Evolutionary Information Diffusion Over Heterogeneous Social Networks

Xuanyu Cao, Yan Chen, Chunxiao Jiang, and K. J. Ray Liu

***Abstract*—A huge amount of information, created and forwarded by millions of people with various characteristics, is propagating through the online social networks every day. Understanding the mechanisms of the information diffusion over the social networks is critical to various applications including online advertisement and website management. Different from most of the existing works, we investigate the information diffusion from an evolutionary game-theoretic perspective and try to reveal the underlying principles dominating the complex information diffusion process over the heterogeneous social networks. Modeling the interactions among the heterogeneous users as a graphical evolutionary game, we derive the evolutionary dynamics and the evolutionarily stable states (ESSs) of the diffusion. The different payoffs of the heterogeneous users lead to different diffusion dynamics and ESSs among them, in accordance with the heterogeneity observed in real-world datasets. The theoretical results are confirmed by simulations. We also test the theory on Twitter hashtag dataset. We observe that the derived evolutionary dynamics fit the data well and can predict the future diffusion data.**

***Index Terms*—Information diffusion, heterogeneous social networks, evolutionary game theory.**

I. Introduction

ONLINE social networks such as Twitter, Facebook and Youtube are ubiquitous in daily life. Billions of people with different characteristics interact on the social networks, not only receiving a lot of information but also creating numerous amount of information. For example, about 500 millions of tweets are sent from Twitter every day [1] while around 300 thousand statuses are updated every minute on Facebook [2]. Each piece of information can either go viral, i.e., become very popular, or disappear quickly with few impact. When the user-generated information such as memes [3] and Twitter hashtags [4] propagates through the social networks, a variety of information diffusion dynamics are observed [5]. The diffusion dynamics or the popularity of the information are determined by the complicated interaction and decision-making of lots of users, which involves users' heterogeneous interests and influences. For instance, a football fan has a higher probability of retweeting a tweet about football and a user tends to post a piece of news if many of his friends have posted it. In practice, many applications are related to the information diffusion over social networks: online advertisements, political statements, rumor detection and control. All these applications call for a better understanding of the information diffusion process over the social networks composed of heterogeneous individuals. Consequently, great efforts have been devoted to studying how the information diffuses in the recent decade.

Existing works on information diffusion can be mainly classified into two categories: i) using machine learning (ML) or data mining approaches to make inference and prediction; ii) devising microscopic mechanisms to explain the information diffusion from the perspective of the individual users' interactions. Among the first category, Pinto *et al.* used early diffusion data to predict future diffusion [6] while the community structure is further exploited to improve the performance of prediction of viral memes in [7]. Yang and Leskovec proposed a clustering algorithm to identify the patterns of the diffusion dynamics of online contents [5]. Given the information diffusion data, efficient algorithms are developed to infer the underlying information diffusion network in [8]–[10]. Alternatively, the authors in [11] estimated the global influence of individuals in the information diffusion process. The interactions between the diffusions of multiple pieces of information are investigated in [12] while the impact of external sources on the information diffusion is considered in [13]. Cheng *et al.* tried to predict the cascades of the information diffusion [14]. Using the data from a real-world experiment, the authors in [15] studied the impact of cluster structure of the social network on the diffusion of behaviors. Similarly, taking an experimental approach, Bakshy *et al.* investigated the role of social ties on the information diffusion [16]. A common limitation of these ML or data mining based approaches is the lack of understanding of the underlying microscopic mechanisms of the individuals' decision making that dominate the information diffusion process, which is the focus of the papers in the second category. In this category, authors in [17] and [18] developed game-theoretic mechanisms to analyze the competitive contagions in networks, such as firms' competing for users' purchase. Under a threshold model, Granovetter studied the diffusion of the collective behaviors, which are defined to be the adoption of one of two alternatives [19]. Assuming each user played the best response to the population's strategies, Morris studied the conditions for global contagion of behaviors [20]. The impact of the network structure on virus propagation was investigated in [21]. Moreover, in [22], algorithms for finding initial targets to maximize

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the future contagions over the networks are presented. The impact of the community structure on information diffusion was studied in a model-based approach in [23].

