This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

Many-to-Many Communications via Space-Time Network Coding

Mohammed W. Baidas [1], Hung-Quoc Lai [2] and K. J. Ray Liu [1]
[1] Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA
[2] US Army RDECOM CERDEC, RDER-STW-NA, Bldg. 245, Aberdeen Proving Ground, MD 21005, USA
(email: baidas@ieee.org, hungquoc.lai@us.army.mil and kjrliu@umd.edu)

Abstract*—In this paper, the mutual cooperative communication between multiple nodes in a wireless network is efficiently achieved through a novel concept of Space-Time Network Coding (STNC). Unlike the conventional point-to-point cooperative communications between two nodes with N relay nodes deployed in between, simultaneous transmissions from the different N nodes acting as source/relay nodes are performed within $2N$ time-slots. In particular, the communication is split into two phases: 1) Broadcasting Phase and 2) Cooperation Phase. In the Broadcasting Phase, each node broadcasts its data symbol to the other nodes in the network in its own time-slot, alternatively; while in the Cooperation Phase, in each time-slot, a set of $(N-1)$ nodes transmit while a single destination node receives the other nodes' transmissions. Specifically, each node employing a selective Decode-and-Forward (DF) cooperative protocol, performs a linear combination of the other nodes data symbols and all the $(N-1)$ nodes simultaneously transmit their signals to a single receiving node; which then performs joint multiuser detection to separate the different nodes' symbols. Exact symbol-error-rate (SER) expressions for arbitrary order M-ary Phase Shift Keying (M-PSK) modulation are derived. In addition, an asymptotic SER approximation is also provided which is shown to be tight at high signal-to-noise ratio (SNR). Finally, the analytical results confirm that for a network of N nodes, a full diversity order of $(N-1)$ per node is achieved by the proposed STNC cooperative communication scheme.

***Index Terms*—Cooperation, decode-and-forward, diversity, many-to-many, symbol-error-rate, wireless network coding**

I. Introduction

NETWORK CODING has recently emerged as an effective approach for efficiently distributing data and increasing network throughput [1] [2]. Cooperative communications have also attracted much attention in the research literature due to their achievable spatial diversity gains, enhanced coverage and improved transmission reliability [3] [4]. In conventional relay networks, a set of N relay nodes are deployed between the source and destination nodes and the available network bandwidth is split into $N+1$ orthogonal channels using TDMA. However, with the increase in the number of relay nodes, the conventional multinode relay networks become excessively bandwidth inefficient. Moreover, the classical cooperative communication protocols are not well suited for distributing information from one or more source nodes to possibly many destination nodes, simultaneously. This in turn suggests the use of the concept of wireless network coding for exchanging data symbols among multiple cooperative nodes over wireless networks.

Recently, there have been several research works aiming at employing wireless network coding in cooperative networks. For instance, in [5], location-aware cooperative wireless network coding through the novel concept of Wireless Network Cocast (WNC) was proposed. In particular, it was illustrated that with WNC, a reduction in aggregate transmission power and delay can be achieved along with incremental diversity for different relaying schemes. In [6], an algebraic superposition of channel codes over a finite field is proposed to allow two nodes to cooperate in transmitting information to a single destination. Bi-directional relaying between two source nodes through a single relay node employing wireless network coding has been introduced in [7]. An outage analysis of network coded communication of multiple users with a single destination node through a set of *dedicated* relay nodes was analyzed in [8].

In this paper, the merits of network coding and cooperative diversity are exploited to allow N nodes "users" to exchange data between each other with the novel concept of *Space-Time Network Coding* (STNC). In this work, the Decode-and-Forward (DF) relaying protocol [3] is studied within the concept of STNC. In particular, the STNC scheme is based on linear wireless network coding over DF nodes and the communication is split into two phases: 1) Broadcasting Phase and 2) Cooperation Phase. In the former phase, each node broadcasts its data to the other nodes in its dedicated time slot; while in the latter phase, in each time-slot, a set of $(N-1)$ nodes transmit linearly network coded signals while a single destination node receives the other nodes' transmissions. A *simple* multiuser detection [9] is then applied at each node to separate the different data symbols received from the different nodes.

Exact analytical derivations of the symbol error rate (SER) performance of the STNC scheme and comparative simulation results are provided in this paper. Moreover, tight asymptotic approximation at high signal-to-noise ratio is also analyzed and the cooperative diversity order achievable with the STNC scheme is verified. It is illustrated that with the STNC scheme, N information symbols of all the N nodes can be exchanged over a total of $2N$ time-slots (i.e. 1/2 symbol per node per channel use) as well as achieving a *full diversity* order of $(N-1)$ per node.

In the remainder of this paper, the system model and communication phases are presented in Section II. The multi-source signal detection and the exact SER analysis are presented in Sections III and IV, respectively. In Section V, the asymptotic upper-bound SER expression is derived while the SER performance evaluation is presented in Section VI. The conclusions are drawn in Section VII.

II. System Model

Consider a wireless network consisting of N nodes $(N \geq 3)$ denoted as $S_1, S_2, \ldots, S_N$. In this model, each node is equipped with only one antenna and can act as a source, relay or destination. Without loss of generality, the Decode-and-Forward (DF) cooperation protocol is considered. The N nodes are assumed to have their own information symbols as $x_1, x_2, \ldots, x_N$, respectively, and each node wishes to exchange its own data symbol with the other nodes.

The communication between all the source nodes is split into two main phases, namely the Broadcasting Phase and the Cooperation Phase, over a total of $2N$ time-slots, N time-slots each. During the broadcasting phase, source node S_j is assigned a time-slot T_j in which it broadcasts its own data symbol to the other nodes S_i for $i \in \{1, 2, \ldots, N\}$ for $i \neq j$. That is, the broadcasting phase is an

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.

