coefficients, which are complex Gaussian random variables, gives

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\Bigg\{\det{}^{-1}\Big[\mathbf{I}_n + \frac{\delta_{s,r}^2 P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}(\mathbf{X}_1-\mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1-\mathbf{X}_2)\text{diag}(|h_{r_1,d}|^2, |h_{r_2,d}|^2, \cdots, |h_{r_n,d}|^2)\Big]\Bigg\}, \tag{18}$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix.

To evaluate the expectation in (18), define the matrix

$$\mathbf{M} = \frac{\delta_{s,r}^2 P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\Phi(\mathbf{X}_1,\mathbf{X}_2)\text{diag}(|h_{r_1,d}|^2, |h_{r_2,d}|^2, \cdots, |h_{r_n,d}|^2),$$

where

$$\Phi(\mathbf{X}_1,\mathbf{X}_2) = (\mathbf{X}_1-\mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1-\mathbf{X}_2).$$

The bound in (18) can be written in terms of the eigenvalues of $\mathbf{M}$ as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\frac{1}{\prod_{i=1}^{n}(1+\lambda_{M_i})}\right\}, \tag{19}$$

where λ_{M_i} is the i-th eigenvalue of the matrix $\mathbf{M}$. If $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where P is the power per symbol for some $\alpha \in (0,1)$ and define $SNR = P/N_0$, the eigenvalues of $\mathbf{M}$ increase with the increase of the SNR. Now assuming that the matrix $\mathbf{M}$ has full rank of order n the following approximations hold at high SNR

$$\begin{aligned}\prod_{i=1}^{n}(1+\lambda_{M_i}) &\simeq 1 + \prod_{i=1}^{n}\lambda_{M_i} \\ &= 1 + \left(\frac{\delta_{s,r}^2 P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\right)^n \prod_{i=1}^{n}\lambda_i \prod_{i=1}^{n}|h_{r_i,d}|^2 \\ &\simeq \prod_{i=1}^{n}\left(1 + \frac{\delta_{s,r}^2 P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\lambda_i|h_{r_i,d}|^2\right),\end{aligned} \tag{20}$$

where λ_i's are the eigenvalues of the matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$. The determinant of a matrix equals the product of the matrix eigenvalues and that the determinant of the multiplication of two matrices equals the product of the individual matrices' determinants.

The PEP in (19) can now be approximated at high SNR as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\frac{1}{\prod_{i=1}^{n}(1+\frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2+N_o+\frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\lambda_i|h_{r_i,d}|^2)}\right\}. \tag{21}$$

Consider now the term $h=\sum_{i=1}^{n}|h_{r_i,d}|^2$ in (21), which can be reasonably approximated as $\sum_{i=1}^{n}|h_{r_i,d}|^2 \approx n\delta_{r,d}^2$, especially for large n [14] (by the strong law of large numbers). Averaging the expression in (21) over the exponential distribution of $|h_{r_i,d}|^2$ gives

$$\begin{aligned}\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq &\prod_{i=1}^{n}\left(\frac{(\delta_{s,r}^2\delta_{r,d}^2 P_1P_2/n)\lambda_i}{4N_0\left(P_1\delta_{s,r}^2+N_o+P_2\delta_{r,d}^2\right)}\right)^{-1}\\ &\times\prod_{i=1}^{n}\left[-\exp\left(-\frac{4N_0\left(P_1\delta_{s,r}^2+N_o+P_2\delta_{r,d}^2\right)}{(\delta_{s,r}^2\delta_{r,d}^2P_1P_2/n)\lambda_i}\right)\mathbf{Ei}\left(-\frac{4N_0\left(P_1\delta_{s,r}^2+N_o+P_2\delta_{r,d}^2\right)}{(\delta_{s,r}^2\delta_{r,d}^2P_1P_2/n)\lambda_i}\right)\right],\end{aligned} \tag{22}$$

where $\mathbf{Ei}(.)$ is the exponential integral function defined as [26]

$$\mathbf{Ei}(\mu) = \int_{-\infty}^{\mu}\frac{\exp(t)}{t}dt, \quad \mu < 0. \tag{23}$$