Recently, the authors of [26], [27] proposed to use an evolutionary game-theoretic framework to model the users' interactions during the information diffusion process. Evolutionary game theory, originating from the evolutionary biology [28], was used as a promising modeling tool in various areas of signal processing such as communication networking and image processing [29]–[33]. In [26], [27], it was found that the dynamics derived under the evolutionary game framework fit the real-world information diffusion dynamics well and could even make predictions on the future diffusion dynamics, suggesting a suitable and tractable paradigm for analyzing the information diffusion.

Most of the existing works treat the network users as homogeneous individuals and do not take the heterogeneity of the users into consideration. However, real-world social networks often exhibit significant heterogeneity. For example, heterogeneous aspects of the Twitter network include: (a) A variety of different topics coexist due to the heterogeneous interests of users; (b) Different users have very different follower counts, indicating different influences [34]; (c) The distribution of tweet counts is highly heterogeneous: the top 15% users account for the 85% of the tweets, suggesting that the user activity strength is heterogeneous [35]. The heterogeneity of the users' interests, influences and activities can have huge impact on information diffusion. For example, when a piece of information related to football reaches a user, whether the user is a football fan or not has huge impact on the decision-making (forwarding or not forwarding that information) of the user.

In this paper, we study the information diffusion over the heterogeneous social networks using a graphical evolutionary game approach. Modeling users' decision making as an evolutionary game, we analyze the information diffusion dynamics. Through the study in this work, we provide a microeconomic framework by using a few utility parameters to describe the mechanisms of the users' decision making in the information diffusion process over the real-world heterogeneous social networks. The main contributions of this work can be epitomized as follows.

1) We propose two mathematically tractable evolutionary game-theoretic models to characterize the impact of users' heterogeneity on the information diffusion over social networks. The two models differ in whether the user type[1] is a private information unknown to others or a publicly known information.
2) For the unknown user type model, we theoretically derive the evolutionary dynamics as well as the evolutionarily stable states (ESSs). The relation between the heterogeneous payoff parameters and the heterogeneous information diffusion dynamics among different types of users is observed. In contrast, the homogeneous model in [26], [27] has to treat all types the same and can only give a mean evolutionary dynamics averaged over all types.
3) For the known user type model, the evolutionary dynamics are derived and a relation between the dynamics is observed, which can be used to further simplify the dynamics. When the users manage to know the types of their neighbors through repeated interactions, the known user type model characterizes the users' decision making process more accurately than the unknown user type model.
4) Using both synthetic data based simulations and real data based experiments, we validate the theoretical results. The good fitting and prediction performance on real-world datasets indicate the effectiveness of the evolutionary game modeling. In particular, our results outperform the homogeneous model in [26], [27] when characterizing the heterogeneous behaviors of different types of users.

[1]The type of a user will be explicitly defined later in Section 2.

The rest of this paper is organized as follows. In Section II, we formally state the evolutionary game-theoretic model for information diffusion. In Section III, we theoretically derive the evolutionary dynamics and the ESSs for the unknown user type model. Then, the evolutionary dynamics of the known user type model are analyzed in Section IV. The experiments on synthetic data and real data are presented in Section V. WE conclude this paper in Section VI.

II. Heterogeneous System Model

In this section, we first give a brief introduction to the preliminary concepts of evolutionary game theory. Then, we elaborate the proposed evolutionary game theoretic formulations of the information diffusion problem over heterogeneous social networks.

A. Basics of Evolutionary Game

The focus of traditional game theory is a game with static players and the solution concept is static Nash equilibrium (NE). On the contrary, evolutionary game theory [28] is concentrated on investigating the dynamics and stable states of a large population of evolving agents who interact with each other. Evolutionary game, as the name suggests, originates from the study of the evolution of species in biology, where animals or plants are modeled as players interacting with each other. Recent works [26], [27] show that it is also a very suitable model to analyze the social interactions among users of social networks.