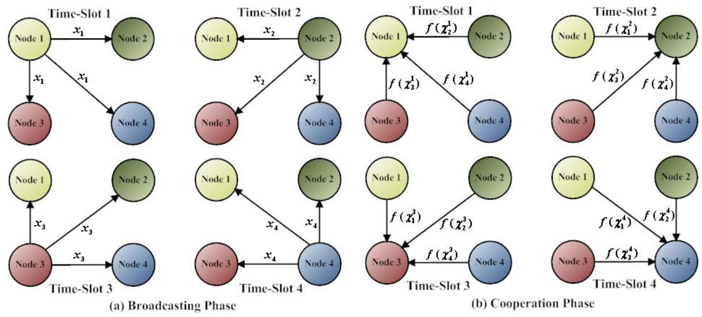

Fig. 1. Space-Time Network Coding - Broadcasting and Cooperation Phases - $N = 4$ Nodes

information exchange phase, which upon completion, each node S_i will have received a set of $(N-1)$ symbols $x_1, \ldots, x_j, \ldots, x_N$ for $j \neq i$ from the other nodes. With respect to the cooperation phase, each node acts as a relay for the other nodes with one node being the destination in each time-slot. In particular, each node, except a single receiving node, forms a linearly-coded signal from the overheard symbols and transmits it to the receiving node, which upon receiving the $(N-1)$ linearly-coded signals, performs a multi-source signal separation to extract the desired symbol from each node. To allow for joint detection/source separation for each of the linearly-coded transmitted symbols of the different nodes at each receiving node, each symbol x_j is spread using a signature waveform $s_j(t)$. The cross-correlation between waveforms $s_j(t)$ and $s_i(t)$ over a symbol duration T_s is given by $\rho_{j,i} = \langle s_j(t), s_i(t)\rangle \triangleq (1/T_s)\int_0^{T_s} s_j(t)s_i^*(t)dt$, where it is assumed that $\rho_{j,j} = ||s_j(t)||^2 = 1$. It is further assumed that each node knows the signature waveform of the other nodes in the network which is required for multi-source detection. The cooperative STNC communication scheme is illustrated in Fig. 1.

A. Broadcasting Phase

In this subsection, the signal model for an arbitrary symbol x_j, $j \in \{1, 2, \ldots, N\}$, transmitted during the broadcasting phase from node S_j to the other nodes is presented. Node S_j broadcasts it own data symbol x_j to the other nodes in its dedicated j^{th} time-slot. Thus, the signal received at each node S_i for $i \neq j$ is given by

$$y_{j,i}(t) = \sqrt{P_{s_j}}h_{j,i}x_j s_j(t) + n_{j,i}(t), \tag{1}$$

where P_{s_j} is the transmitted power by node S_j (the power allocation will be discussed in a later section), $s_j(t)$ is the signature waveform of node S_j, and $n_{j,i}(t)$ is the additive white Gaussian noise sample at node S_i due to the signal transmitted by node S_j. In (1), $n_{j,i}(t)$ is modeled as a zero-mean complex Gaussian random variable with variance N_0. Moreover, $h_{j,i}$ is the flat fading channel coefficient between nodes S_j and S_i that is distributed as a zero-mean complex Gaussian random variable $h_{j,i} \sim \mathcal{CN}(0, \sigma_{j,i}^2)$, where $\sigma_{j,i}^2$ is the channel gain. In addition, $h_{j,i}$ can be expressed as $h_{j,i} = |h_{j,i}|e^{j\phi_{j,i}}$, where $|h_{j,i}| = \alpha_{j,i}$ is the Rayleigh distributed magnitude as

$$f_{\alpha_{j,i}}(\alpha) = \frac{2\alpha}{\sigma_{j,i}^2}\exp\left(-\frac{\alpha^2}{\sigma_{j,i}^2}\right), \quad \alpha \geq 0, \tag{2}$$

with $\phi_{j,i}$ being the phase response, uniformly distributed over the interval $[-\pi, \pi)$. Moreover, it is assumed that the receiving node S_i can perfectly estimate the channel coefficient $h_{j,i}$ from the received signal $y_{j,i}$. Also, the channels are assumed to be reciprocal (i.e. $h_{i,j} = h_{j,i}$) as in Time Division Duplexing (TDD) systems.

The broadcasting phase can be put in matrix form as follows

$$\begin{array}{c} \\ T_1 \\ \vdots \\ T_j \\ \vdots \\ T_N \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & \cdots & S_j & \cdots & S_N \end{array} \\ \begin{bmatrix} \sqrt{P_{s_1}}x_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & \sqrt{P_{s_j}}x_j & \cdots & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{P_{s_N}}x_N \end{bmatrix} \end{array}. \tag{3}$$

The detection of the data symbol x_j at the node S_i can be achieved by cross-correlating the received signal $y_{j,i}(t)$ in (1) with the signature waveform $s_j(t)$ as

$$y_{j,i} = \langle y_{j,i}(t), s_j(t)\rangle = \sqrt{P_{s_j}}h_{j,i}x_j + n_{j,i}, \tag{4}$$

where $n_{j,i} \sim \mathcal{CN}(0, N_0)$. Upon the completion of broadcasting phase (i.e. after N time-slots), each node S_i will have received a set of $(N-1)$ symbols $\{y_{j,i}\}_{j=1,j\neq i}^N$ from all the other nodes in the network. With the knowledge of the channel coefficients at the i^{th} node, a matched filtering operation is applied on each of the received signals $y_{j,i}$, in the form of $\left(\sqrt{P_{s_j}}h_{j,i}^*/N_0\right)y_{j,i}$. Therefore, the SNR at the output of the matched-filter is expressed as

$$\gamma_{j,i} = \frac{P_{s_j}|h_{j,i}|^2}{N_0}. \tag{5}$$

After each source S_i has decoded its $(N-1)$ received symbols $\{y_{j,i}\}_{j=1,j\neq i}^N$, the set of available decoded data symbols at each node in the network is given by the matrix

$$\mathbf{X} = \left[\begin{array}{ccccc} * & x_2\mathcal{I}_{2,1} & \cdots & x_{N-1}\mathcal{I}_{N-1,1} & x_N\mathcal{I}_{N,1} \\ x_1\mathcal{I}_{1,2} & * & \cdots & x_{N-1}\mathcal{I}_{N-1,2} & x_N\mathcal{I}_{N,2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_1\mathcal{I}_{1,N-1} & x_2\mathcal{I}_{2,N-1} & \cdots & * & x_N\mathcal{I}_{N,N-1} \\ x_1\mathcal{I}_{1,N} & x_2\mathcal{I}_{2,N} & \cdots & x_{N-1}\mathcal{I}_{N-1,N} & * \end{array}\right] \begin{array}{c} S_1 \\ S_2 \\ \vdots \\ S_{N-1} \\ S_N \end{array}, \tag{6}$$

where $\mathcal{I}_{j,i}$ acts as a binary indicator function as follows

$$\mathcal{I}_{j,i} = \begin{cases} 1, & \text{if node } S_i \text{ decodes } x_j \text{ correctly} \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