The exponential integral function can be approximated as μ tends to 0 as $-\mathbf{Ei}(\mu) \approx \ln\left(-\frac{1}{\mu}\right)$, $\mu < 0$ [26]. At high SNR (high P) $\exp\left(-\frac{4N_0\left(P_1\delta_{s,r}^2+N_o+P_2\delta_{r,d}^2\right)}{(\delta_{s,r}^2\delta_{r,d}^2P_1P_2/n)\lambda_i}\right) \approx 1$, and using the approximation for the $\mathbf{Ei}(.)$ function provides the bound in (22) as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq \prod_{i=1}^{n}\left(\frac{(\delta_{s,r}^2\delta_{r,d}^2P_1P_2/n)\lambda_i}{4N_0\left(P_1\delta_{s,r}^2+P_2\delta_{r,d}^2\right)}\right)^{-1}\prod_{i=1}^{n}\ln\left(\frac{(\delta_{s,r}^2\delta_{r,d}^2P_1P_2/n)\lambda_i}{4N_0\left(P_1\delta_{s,r}^2+P_2\delta_{r,d}^2\right)}\right). \tag{24}$$

Let $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where P is the power per symbol, for some $\alpha \in (0,1)$. With the definition of the SNR as $SNR = P/N_0$, the bound in (24) can be given as

$$\begin{aligned}\Pr(\mathbf{X}_1 \to \mathbf{X}_2) &\leq a_{AF}\frac{1}{\prod_{i=1}^{n}\lambda_i}SNR^{-n}\prod_{i=1}^{n}\left(\ln(SNR)+\ln\left(C_i\right)\right)\\ &\backsimeq a_{AF}\frac{1}{\prod_{i=1}^{n}\lambda_i}SNR^{-n}\left(\ln(SNR)\right)^n,\end{aligned} \tag{25}$$

where

$$C_i = \frac{(\delta_{s,r}^2\delta_{r,d}^2\alpha(1-\alpha)/n)\lambda_i}{4\left(\alpha\delta_{s,r}^2+(1-\alpha)\delta_{r,d}^2\right)},\ i=1,\cdots,n,$$

are constant terms that do not depend on the SNR and a_{AF} is a constant that depends on the power allocation parameter α and the variances of the channels. The $\ln(C_i)$ terms are neglected at high SNRs resulting in the last bound in (25). The diversity order of the system can be calculated as $d_{AF} = \lim_{SNR\to\infty} -\frac{\log(PEP)}{\log(SNR)} = n$. The system will achieve a full diversity of order n if the matrix $\mathbf{M}$ is full rank, that is the code matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ must be full rank of order n over all distinct pairs of codewords $\mathbf{X}_1$ and $\mathbf{X}_2$. It can be easily shown, following the same approach, that if the code matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ is rank deficient, then the system will not achieve full diversity. So any code that is designed to achieve full diversity over MIMO channels will achieve full diversity in the case of amplify-and-forward distributed space-time coding scheme.

If full diversity is achieved, the coding gain is given as

$$C_{AF} = \left(a_{AF} \frac{1}{\prod_{i=1}^{n} \lambda_i}\right)^{-\frac{1}{n}}.$$

To maximize the coding gain of the amplify-and-forward distributed space-time codes the product $\prod_{i=1}^{n} \lambda_i$ needs to be maximized, which is the same as the determinant criterion used over MIMO channels [23]. So if a space-time code is designed to maximize the coding gain over MIMO channels, it will also maximize the coding gain if it can be used in a distributed fashion with the amplify-and-forward protocol.

IV. Synchronization-Aware Distributed Space-Time Codes

In this section, the design of distributed space-time codes that relax the stringent synchronization requirement is considered. Most of the previous work on cooperative transmission assumed perfect synchronization between the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays are identical. To simplify the synchronization in the network a diagonal structure is imposed on the space-time code used. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay to transmit at any time slot. Hence, synchronizing simultaneous in-phase transmissions of randomly distributed relay nodes is not necessary.

This greatly simplifies the synchronization since nodes can maintain slot synchronization, which means that coarse slot synchronization is available [27][2]. However, fine synchronization is more difficult to be achieved. Guard intervals are introduced to ensure that the transmissions from different relays are not overlapped. One relay is allowed to consecutively transmit its part of the space-time code from different data packets. This allows the overhead introduced by the guard intervals to be neglected. Fig 3 shows

[2]For example, any synchronization scheme that is used for TDMA systems can be employed to achieve synchronization in the network.