A very important solution concept of evolutionary game theory is *evolutionarily stable state (ESS)*, which predicts the ultimate equilibrium of the evolutionary dynamics in a evolutionary game. Consider an evolutionary game with a large population of players. Suppose we have m strategies $\{1, \ldots, m\}$ an m by m payoff matrix U whose (i, j)-th entry u_{ij} is the payoff for strategy i verse strategy j (i.e., when a player with strategy i interacts with a player with strategy j, he will get a payoff of u_{ij}). Denote p_i the proportion of players adopting strategy i and $p = [p_1, p_2, \ldots, p_m]^T$ is the system state of the evolutionary game. Thus, the payoff of any sub-population with state q when interacting the whole population with state p is $q^T Up$. We call a state p^* an ESS if for any $q \neq p^*$, the following two conditions hold [28]:

1) $q^T U p^* \leq p^{*^T} U p^*$,
2) if $q^T U p^* = p^{*^T} U p^*$, then $p^{*^T} U q > q^T U q$.

The first condition is an NE condition, stating that any mutant (deviation from the ESS p^*) of any sub-population cannot make the payoff better off. The second condition guarantees that if deviation remains the payoff unchanged, then within the mutated sub-population (i.e., interacting with the sub-population state q), the ESS is strictly better than the deviated state q. This further ensures the stability of the state p^*. An important issue of evolutionary game theory is to compute the ESSs. A prevalent approach is to find the locally stable state of the evolutionary dynamics as a dynamical system $\dot{p} = f(p)$, where f is some function.

Classical evolutionary game assumes that every two players can interact with each other, implicitly making the hypothesis that the underlying interaction network is a complete graph. A useful generalization of the classical evolutionary game is the graphical evolutionary game, in which the interaction network is possibly incomplete. In graphical evolutionary game theory [38], [39], the player strategy update rule directly depends on the *fitness* of the users, which can be defined as a convex combination of the baseline fitness B and the payoff U, i.e.,

$$\pi = (1 - \alpha)B + \alpha U, \tag{1}$$

where π is the fitness. Here $0 < \alpha < 1$ is the selection strength, controlling the impact of the payoff on the fitness. In the literature of graphical evolutionary game theory [24]–[27], α is generally assumed to be very small and we also make this assumption in the rest of the paper. The reason of assuming a small α is that we expect evolutions/adaptations to occur gradually and slowly. For instance, in biology, the evolution of species takes place very slowly; in adaptive signal processing (e.g., LMS algorithm), we usually adopt a small step size to inhibit abrupt intense change or instability. A small α limits the impact of payoff differences on the values of fitness, and thus reduces the gaps between the fitness of different players, which slows down the evolution. In fact, later we will see that the evolution dynamics are often proportional to α. After defining fitness, we can introduce three most prevalent strategy update rules in the literature of graphical evolutionary game theory, namely birth-death (BD), death-birth (DB) and imitation (IM).

1) BD update rule: one player is chosen for reproduction with probability proportional to fitness. The chosen player's strategy replaces one of its neighbor's strategy with uniform probability.
2) DB update rule: one player is chosen to abandon its strategy with uniform probability. He/she will adopt one of its neighbors' strategies with probability proportional to their fitness.
3) IM update rule: one player is chosen to update its strategy with uniform probability. He/she may maintain his/her current strategy or adopt one of his/her neighbors' strategies, with probability proportional to fitness.

In this paper, we adopt the DB update rule. The other update rules can be similarly analyzed under our framework. In the following, we elaborate how to model the information diffusion over heterogeneous social networks by using evolutionary game theory.

A social network can be generally modeled as a graph, with nodes representing users and edges representing relationships. We assume there are N nodes (users) in the network and each node has some neighbors with whom it interacts. The number of neighbors k exhibits certain distributions $\lambda(k)$ (the fraction of nodes whose degree is k) in real social networks, e.g. Poisson distribution in Erdos-Renyi networks [36] and power law distribution in Barabasi-Albert scale-free networks [37]. In addition, real-world social networks usually consist of groups of users with different interests, influences and activities. To capture this heterogeneity, we categorize the users into M types, whereas the proportion of type-i users is $q(i), i = 1, 2, \ldots, M$. In the game-theoretic formulation, the N users are regarded as players. When a piece of information (e.g., a hashtag, a status or a meme) is generated, each user has two possible strategies: forwarding the information ($\mathcal{S}_f$) or not forwarding it ($\mathcal{S}_n$). We denote $p_f(i)$ the proportion of users adopting $\mathcal{S}_f$ among all the type-i users and p_f the proportion of users adopting $\mathcal{S}_f$ among users of all types. We shall call $p_f(i)$ and p_f *population dynamics* or *popularity dynamics* in the rest of the paper.