B. Cooperation Phase

The analysis of the Cooperation Phase is considered in this subsection, with the assumption that the nodes are perfectly synchronized by a distributed algorithm. In the Cooperation Phase, each node S_i in its assigned time-slot receives a signal of the $N-1$ source nodes. In particular, each node S_j other than the destination node S_i forms a linearly-coded signal of the other source nodes' received symbols and transmits it to node S_i during the i^{th} time-slot, simultaneously. Moreover, at each node S_j, each linearly-coded signal contains at most the received data symbols from $(N-2)$, since each node aims at relaying the remaining $(N-2)$ nodes' symbols to node S_i. Specifically, the signal transmitted to node S_i from node S_j is composed from the received data symbols of the j^{th} row in the matrix $\mathbf{X}$ excluding the data symbol $x_i\mathcal{I}_{i,j}$ (since that symbol x_i was originally generated at node S_i) as follows

$$\mathcal{X}_j^i = \{x_k\mathcal{I}_{k,j}\}_{k=1,k\neq j}^N \setminus x_i\mathcal{I}_{i,j}. \tag{8}$$

Based on equation (6), during the i^{th} time-slot, the signal transmitted from the j^{th} node is given by

$$f(\mathcal{X}_j^i)(t) = \frac{h_{j,i}^*}{|h_{j,i}|}\sum_{\substack{k=1 \\ k\neq i, k\neq j}}^N \sqrt{P_{k,j}}x_k\mathcal{I}_{k,j}s_k(t), \tag{9}$$

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

where $s_k(t)$ is the signature waveform of the k^{th} node and $P_{k,j}$ is the power at j^{th} node used to transmit the symbol x_k. Moreover, $h_{j,i}$ is the channel coefficient between source nodes S_j and S_i; which has already been estimated in the Broadcasting Phase during the j^{th} time-slot. Clearly, the functions $f(\mathcal{X}_j^i)$ at each node are linear combinations of symbols received from other nodes. The operation of the Cooperation Phase can be expressed in matrix form as follows

$$\begin{array}{c} \\ T_1 \\ \vdots \\ T_i \\ \vdots \\ T_N \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & \cdots & S_i & \cdots & S_N \end{array} \\ \begin{bmatrix} 0 & \cdots & f(\mathcal{X}_i^1) & \cdots & f(\mathcal{X}_N^1) \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ f(\mathcal{X}_1^i) & \cdots & 0 & \cdots & f(\mathcal{X}_N^i) \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ f(\mathcal{X}_1^N) & \cdots & f(\mathcal{X}_i^N) & \cdots & 0 \end{bmatrix} \end{array}. \tag{10}$$

The received signal at the i^{th} node during the i^{th} time-slot from the $(N-1)$ other nodes is given by

$$\mathcal{Y}_i(t) = \sum_{\substack{m=1 \\ m \neq i}}^{N} h_{m,i} f(\mathcal{X}_m^i)(t) + w_i(t) = \sum_{\substack{m=1 \\ m \neq i}}^{N} x_m a_{i,m} s_m(t) + w_i(t), \tag{11}$$

where $w_i(t)$ is the additive white Gaussian noise at node S_i and

$$a_{i,m} = \sum_{\substack{k=1 \\ k \neq i, k \neq m}}^{N} |h_{k,i}| \sqrt{P_{m,k}} \mathcal{I}_{m,k}, \tag{12}$$

where the summation in (12) contains at most $(N-2)$ terms, depending on how many data symbols have been decoded correctly.

III. Multi-source Signal Detection

Based on the received signal $\mathcal{Y}_i(t)$, node S_i performs a multi-source detection operation to extract the $(N-1)$ symbols of the other nodes. Each soft symbol x_j, $j \in \{1, 2, \ldots, N\}_{i \neq j}$ is detected by passing the received signal $\mathcal{Y}_i(t)$ through a Matched Filter Bank (MFB) of $(N-1)$ branches, matched to the corresponding set of the nodes' signature waveforms $s_j(t)$ for $j \in \{1, 2, \ldots, N\}_{i \neq j}$ and sampling them at the end of the symbol duration T_s to obtain

$$\mathcal{Y}_{i,j} = \langle \mathcal{Y}_i(t), s_j(t) \rangle = \sum_{\substack{m=1 \\ m \neq i}}^{N} x_m a_{i,m} \rho_{m,j} + w_{i,j}, \tag{13}$$

where $\rho_{m,j}$ is the correlation coefficient between the signature waveforms $s_m(t)$ and $s_j(t)$. The matched-filtered signal forms an $(N-1) \times 1$ vector comprising all the $\mathcal{Y}_{i,j}$'s signals as

$$\boldsymbol{\mathcal{Y}}_i = \boldsymbol{R}_i \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{w}_i, \tag{14}$$

where

$$\boldsymbol{\mathcal{Y}}_i = \left[\mathcal{Y}_{i,1}, \ldots, \mathcal{Y}_{i,i-1}, \mathcal{Y}_{i,i+1}, \ldots, \mathcal{Y}_{i,N}\right]^T, \tag{15}$$

$$\boldsymbol{x}_i = \left[x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N\right]^T, \tag{16}$$

$\boldsymbol{w}_i = \left[w_{i,1}, \ldots, w_{i,i-1}, w_{i,i+1}, \ldots, w_{i,N}\right]^T \sim \mathcal{CN}(\boldsymbol{0}, N_0 \boldsymbol{R}_i)$, with

$$\boldsymbol{R}_i = \begin{bmatrix} 1 & \cdots & \rho_{1,(i-1)} & \rho_{1,(i+1)} & \cdots & \rho_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ \rho_{(i-1),1} & \cdots & 1 & \rho_{(i-1),(i+1)} & \cdots & \rho_{(i-1),N} \\ \rho_{(i+1),1} & \cdots & \rho_{(i+1),(i-1)} & 1 & \cdots & \rho_{(i+1),N} \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{N,1} & \cdots & \rho_{N,(i-1)} & \rho_{N,(i+1)} & \cdots & 1 \end{bmatrix}, \tag{17}$$

and

$$\boldsymbol{A}_i = \begin{bmatrix} a_{i,1} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & a_{i,(i-1)} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & a_{i,(i+1)} & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & a_{i,N} \end{bmatrix}, \tag{18}$$

where both matrices $\boldsymbol{R}_i$ and $\boldsymbol{A}_i$ have dimensions $(N-1) \times (N-1)$. The signal vector $\boldsymbol{\mathcal{Y}}_i$ can then be decorrelated with the assumption that matrix $\boldsymbol{R}_i$ is invertible with the inverse matrix $\boldsymbol{R}_i^{-1}$ which yields