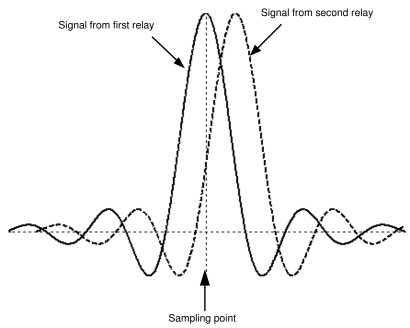

Fig. 3. Baseband signals (each is raised cosine pulse-shaped) from two relays at the receiver.

the effect of propagation delay on the received signal from two relays. The sampling time in Fig 3 is the optimum sampling time for the first relay signal, but clearly it is not optimal for the second relay signal. Some work has been done on selecting the optimal sampling time [28] but this only works for the case of two relays. The code design criterion for the DDSTC is derived. An amplify-and-forward system model is considered, which simplifies the relay node design and prevents the propagation of relay errors.

A. DDSTC System Model

In this subsection, the system model with n relay nodes, which helps the source by emulating a diagonal STC, is introduced. The system has two phases with the time frame structure shown in Fig. 2(b). In phase 1, the received signals at the relay nodes are modeled as in (1) with $L = n$.

In phase 2, the k-th relay applies a linear transformation $\mathbf{t}_k$ to the received data vector, where $\mathbf{t}_k$ is an $1 \times n$ row vector, as

$$\begin{aligned} y_{r_k} &= \mathbf{t}_k \mathbf{y}_{s,r_k} \\ &= \sqrt{P_1} h_{s,r_k} \mathbf{t}_k \mathbf{s} + \mathbf{t}_k \mathbf{v}_{s,r_k} \\ &= \sqrt{P_1} h_{s,r_k} x_k + v_{r_k}, \end{aligned} \tag{26}$$

where $x_k = \mathbf{t}_k \mathbf{s}$ and $v_{r_k} = \mathbf{t}_k \mathbf{v}_{s,r_k}$. If the linear transformations are restricted to have unit norm, i.e., $||\mathbf{t}_k||^2 = 1$ for all k, then v_{r_k} is $\mathcal{CN}(0, N_o)$. The relay then multiplies y_{r_k} by the factor

$$\beta_k \leq \sqrt{\frac{P_2}{P_1 |h_{s,r_k}|^2}} \tag{27}$$

to satisfy a power constraint of $P = P_1 + P_2$ transmitted power per source symbol [3]. The received signal at the destination due to the k-th relay transmission is given by

$$\begin{aligned} y_k &= h_{r_k,d}\beta_k\sqrt{P_1}h_{s,r_k}x_k + h_{r_k,d}\beta_k v_{r_k} + \tilde{v}_k \\ &= h_{r_k,d}\beta_k\sqrt{P_1}h_{s,r_k}x_k + z_k, \qquad k = 1,\cdots,n, \end{aligned} \tag{28}$$

where $\tilde{v}_k$ is modeled as $\mathcal{CN}(0, N_0)$ and hence, z_k, given the channel coefficients is $\mathcal{CN}(0, (\beta_k^2|h_{r_k,d}|^2 + 1)N_0),\ k = 1,\cdots,n$.

B. DDSTC Performance Analysis

In this subsection, the code design criterion of the DDSTC based on the PEP analysis is derived. In the following, the power constraint in (27) is set to be satisfied with equality.

Now, we start deriving a PEP upper bound to derive the code design criterion. Let σ_k^2 denote the variance of z_k in (28) and is given as

$$\sigma_k^2 = \left(\frac{P_2|h_{r_k,d}|^2}{P_1|h_{s,r_k}|^2} + 1\right)N_0,\ k = 1,\cdots,n. \tag{29}$$

Then, define the codeword vector $\mathbf{x}$ from (26) as

$$\mathbf{x} = \underbrace{\left[\mathbf{t}_1^T, \mathbf{t}_2^T, \cdots, \mathbf{t}_n^T\right]^T}_{\mathbf{T}}\mathbf{s} = \mathbf{T}\mathbf{s}, \tag{30}$$

where $\mathbf{T}$ is an $n \times n$ linear transformation matrix. From $\mathbf{x}$ define the $n \times n$ code matrix $\mathbf{X} = \text{diag}(\mathbf{x})$, which is a diagonal matrix with the elements of $\mathbf{x}$ on its diagonal. Let $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T$ denote the received data vector at the destination node as given from (28).