$$\tilde{\boldsymbol{\mathcal{Y}}}_i = \boldsymbol{R}_i^{-1} \boldsymbol{\mathcal{Y}}_i = \boldsymbol{A}_i \boldsymbol{x}_i + \tilde{\boldsymbol{w}}_i, \tag{19}$$

where $\tilde{\boldsymbol{w}}_i \sim \mathcal{CN}(\boldsymbol{0}, N_0 \boldsymbol{R}_i^{-1})$. Therefore, the soft symbol of x_j can be obtained from detected signal vector $\tilde{\boldsymbol{\mathcal{Y}}}_i$ at node S_i at the output of the j^{th} branch of the MFB and is expressed as

$$\tilde{\mathcal{Y}}_{i,j} = a_{i,j} x_j + \tilde{w}_{i,j}, \tag{20}$$

where $\tilde{w}_{i,j} \sim \mathcal{CN}(0, N_0 r_{i,j})$ with $r_{i,j}$ being the j^{th} diagonal element of the inverse matrix R_i^{-1} associated with the data symbol x_j and $a_{i,j}$ is given by (12). Without loss of generality, let $\rho_{j,i} = \rho$ for all $j \neq i$. Thus, it can be easily shown that

$$r_{i,j} = \frac{1 + (N-3)\rho}{1 + (N-3)\rho - (N-2)\rho^2} \triangleq r_{N-1}. \tag{21}$$

It should be noted that upon the completion of the Broadcasting Cooperation Phases, the j^{th} data symbol x_j, $j \in \{1, 2, \ldots, N\}_{i \neq j}$ is relayed at most $(N-2)$ times before reaching node S_i. In addition, since in the Broadcasting Phase, the source node S_j has already broadcasted its data symbol x_j to all the other nodes including S_i, this implies that node S_i received at most a total of $(N-1)$ copies of the symbol x_j. This fact will be used later to prove that a full diversity order of $(N-1)$ per node is achieved at a high SNR.

On the other hand, since $\mathcal{I}_{j,i} \in \{0, 1\}$ for $i \neq j$, represents the detection state at S_i of the data symbol x_j; then, in general, node S_k for $k \in \{1, 2, \ldots, N\}_{k \neq i, k \neq j}$, forwards the symbol x_j to node S_i only if it has successfully detected it. Therefore, at node S_i, all the $\mathcal{I}_{j,k}$'s form a binary (base-2) number

$$\mathcal{I}_j^i = [\mathcal{I}_{j,N} \ \ldots \ \mathcal{I}_{j,k=(i+1)} \ \mathcal{I}_{j,k=(i-1)} \ \ldots \ \mathcal{I}_{j,1}]_2 \setminus \mathcal{I}_{j,j}, \tag{22}$$

that represents one of the $2^{(N-2)}$ detection states of the $(N-2)$ nodes S_j's acting as relay nodes. In other words, $|\mathcal{I}_j^i|$ contains at most $(N-2)$ 1's. For example, for $N = 4$, the detection state of x_1 at node S_2 is given by $\mathcal{I}_1^2 = [\mathcal{I}_{1,4} \ \mathcal{I}_{1,3}]_2$ with $|\mathcal{I}_1^2|$ taking decimal values in $\{0, 1, 2, 3\}$ in the form of $\mathcal{I}_1^2 = [0 \ 0]_2$, $[0 \ 1]_2$, $[1 \ 0]_2$, or $[1 \ 1]_2$, respectively.

The detection of data symbol x_j at the node S_i can be achieved through combining the signals received in the Broadcasting and Cooperation Phases. However, during the Cooperation Phase, it might occur that $a_{i,j} = 0$ (i.e. the receiving node might not receive any linear combination for some symbol x_j and this occurs when all the other nodes decode x_j incorrectly, simultaneously). This in turn implies that the detection state for symbol x_j at node S_i is $|\mathcal{I}_j^i| = 0$. Thus, the j^{th} branch of the MFB $\tilde{\mathcal{Y}}_{i,j}$ is not added into the detected $\tilde{x}_j^i$. Hence, for $|\mathcal{I}_j^i| = 0$, a simple phase correction is applied to the received signal $y_{j,i}$ during the j^{th} time-slot in the Broadcasting Phase by multiplying it with the deterministic conjugate of the known channel phase response (i.e. $\tilde{x}_j^i = e^{-j\phi_{j,i}} y_{j,i}$). However, for $|\mathcal{I}_j^i| > 0$ (i.e. $a_{i,j} \neq 0$), the phase-corrected signal is also combined with $\tilde{\mathcal{Y}}_{i,j}$ which yields

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

$$\tilde{x}_j^i = e^{-j\phi_{j,i}} y_{j,i} + \tilde{\mathcal{Y}}_{i,j}, \tag{23}$$

where $y_{j,i} = \sqrt{P_{s_j}} h_{j,i} x_j + n_{j,i}$. Upon substitution of $y_{j,i}$, (12) and (20) into (23), the combined symbol $\tilde{x}_j$ at node S_i becomes

$$\tilde{x}_j^i = \left(|h_{j,i}| \sqrt{P_{s_j}} + \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} |h_{k,i}| \sqrt{P_{j,k}} \mathcal{I}_{j,k} \right) x_j + \eta_{i,j}, \tag{24}$$

where $\eta_{i,j} = e^{-j\phi_{j,i}} n_{j,i} + \tilde{w}_{i,j}$ is the zero-mean equivalent noise with variance $(1 + r_{N-1})N_0$, with the assumption that $n_{j,i}$ and $\tilde{w}_{i,j}$ are statistically independent. Hence, the conditional signal-to-noise ratio given the detection state $\mathcal{I}_j^i$ of the combined data symbol $\tilde{x}_j^i$ at node S_i can be expressed as

$$\gamma^i_{j|\mathcal{I}_j^i} = \begin{cases} \frac{|h_{j,i}|^2 P_{s_j}}{N_0}, & \text{if } |\mathcal{I}_j^i| = 0 \\ \frac{\left(|h_{j,i}| \sqrt{P_{s_j}} + \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} |h_{j,i}| \sqrt{P_{j,k}} \mathcal{I}_{j,k} \right)^2}{(1 + r_{N-1})N_0}, & \text{if } |\mathcal{I}_j^i| > 0 \end{cases}. \tag{25}$$

The total transmit power P_j associated with transmitting symbol x_j is distributed among the $(N-1)$ transmissions. In particular, $P_j = P_{s_j} + \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} P_{j,k}$, where P_{s_j} is the transmit power at node S_j; while the $P_{j,k}$'s are the power allocations for the remaining transmissions at the other nodes. Without any loss of generality, it is assumed that all the symbols are assigned the same total power, such that $P = P_j, \forall j \in \{1, 2, \ldots, N\}$. For simplicity, it is further assumed that $P_s = P_{s_j} = P_{j,k} = P/(N-1)$, $\forall k \in \{1, 2, \ldots, N\}_{k \neq i, k \neq j}$. However, it should be noted that optimal power allocation among the different nodes is also possible; however is beyond the scope of this paper as it would deviate from the main thrust of this work.