Using our system model assumptions, the pdf of $\mathbf{y}$ given the source data vector $\mathbf{s}$ and the channel state information (CSI) is given by

$$p(\mathbf{y}|\mathbf{s}, CSI) = \left(\prod_{i=1}^{n}\frac{1}{\pi\sigma_i^2}\right)\exp\left(-\sum_{i=1}^{n}\frac{1}{\sigma_i^2}\left|y_i - \sqrt{\frac{P_1P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}h_{r_i,d}x_i\right|^2\right). \tag{31}$$

From which, the maximum likelihood (ML) decoder can be expressed as

$$\arg\max_{\mathbf{s}\in\mathcal{S}} p(\mathbf{y}|\mathbf{s}, CSI) = \arg\min_{\mathbf{s}\in\mathcal{S}}\sum_{i=1}^{n}\frac{1}{\sigma_i^2}\left|y_i - \sqrt{\frac{P_1P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}h_{r_i,d}x_i\right|^2, \tag{32}$$

where $\mathcal{S}$ is the set of all possible transmitted source data vectors.

The PEP of mistaking $\mathbf{X}_1$ by $\mathbf{X}_2$ can be upper bounded as [29]

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \le E\left\{\exp\left(\lambda[\ln p(\mathbf{y}|\mathbf{s}_2) - \ln p(\mathbf{y}|\mathbf{s}_1)]\right)\right\}, \tag{33}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are the code matrices corresponding to the source data vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. Equation (33) applies for any λ which is a parameter that can be adjusted to get the tightest bound. Now, the PEP can be written as

$$\begin{aligned}\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\Big\{ \exp\Bigg(-\lambda\Big[\sum_{i=1}^{n} \frac{1}{\sigma_i^2}\Bigg(\sqrt{\frac{P_1P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}h_{r_i,d}(x_{1i}-x_{2i})z_i^* \\ + \sqrt{\frac{P_1P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}^*h_{r_i,d}^*(x_{1i}-x_{2i})^*z_i + \frac{P_1P_2}{P_1|h_{s,r_i}|^2}|h_{s,r_i}|^2|h_{r_i,d}|^2|x_{1i}-x_{2i}|^2\Bigg)\Big]\Bigg)\Big\},\end{aligned} \tag{34}$$

where the expectation is over the noise and channel coefficients statistics and x_{ij} is the j-th element of the i-th code vector.

To average the expression in (34) over the noise statistics, define the receiver noise vector $\mathbf{z} = [z_1, z_2, \cdots, z_n]^T$, where z_i's are as defined in (28). The pdf of $\mathbf{z}$ given the channel state information is given by

$$p(\mathbf{z}|CSI) = \left(\prod_{i=1}^{n}\frac{1}{\pi\sigma_i^2}\right)\exp\left(-\sum_{i=1}^{n}\frac{1}{\sigma_i^2}z_iz_i^*\right). \tag{35}$$

Taking the expectation in (34) over $\mathbf{z}$ given the channel coefficients yields

$$\begin{aligned}\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\Big\{ \exp\left(-\lambda(1-\lambda)\sum_{i=1}^{n}\frac{1}{\sigma_i^2}\frac{P_1P_2}{P_1|h_{s,r_i}|^2}\left(|h_{s,r_i}|^2|h_{r_i,d}|^2|x_{1i}-x_{2i}|^2\right)\right) \\ \int_{\mathbf{z}}\left(\prod_{i=1}^{n}\frac{1}{\pi\sigma_i^2}\right)\exp\left(-\sum_{i=1}^{n}\frac{1}{\sigma_i^2}|z_i+\lambda\sqrt{\frac{P_1P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}h_{r_i,d}(x_{1i}-x_{2i})|^2\right)d\mathbf{z}\Big\} \\ = E\left\{\exp\left(-\lambda(1-\lambda)\sum_{i=1}^{n}\frac{1}{\sigma_i^2}\frac{P_1P_2}{P_1|h_{s,r_i}|^2}\left(|h_{s,r_i}|^2|h_{r_i,d}|^2|x_{1i}-x_{2i}|^2\right)\right)\right\}.\end{aligned} \tag{36}$$

Choose $\lambda = 1/2$ that maximizes the term $\lambda(1-\lambda)$, i.e., minimizes the PEP upper bound. Substituting for σ_i^2's from (29), the PEP can be upper bounded as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\exp\left(-\frac{1}{4}\sum_{i=1}^{n}\frac{P_1|h_{s,r_i}|^2P_2|h_{r_i,d}|^2}{(P_1|h_{s,r_i}|^2+P_2|h_{r_i,d}|^2)N_0}|x_{1i}-x_{2i}|^2\right)\right\}. \tag{37}$$