IV. Symbol Error Rate (SER) Performance Analysis

In this section, the exact SER expression for the M-ary Phase Shift Keying (M-PSK) modulation for the DF protocol of the symbol x_j detected at node S_i is derived. Since the detection at each node is statistically independent from the others, $\mathcal{I}_{j,k}$'s for $k \in \{1, 2, \ldots, N\}_{k \neq i, k \neq j}$ are defined as independent Bernoulli random variables with a distribution expressed as [4]

$$\Upsilon(\mathcal{I}_{j,k}) = \begin{cases} 1 - \mathcal{P}_{j,k}, & \text{if } \mathcal{I}_{j,k} = 1 \\ \mathcal{P}_{j,k}, & \text{if } \mathcal{I}_{j,k} = 0 \end{cases}, \tag{26}$$

where $\mathcal{P}_{j,k}$ is the SER of detecting x_j at the S_k node. Thus, the probability of x_j detection in state $\mathcal{I}_j^i$ at node S_i is written as

$$\Pr(|\mathcal{I}_j^i|) = \prod_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} \Upsilon(\mathcal{I}_{j,k}), \tag{27}$$

where in the example of $N = 4$, for $|\mathcal{I}_1^2| = 1$ (i.e. $\mathcal{I}_1^2 = [0\ 1]_2$), $\Pr\left(|\mathcal{I}_1^2| = 1\right) = \mathcal{P}_{1,4}\left(1 - \mathcal{P}_{1,3}\right)$. In general, the conditional SER of M-PSK systems with the instantaneous signal-to-noise ratio (SNR) γ given a generic set of channel coefficients $\{h\}$ is expressed as [10]

$$\Psi_{|\{h\}}(\gamma) \triangleq \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}\gamma}{\sin^2\theta} \right) d\theta, \tag{28}$$

where $b_{PSK} = \sin^2(\pi/M)$. Based on (4 - 5), the SNR in detecting the symbol x_j at node S_k given the channel gain is $\gamma_{j,k} = P_{s_j}|h_{j,k}|^2/N_0$. In general, the magnitude squared of a circularly symmetric Gaussian random variable $h_{j,k}$ is modeled as an exponential random variable with rate $1/\sigma_{j,k}^2$ (i.e. $|h_{j,k}|^2 \sim \mathcal{E}xp(1/\sigma_{j,k}^2)$), where $\sigma_{j,k}^2$ is the channel gain. Thus, by averaging the expression in (28) with respect to $|h_{j,k}|^2$, the SER of detecting x_j at S_k is obtained as

$$\mathcal{P}_{j,k} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} G\left(\frac{b_{PSK} P_s}{N_0 \sin^2\theta}, \sigma_{j,k}^2 \right) d\theta, \tag{29}$$

where $G(w(\theta), \sigma^2)$ is defined in (54) in Appendix I. On the other hand, based on the conditional SNR $\gamma^i_{j|\mathcal{I}_j^i}$ expression in (24), the conditional SER of symbol x_j at node S_i can be shown to be

$$\Psi_{|\{h_{k,i}\}_{k=1,k\neq i}^N}(\gamma^i_{j|\mathcal{I}_j^i}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}\gamma^i_{j|\mathcal{I}_j^i}}{\sin^2\theta} \right) d\theta, \tag{30}$$

which reduces to the following two cases:

1) $|\mathcal{I}_j^i| = 0$: After averaging over the exponential random variable $|h_{j,i}|^2$, the conditional SER can be expressed as

$$\Psi(\gamma^i_{j||\mathcal{I}_j^i|=0}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} G\left(\frac{b_{PSK} P_s}{N_0 \sin^2\theta}, \sigma_{j,i}^2 \right) d\theta. \tag{31}$$

2) $|\mathcal{I}_j^i| > 0$: Similarly, the conditional SER can be obtained as

$$\Psi_{|\{h_{k,i}\}_{k=1,k\neq i}^N}(\gamma^i_{j||\mathcal{I}_j^i|>0}) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{C_{PSK}^2}{2\sin^2\theta} \mathcal{H}_{i,j}^2 \right) d\theta, \tag{32}$$

where

$$\mathcal{H}_{i,j}^2 = \left(|h_{j,i}| + \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} |h_{k,i}| \mathcal{I}_{j,k} \right)^2, \tag{33}$$

and

$$C_{PSK} = \sqrt{\frac{2 b_{PSK} P_s}{(1 + r_{N-1}) N_0}}. \tag{34}$$

Clearly, $\mathcal{H}_{i,j}^2$ is a sum of $1 + |\mathcal{I}_j^i|$ Rayleigh random variables. The analysis for the conditional SER is analogous to that of the equal gain combining in [11], using the Gauss-Hermite formula [12, p. 890, eq.(25.4.46)]. Thus, after averaging over the channel statistics, the conditional SER is given by [11]

$$\Psi(\gamma^i_{j||\mathcal{I}_j^i|>0}) = \frac{1}{2\pi^2} \int_0^{(M-1)\pi/M} \frac{1}{\sqrt{{}_i\eta_j(\theta)}} \sum_{n=1}^{N_p} w_{ni} \mathcal{F}_j \left(\frac{\kappa_n}{\sqrt{{}_i\eta_j(\theta)}}, \theta \right) d\theta, \tag{35}$$

where κ_n, w_n are the zeros and weight factors as given in [12, p. 924, table (25.10)] and N_p is the order of the Hermite polynomial $H_{N_p}(\cdot)$, respectively. It was verified that $N_p = 20$ results in excellent accuracy. In addition [11]

$${}_i\eta_j(\theta) = \frac{\sin^2(\theta)}{2C_{PSK}^2} + \frac{1}{4} \left(\sigma_{j,i}^2 + \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} \sigma_{j,k}^2 \mathcal{I}_{j,k} \right), \tag{36}$$

and ${}_i\mathcal{F}_j(\nu, \theta) = {}_iR_j(\nu, \theta) \cos({}_i\Theta_j(\nu, \theta))$, with

$${}_iR_j(\nu, \theta) = \sqrt{X^2(\theta) + Y^2(\nu, \theta)} \times D_{j,i} \prod_{\substack{k=1 \\ k \neq i, k \neq j}}^{N} \sqrt{A^2(\nu, \sigma_{j,k}^2 \mathcal{I}_{j,k}) + B^2(\nu, \sigma_{j,k}^2 \mathcal{I}_{j,k})}, \tag{37}$$

and ${}_i\Theta_j(\nu, \theta)$ is defined in (38) (top of next page). On the other hand, $X(\theta)$, $Y(\nu, \theta)$ are defined as

$$X(\theta) = \sqrt{\frac{\pi}{2}} \frac{\sin(\theta)}{C_{PSK}}, \text{ and } Y(\nu, \theta) = \frac{\nu \sin^2(\theta)}{C_{PSK}^2} {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; \frac{\nu^2 \sin^2(\theta)}{2C_{PSK}} \right) \tag{39}$$

respectively, where $sgn(\cdot)$ is the sign function and ${}_1F_1(\cdot; \cdot; \cdot)$ is the Kummer confluent hypergeometric function [12]. Moreover, $A(\nu, \tau)$ and $B(\nu, \tau)$ are defined as

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.

$$
{}_i\Theta_j(\nu,\theta) = \arctan\left(\frac{Y(\nu,\theta)}{X(\theta)}\right) + \arctan\left(\frac{B(\nu,\sigma_{j,i}^2)}{A(\nu,\sigma_{j,i}^2)}\right) + \sum_{\substack{k=1\\k\neq i,k\neq j}}^{N} \arctan\left(\frac{B(\nu,\sigma_{j,k}^2\mathcal{I}_{j,k})}{A(\nu,\sigma_{j,k}^2\mathcal{I}_{j,k})}\right) + \frac{\pi}{2}\left(N - \mathrm{sgn}\left(Y(\nu,\theta)\right) - \mathrm{sgn}\left(B(\nu,\sigma_{j,i}^2)\right) - \sum_{\substack{k=1\\k\neq i,k\neq j}}^{N} \mathrm{sgn}\left(B(\nu,\sigma_{j,k}^2\mathcal{I}_{j,k})\right)\right). \quad (38)
$$

$$
\bar{\Psi}(\gamma^i_{j||\mathcal{I}^i_j|=\ell}) \approx \left(\frac{N_0}{P}\right)^{\ell+1}\left(\frac{(1+r_{N-1})}{\mu b_{PSK}\sigma_i^2\left(1+\frac{\pi}{4}\ell\right)}\right)^{\ell+1} \frac{F_1\left(\ell+\frac{3}{2},\frac{1}{2},\ell+1,\ell+\frac{5}{2},\sin^2\left(\frac{(M-1)\pi}{M}\right),-\frac{(1+r_{N-1})N_0}{\mu P b_{PSK}\sigma_i^2(1+\frac{\pi}{4}\ell)}\sin^2\left(\frac{(M-1)\pi}{M}\right)\right)}{\pi(2\ell+3)}\left(\sin\left(\frac{(M-1)\pi}{M}\right)\right)^{2\ell+3}. \quad (50)
$$

$$
A(\nu,\tau) = {}_1F_1\left(-\frac{1}{2};\frac{1}{2};\frac{\nu^2\tau}{4}\right), \text{ and } B(\nu,\tau) = \Gamma\left(\frac{3}{2}\right)\sqrt{\tau}\nu, \quad (40)
$$

respectively; with $\Gamma(\cdot)$ being the gamma function [12]. Also, $D_{j,i}$ is defined as

$$
D_{j,i} = \sqrt{A^2(\nu,\sigma_{j,i}^2) + B^2(\nu,\sigma_{j,i}^2)}. \quad (41)
$$

Given the detection state $\mathcal{I}^i_j$, which takes $2^{(N-2)}$ values, the SER for detecting the data symbol x_j at the i^{th} node can be calculated using the law of total probability as

$$
\begin{aligned}
P^j_{SER} &= \sum_{\ell=0}^{2^{(N-2)}-1} \Pr(\tilde{x}^i_j \neq x_j||\mathcal{I}^i_j| = \ell)\cdot\Pr(|\mathcal{I}^i_j| = \ell) \\
&= \Psi(\gamma^i_{j||\mathcal{I}^i_j|=0})\cdot\Pr(|\mathcal{I}^i_j|=0) + \sum_{\ell=1}^{2^{(N-2)}-1}\Psi(\gamma^i_{j||\mathcal{I}^i_j|=\ell})\cdot\Pr(|\mathcal{I}^i_j|=\ell)
\end{aligned} \quad (42)
$$

where $\Pr(\tilde{x}^i_j \neq x_j|\mathcal{I}^i_j)$ is the probability of making a symbol error for a particular detection state, and $\Pr(|\mathcal{I}^i_j|)$ is as defined (27).

V. Asymptotic Upper Bound SER Analysis

The asymptotic upper-bound SER performance is obtained at a high SNR by performing a series of approximations to the term $\Psi(\gamma^i_{j||\mathcal{I}^i_j|=\ell})$, for $\ell \in \{0,1,\ldots,2^{(N-2)}-1\}$ and also the term $\Pr(|\mathcal{I}^i_j|)$. It should be noted that finding the distribution of the sum of independent but not identical Rayleigh random variables as given by $\mathcal{H}^2_{i,j}$ in (33) can be extremely difficult. Therefore, it is assumed that the channel gains are identical (i.e. $\sigma^2_{j,i} = \sigma^2_{k,i} = \sigma^2_i$, $\forall k \in \{1,2,\ldots,N\}_{k\neq i}$). This implies that the same average SNR/symbol/path is assumed between the nodes in the network.