To get the expression in (37), let us define the variable

$$\gamma_i = \frac{P_1|h_{s,r_i}|^2P_2|h_{r_i,d}|^2}{(P_1|h_{s,r_i}|^2+P_2|h_{r_i,d}|^2)N_0},\ i = 1, ..., n,$$

which is the scaled harmonic mean[3] of the two exponential random variables $\frac{P_1|h_{s,r_i}|^2}{N_0}$ and $\frac{P_2|h_{r_i,d}|^2}{N_0}$. Averaging the expression in (37) over the channel coefficients, the upper bound on the PEP can be

[3]The scaling factor is 1/2 since the harmonic mean of two numbers, g_1 and g_2, is $\frac{2g_1g_2}{g_1+g_2}$.

expressed as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq \prod_{i=1, x_{1i} \neq x_{2i}}^{n} M_{\gamma_i}\left(\frac{1}{4}|x_{1i} - x_{2i}|^2\right), \tag{38}$$

where $M_{\gamma_i}(.)$ is the moment generating function (MGF) of the random variable γ_i. The problem now is to get an expression for $M_{\gamma_i}(.)$. To get $M_{\gamma_i}(.)$, let y_1 and y_2 be two independent exponential random variables with parameters α_1 and α_2, respectively. Let $y = \frac{y_1 y_2}{y_1 + y_2}$ be the scaled harmonic mean of y_1 and y_2. Then the MGF of y is [4]

$$M_y(s) = \frac{(\alpha_1 - \alpha_2)^2 + (\alpha_1 + \alpha_2)s}{\Delta^2} + \frac{2\alpha_1\alpha_2 s}{\Delta^3} \ln \frac{(\alpha_1 + \alpha_2 + s + \Delta)^2}{4\alpha_1\alpha_2}, \tag{39}$$

where

$$\Delta = \sqrt{(\alpha_1 - \alpha_2)^2 + 2(\alpha_1 + \alpha_2)s + s^2}.$$

Using the expression in (39), the MGF for γ_i can be approximated at high enough SNR to be [4]

$$M_{\gamma_i}(s) \simeq \frac{\zeta_i}{s}, \tag{40}$$

where

$$\zeta_i = \frac{N_0}{P_1 \delta_{s,r}^2} + \frac{N_0}{P_2 \delta_{r,d}^2}.$$

The PEP can now be upper bounded as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq N_0^n \left(\prod_{i=1, x_{1i} \neq x_{2i}}^{n} \left(\frac{1}{P_1 \delta_{s,r}^2} + \frac{1}{P_2 \delta_{r,d}^2}\right)\right) \left(\prod_{i=1, x_{1i} \neq x_{2i}}^{n} \frac{1}{4}|x_{1i} - x_{2i}|^2\right)^{-1}. \tag{41}$$

Let $P_1 = \alpha P$ and $P_2 = (1 - \alpha)P$, where P is the power per symbol, for some $\alpha \in (0, 1)$ and define $SNR = P/N_0$. The diversity order d_{DDSTC} of the system is

$$d_{DDSTC} = \lim_{SNR \to \infty} -\frac{\log(PEP)}{\log(SNR)} = \min_{m \neq j} rank(\mathbf{X}_m - \mathbf{X}_j), \tag{42}$$

where $\mathbf{X}_m$ and $\mathbf{X}_j$ are two possible code matrices. To achieve a diversity order of n, the matrix $\mathbf{X}_m - \mathbf{X}_j$ should be of full rank for any $m \neq j$ (that is $x_{mi} \neq x_{ji}\ \forall m \neq j,\ \forall i = 1, \cdots, n$). Intuitively, if two code matrices exist for which the rank of the matrix $\mathbf{X}_m - \mathbf{X}_j$ is not n this means that they have at least one diagonal element that is the same in both matrices. Clearly, this element can not be used to decide between these two possible transmitted code matrices and hence, the diversity order of the system is reduced. This criterion implies that each element in the code matrix is unique to that matrix and any other matrix will have a different element at that same location and this is really the source of diversity. Furthermore, to minimize the PEP bound in (41) we need to maximize