At high SNR, it is expected that the SER $\mathcal{P}_{j,k}$ of detecting x_j at node S_k for $k \in \{1,2,\ldots,N\}_{k\neq i,k\neq j}$, becomes sufficiently small such that $1-\mathcal{P}_{j,k} \approx 1$. Thus, only the terms in the quantity $\Pr(|\mathcal{I}^i_j|) = \prod_{k=1, k\neq i,k\neq j}^{N}\Upsilon(\mathcal{I}_{j,k})$ that will count are those corresponding to the nodes that have decoded their received x_j symbol incorrectly [4]. Hence, let ${}_0\Phi_j$ and ${}_1\Phi_j$ denote the subsets of the indices of the nodes that decode x_j erroneously and correctly, respectively. That is, ${}_0\Phi_j = \{k : \mathcal{I}_{j,k} = 0\}$ and ${}_1\Phi_j = \{k : \mathcal{I}_{j,k} = 1\}$, for $k \in \{1,2,\ldots,N\}_{k\neq i,k\neq j}$. Moreover, $|{}_0\Phi_j|$ and $|{}_1\Phi_j| \in \{0,1,\ldots,N-2\}$. In addition, it should be noted that $|{}_0\Phi_j| + |{}_1\Phi_j| = (N-2)$ for any detection state $\mathcal{I}^i_j$. Also, since, the transmitter powers allocated so source node S_j and the remaining nodes for the transmission of symbol x_j is defined as a ratio of the total power P as $P_s = P_{s_j} = P_{j,k} = \mu P$, $\forall k \in \{1,2,\ldots,N\}_{k\neq i,k\neq j}$ with $\mu = 1/(N-1)$, the expression for $\Pr(|\mathcal{I}^i_j|)$ given in (27) can then be expressed with the aid of (55) (see Appendix I) as

$$
\Pr(|\mathcal{I}^i_j|) \simeq \prod_{k\in{}_0\Phi_j}\Upsilon(\mathcal{I}_{j,k}) = \left(\frac{N_0}{P}\right)^{|{}_0\Phi_j|}\left(\frac{\varphi}{\mu b_{PSK}\sigma_i^2}\right)^{|{}_0\Phi_j|}, \quad (43)
$$

where

$$
\varphi = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\sin^2(\theta)\,d\theta = \frac{2\pi(M-1)+M\sin\left(\frac{2\pi}{M}\right)}{4\pi M}. \quad (44)
$$

With respect to $\Psi(\gamma^i_{j||\mathcal{I}^i_j|=0})$, the upper-bound approximation at high SNR can be expressed with the aid of (55) as

$$
\bar{\Psi}(\gamma^i_{j||\mathcal{I}^i_j|=0}) = \tilde{F}\left(\frac{b_{PSK}P_s}{N_0\sin^2\theta},\sigma_i^2\right) \lesssim \left(\frac{N_0}{P}\right)\frac{\varphi}{\mu b_{PSK}\sigma_i^2}. \quad (45)
$$

In order to find an upper-bound approximation for $\bar{\Psi}(\gamma^i_{j||\mathcal{I}^i_j|=\ell})$, the expression (33) must be approximated. Based on (25), the conditional SNR for $|\mathcal{I}^i_j| = \ell$ is given by

$$
\gamma^i_{j||\mathcal{I}^i_j|=\ell} = \frac{P_s}{(1+r_{N-1})N_0}\left(|h_{j,i}| + \sum_{\substack{k=1\\k\neq i,k\neq j}}^{N}|h_{j,i}|\mathcal{I}_{j,k}\right)^2. \quad (46)
$$

Thus, the average SNR conditioned on the network detection state can be shown to be [11]

$$
\gamma_{av,(\ell+1)} = \gamma_{s_i}\left(1+\frac{\pi}{4}\ell\right), \quad (47)
$$

where $\gamma_{s_i} = P_s\sigma_i^2/(1+r_{N-1})N_0$. An accurate approximation to the conditional SER has been determined in [11] and is expressed as

$$
\bar{\Psi}(\gamma^i_{j||\mathcal{I}^i_j|=\ell}) = I_{(\ell+1)}\left(b_{PSK}\gamma_{av,(\ell+1)},\frac{(M-1)\pi}{M}\right). \quad (48)
$$

In general

$$
I_L(\zeta,\phi) = \frac{1}{\pi}\int_0^{\phi}\left(\frac{\sin^2\phi}{\sin^2\phi+\zeta}\right)^L d\phi. \quad (49)
$$

Therefore, $\bar{\Psi}(\gamma^i_{j||\mathcal{I}^i_j|=\ell})$ with the aid of (56) (see Appendix II) can be shown to be as expressed in (50) (top of page). Therefore, the asymptotic SER expression, after a series of manipulations, can be written as (51) (top of next page), where it should be noted that $1+|{}_0\Phi_j|+|{}_1\Phi_j| = (N-1)$.

The cooperative diversity order of a wireless system is identified from the SER expression as follows [4]

$$
P^j_{SER} \sim (SNR\cdot\Delta)^{-d}, \quad (52)
$$

where $SNR \triangleq P/N_0$ is the signal-to-noise ratio term, the exponent d denotes the diversity order and Δ defines the cooperation gain. Thus, it is clear that $d = N-1$ and the STNC achieves full diversity.

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.

$$P_{SER}^{j} \lesssim \left(\frac{N_0}{P}\right)^{N-1} \left(\begin{array}{l} \left(\frac{\varphi}{\mu b_{PSK}\sigma_i^2}\right)^{N-1} + \\ \sum_{\ell=1}^{2^{(N-2)}-1} \left(\frac{1+r_{N-1}}{1+\frac{\pi}{4}|_1\Phi_j|}\right)^{|_1\Phi_j|+1} \frac{{}_2F_1\left(|_1\Phi_j|+\frac{3}{2},\frac{1}{2},|_1\Phi_j|+\frac{5}{2},\sin^2\left(\frac{(M-1)\pi}{M}\right)\right)}{\pi(\mu b_{PSK}\sigma_i^2)^{N-1}(2|_1\Phi_j|+3)} \left(\sin\left(\frac{(M-1)\pi}{M}\right)\right)^{2|_1\Phi_j|+3} (\varphi)^{|_0\Phi_j|} \end{array} \right). \quad (51)$$

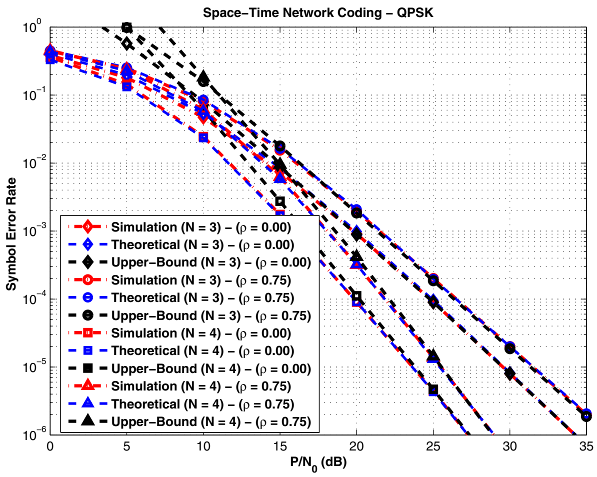

Fig. 2. QPSK SER Performance of the STNC for $N = 3$ and $N = 4$ Nodes

VI. SER Performance Evaluation

In this section, the simulated QPSK SER performance and the theoretical exact and upper-bound SER expressions of the STNC scheme with $N = 3$ and $N = 4$ nodes are evaluated. In the following simulations, all the channel coefficients are i.i.d. with equal unity gain (i.e. $h_{i,j} \backsim \mathcal{CN}(0,1)$, $\forall i,j \in \{1,2,\ldots,N\}$ and $i \neq j$).