$$\min_{m \neq j} \left(\prod_{i=1}^{n} |x_{mi} - x_{ji}|^2\right)^{1/n}, \tag{43}$$

which is called the minimum product distance of the set of symbols $\mathbf{s} = [s_1, s_2, ..., s_n]^T$ [30], [31]. A linear mapping is used to form the transmitted codeword, that is

$$\mathbf{x} = \mathbf{T}\mathbf{s}. \tag{44}$$

Several works have considered the design of the $n \times n$ transformation matrix $\mathbf{T}$ to maximize the minimum product distance. It was proposed in [32] and [33] to use both Hadamard transforms and Vandermonde matrices to design the $\mathbf{T}$ matrix. The transforms based on the Vandermonde matrices were shown to give larger minimum product distance than the Hadamard-based transforms. Some of the best known transforms based on the Vandermonde matrices [18] are summarized. Two classes of optimum transforms were proposed in [32]

1) If $n = 2^k$ $(k \geq 1)$, the optimum transform is given by $\mathbf{T}_{opt} = \frac{1}{\sqrt{n}}\text{vander}(\theta_1, \theta_2, ..., \theta_n)$, where $\theta_1, \theta_2, ..., \theta_n$ are the roots of the polynomial $\theta^n - j$ over the field $\mathbf{Q}[j] \triangleq \{c+dj$: both c and d are rational numbers$\}$ and they are determined as $\theta_i = e^{j\frac{4i-3}{2n}\pi}, \quad i = 1, 2, ..., n.$
2) If $n = 3.2^k$ $(k \geq 0)$, the optimum transform is given by $\mathbf{T}_{opt} = \frac{1}{\sqrt{n}}\text{vander}(\theta_1, \theta_2, ..., \theta_n)$, where $\theta_1, \theta_2, ..., \theta_n$ are the roots of the polynomial $\theta^n + w$ over the field $\mathbf{Q}[w] \triangleq \{c + dw$: both c and d are rational numbers$\}$ and they are determined as $\theta_i = e^{j\frac{6i-1}{3n}\pi}, \quad i = 1, 2, ..., n.$

The signal constellation from $\mathbf{Z}[j]$ such as QAM, MPSK and PAM constellations are of practical interest. Moreover, in [33], some non-optimal transforms were proposed for some n's not satisfying any of the above two cases.

V. Simulation Results

In this section, simulation results for the distributed space-time coding schemes from the previous sections are presented. In the simulations, the variance of any source-relay or relay-destination channel is taken to be 1. The performance of the different schemes with two relays helping the source are compared. Fig. 4 shows the simulations for two decode-and-forward systems using the Alamouti scheme (DAF Alamouti) and the diagonal STC (DAF DAST), distributed space-time codes based on the linear dispersion (LD) space-time codes (LD-DSTC) [14] which are based on the AAF scheme, the orthogonal distributed space-time codes (O-DSTC) proposed in [34] and [35], and DDSTC. The O-DSTCs are based on a generalized AAF scheme where relay nodes apply linear transformation to the received data as well as their complex conjugate. All of these systems have a data rate of (1/2). QPSK modulation is used, which means that a rate of one transmitted bit per symbol (1 bit/sym) is achieved. For the decode-and-forward system the power of the relay nodes that have decoded erroneously is not re-allocated to other relay nodes. Clearly, decode-and-forward based systems outperform amplify-and-forward based systems [4] but this is under the assumption that each relay node can decide whether it has decoded correctly or not. Intuitively, the decode-and-forward will deliver signals that are less noisy to the destination. The noise is suppressed at the relay nodes by transmitting a noise-free version of the signal. The amplify-and-forward delivers more noise to the destination due to noise propagation from the relay nodes. However, the assumption of correct decision at the relay nodes imposes practical limitations on the decode-and-forward systems, otherwise, error propagation [3] may occur caused by errors at the relay nodes. Error propagation would highly degrade the system BER performance.

Fig. 5 shows the simulation results for two decode-and-forward systems using the $\mathcal{G}_3$ ST block code of [25] and the diagonal STC (DAF DAST), LD-DSTC, and the DDSTC. For fair comparison the number of transmitted bits per symbol is fixed to be 1 bit/sym. The $\mathcal{G}_3$ ST block code has a data rate of (1/2) [25], which results in an overall system data rate of (1/3). Therefore, 8-PSK modulation is employed for the system that uses the $\mathcal{G}_3$ ST block code. For the other three systems QPSK modulation is used as these systems have a data rate of (1/2). For the decode-and-forward system the power of the relay nodes that decoded erroneously is not re-allocated. Clearly, decode-and-forward based systems outperform amplify-and-forward based systems under the same constraints stated previously. It is noteworthy that the performance of the LD-DSTC is not optimized since the LD matrices are randomly selected based on the isotropic distribution on the space of $n \times n$ unitary matrices as in [14]. The performance of the LD based codes can be improved by trying to optimize the selection of the LD matrices, which is out of the scope of this paper.

In the sequel, the effect of the synchronization errors on the system BER performance is investigated. Fig. 6 shows the case of having two relays helping the source and propagation delay mismatches of $T_2 = 0.2T,\ 0.4T$ and $0.6T$, where T is the time slot duration. Raised cosine pulse-shaped waveforms were used with roll-off factor of 0.2 and QPSK modulation. Clearly, the BER performance of the system highly deteriorates as the propagation delay mismatch becomes larger. Fig. 7 shows the case of having three relays helping the source for different propagation delay mismatches. Decode-and-forward (DAF) system using the $\mathcal{G}_3$ ST block code of [25] and the DDSTC were compared. For fair comparison the number of transmitted bits per symbol is fixed to be 1 bit/sym. Again, the $\mathcal{G}_3$ ST block code has a data rate of (1/2) [25], which results in an overall system data rate to be (1/3). Therefore, 8-PSK modulation is employed for the system that uses the $\mathcal{G}_3$ ST block code. For the DDSTC, QPSK modulation is used

[4]DDSTC is based on amplify-and-forward protocol.

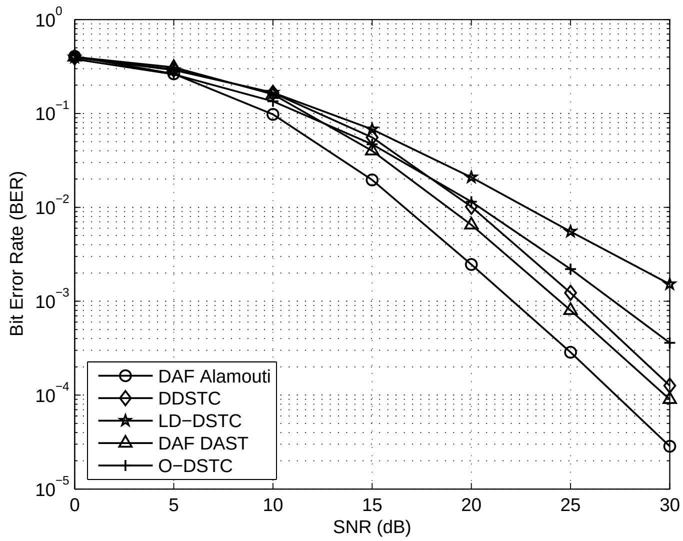

Fig. 4. BER for two relays with data rate 1 bit/sym.

as the system has a data rate of (1/2). Raised cosine pulse-shaped waveforms with roll-off factor of 0.2 are used. Clearly, the system performance is highly degraded as the propagation delay mismatch becomes larger. From Figures 6 and 7 it is clear that the synchronization errors can highly deteriorate the system BER performance. The DDSTC bypasses this problem by allowing only one relay transmission at any time slot.

VI. Conclusions

The design of distributed space-time codes in wireless relay networks is considered for different schemes, which vary in the processing performed at the relay nodes. For the decode-and-forward distributed space-time codes, any space-time code that is designed to achieve full diversity over MIMO channels can achieve full diversity under the assumption that the relay nodes can decide whether they have decoded correctly or not. A code that maximizes the coding gain over MIMO channels is not guaranteed to maximize the coding gain in the decode-and-forward distributed space-time coding. This is due to the fact that not all of the relays will always transmit their code columns in the second phase. Then, the code design criteria for the amplify-and-forward distributed space-time codes were considered. In this case, a code designed to achieve full diversity over MIMO channels will also achieve full diversity. Furthermore, a code that maximizes the coding gain over MIMO channels will also maximize the coding