It is clear from Fig. 2, that as the number of nodes increases, the better the performance. This is due to the fact that with the increase in N, higher diversity gains are achieved. In addition, it is noticed that the best SER performance is achieved when $\rho = 0$, that is the signature waveforms are perfectly orthogonal and there is no interference between the nodes. However, when $\rho = 0.75$, there is about $2\ dB$ SER performance degradation. This in turn signifies the importance of the STNC scheme as it allows N nodes to communicate simultaneously and achieves $N-1$ diversity order with only a slight degradation for non-orthogonal signature waveforms.

VII. Conclusions

In this paper, the novel Space-Time Network Coding (STNC) scheme that allows N nodes to exchange their data symbols over a total of $2N$ time-slots, was presented. The exact and asymptotic SER expressions were derived and it was shown that with the STNC scheme, each node achieves a full diversity order of $N-1$. Finally, it is concluded that the STNC serves as a potential many-to-many cooperative communication scheme and its scope goes much further beyond the generic source-relay-destination communications.

VIII. Appendix I

Let Y be an exponentially distributed random variable with probability density function

$$f_Y(y) = \frac{1}{\sigma^2}\exp\left(-\frac{y}{\sigma^2}\right), \quad y \geq 0, \quad (53)$$

where σ^2 is the rate parameter. Averaging the function $\exp(-w(\theta)y)$ over the distribution of the exponential random variable Y, yields

$$G(w(\theta),\sigma^2) = \int_0^{\infty} \exp\left(-w(\theta)y\right)\left(\frac{1}{\sigma^2}\right)\exp\left(-\frac{y}{\sigma^2}\right)\,dy = \frac{1}{1+w(\theta)\sigma^2}, \quad (54)$$

where $w(\theta)$ is some function of θ.

For large values of $w(\theta)$, the denominator of $G(w(\theta),\sigma^2)$ can be approximated as $1 + w(\theta)\sigma^2 \approx w(\theta)\sigma^2$. Thus,

$$\bar{G}(w(\theta),\sigma^2) \lesssim \frac{1}{w(\theta)\sigma^2}. \quad (55)$$

IX. Appendix II

The term $I_L(\xi,\phi)$ can be shown to be

$$\begin{aligned} I_L(\xi,\phi) &= \frac{1}{\pi}\int_0^{\phi}\left(\frac{\sin^2\phi}{\sin^2\phi+\xi}\right)^L d\phi \\ &= \frac{(1/\xi)^L(\sin(\phi))^{2L+1}}{\pi(2L+1)} F_1\left(L+\frac{1}{2},\frac{1}{2},L,L+\frac{3}{2},\sin^2(\phi),-\frac{\sin^2(\phi)}{\xi}\right), \end{aligned} \quad (56)$$

where

$$F_1(\alpha;\beta,\beta';\zeta;x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m(\beta')_n}{m!n!(\zeta)_{m+n}}x^m y^n, \quad (57)$$

is the Appell hypergeometric function of the first kind [13]. It can be shown that for $y \rightarrow 0$, the Appell hypergeometric function can be approximated as [13]

$$F_1(\alpha;\beta,\beta';\zeta;x,y) \approx {}_2F_1(\alpha,\beta,\gamma;x), \quad (58)$$

where ${}_2F_1(\alpha,\beta;\gamma;x)$ is the hypergeometric function [12].

References

[1] R. Ahlswede, N. Cai, S.-Y. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. on Info. Theory*, vol. 46, pp. 1204 – 1216, July 2000.

[2] S.-Y. Li, R. W. Yeung, and N. Cai, "Linear network coding," *IEEE Trans. on Info. Theory*, vol. 49, pp. 371 – 381, Feb. 2003.

[3] J. N. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, pp. 3062–3080, 2004.

[4] K. J. R. Liu, A. Sadek, W. Su, and A. Kwasinski, *Cooperative Communications and Networking*. Cambridge University Press, Jan. 2009.

[5] H. Q. Lai, A. S. Ibrahim, and K. J. R. Liu, "Wireless network cocast: Location-aware cooperative communications using network coding," *IEEE Trans. on Wireless Communications*, vol. 8, pp. 3844 – 3854, Jul. 2009.

[6] L. Xiao, T. E. Fuja, J. Kliewer, and D. J. C. Jr., "A network coding approach to cooperative diversity," *IEEE Trans. on Info. Theory*, vol. 53, pp. 3714 – 3722, Oct. 2007.

[7] P. Poposki and H. Yomo, "Wireless network coding by amplify-and-forward for bi-directional traffic flows," *IEEE Comms. Letters*, vol. 11, pp. 16 – 18, 2007.

[8] Y. Chen, S. Kishore, and J. Li, "Wireless diversity through network coding," *Proc. IEEE WCNC 2006*, vol. 3, pp. 1681–1686, April 2006.

[9] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.

[10] M. K. Simon and M. S. Alouini, "A unified approach to the performance analysis of digital communications over generalized fading channels," *Proc. IEEE*, vol. 86, no. 9, pp. 1860–1877, Sep. 1998.

[11] M.-S. Alouini and M. K. Simon, "Performance analysis of coherent equal gain combining over nakami-m fading channels," *IEEE Trans. on Vehicular Technology*, vol. 50, pp. 1449 – 1462, 2001.

[12] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions, with Formulas, Graphs and Mathematical Tables*. New York: Dover publications, 9th Edition, 1970.

[13] E. T. Whittaker and G. N. Watson, *A Course in Modern Analysis, 4th Edition*. Cambridge University Press, 1990.

Authorized licensed use limited to: University of Maryland College Park. Downloaded on July 14,2010 at 19:17:49 UTC from IEEE Xplore. Restrictions apply.