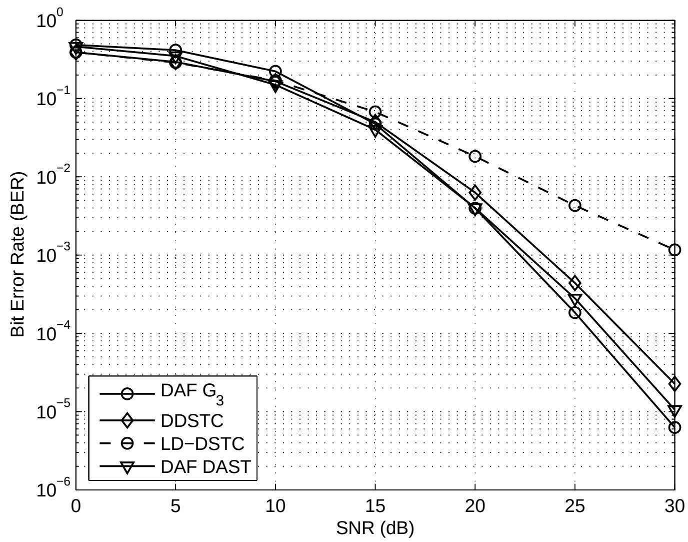

Fig. 5. BER for three relays with data rate 1 bit/sym.

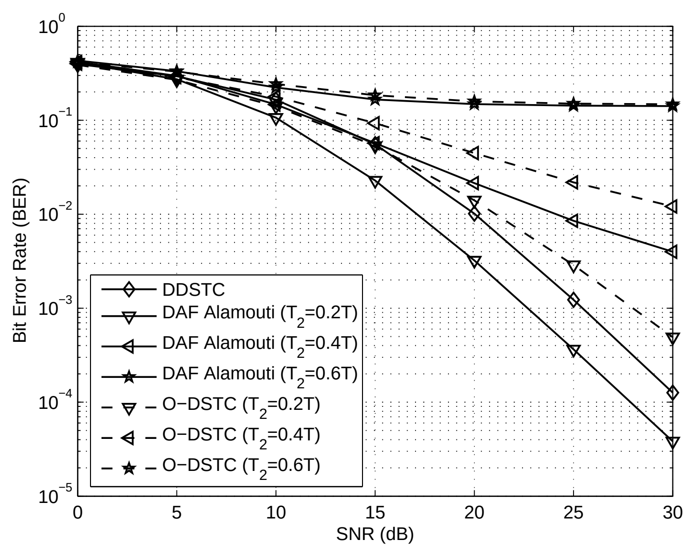

Fig. 6. BER performance with propagation delay mismatch: two relays case.

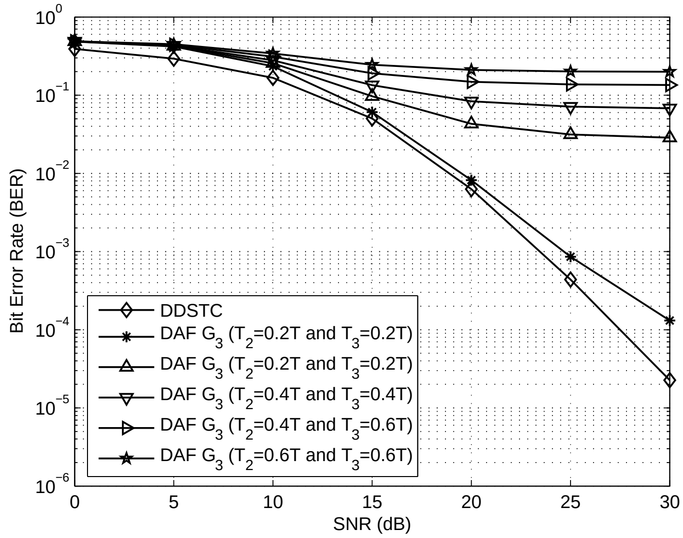

Fig. 7. BER performance with propagation delay mismatch: three relays case.

gain in the amplify-and-forward distributed space-time scheme.

The design of DDSTC for wireless relay networks was investigated. In DDSTC, the diagonal structure of the code was imposed to simplify the synchronization between randomly located relay nodes. Synchronization mismatches between the relay nodes causes inter-symbol interference, which can highly degrade the system performance. DDSTC relaxes the stringent synchronization requirement by allowing only one relay to transmit at any time slot. The code design criterion for the DDSTC based on minimizing the PEP was derived and the design criterion is found to be maximizing the minimum product distance. This is the same criterion used for designing DAST codes and full-rate full-diversity space frequency codes.

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