B. Unknown User Type Model

In real-world social networks, users often do not know the types of their neighbors/friends. For example, a user may not know whether his friend is fan of a singer or not. In this subsection, we present a model where the user type is private information that is unknown to others. Consider one social interaction where a type-i user A is interacting with one of its neighbors, a type-j user B. Because A does not know the type of B, the payoff of A should not depend on the type of B in this social interaction. Specifically, the payoff matrix of the type-i node A is:

$$\begin{array}{cc} & \begin{array}{cc} S_f & \quad S_n \end{array} \\ \begin{array}{c} S_f \\ S_n \end{array} & \begin{pmatrix} ru_{ff}(i) & u_{fn}(i) \\ u_{fn}(i) & u_{nn}(i) \end{pmatrix} \end{array}.$$

When A and B both adopt $\mathcal{S}_f$, the payoff of A is $u_{ff}(i)$ regardless of the type of B. Both $u_{fn}(i)$ and $u_{nn}(i)$ are similarly defined. Here, a symmetric payoff structure is considered as in [26], [27]. In other words, when a type-i user with strategy $\mathcal{S}_f$ ($\mathcal{S}_n$) meets a user with strategy $\mathcal{S}_n$ ($\mathcal{S}_f$), its payoff is $u_{fn}(i)$. The reason of this symmetric payoff assumption is that often disagreement (one with strategy $\mathcal{S}_f$ while the other with strategy $\mathcal{S}_n$) leads to the same payoff to both sides. For instance, if a user mentions a hashtag while another user does not, then when they interact none of them can find common topic to discuss and both get the same payoff. The physical meaning of the payoff depends on the applications: if the social network nodes are social network users, then their payoffs may be their popularity; if the social network nodes are websites, then their payoffs may be their hit rates. The values of the payoff matrix depend on both the content of the information and the types of the users. For example, if the information is a recent hot topic (e.g., world cup in the summer of 2014) and forwarding it can increase users' popularity, then $u_{ff}(i)$ is big and $u_{nn}(i)$ is small. And if a group of users are very interested in that hot topic (e.g., football fans), then they may have even larger $u_{ff}(i)$ and smaller $u_{nn}(i)$ compared to other groups of users. By taking the baseline fitness to be 1 in Eq. (1), we can write the fitness as $\pi = 1 - \alpha + \alpha U$ (π is the fitness and U is the payoff). Here $0 < \alpha < 1$ is the selection strength, which is assumed very small conventionally. We note that different from payoff, fitness represents the level of fitting of a user in the social network. This fitting level contains not only the payoff obtained from extrinsic interactions but also a baseline fitness which encompasses intrinsic attributes of users, such as the satisfaction of the social network/website. Suppose A has k_f neighbors adopting $\mathcal{S}_f$, then the fitness of A is:

$$\pi_f(i,k_f) = 1 - \alpha + \alpha[k_f u_{ff}(i) + (k-k_f)u_{fn}(i)]. \quad (2)$$

One can similarly obtain $\pi_n(i,k_f)$, the fitness of A when A adopts $\mathcal{S}_n$ as follows:

$$\pi_n(i,k_f) = 1 - \alpha + \alpha[k_f u_{fn}(i) + (k-k_f)u_{nn}(i)]. \quad (3)$$

Furthermore, since A only knows the strategies of its neighbors but not the types of its neighbors, it regards the type of all of its neighbors the same as itself, i.e., type i. In other words, if one neighbor is adopting strategy $\mathcal{S}_f$, A consider its fitness to be $\pi_f(i,k_f)$. Otherwise, A considers its fitness to be $\pi_n(i,k_f)$.

C. Known User Type Model

Sometimes, through repeated interactions, users may somehow manage to know its neighbors' types. For instance, when a user observes that one of his friends frequently post news about football match, he may gradually know that this friend is a football fan. In this subsection, we present a model where the user types are publicly known information. Consider a social interaction where a Type-i user A is interacting with one of its neighbors, Type-j user B. Here, different from the unknown user type model, A knows the type of B. Hence the payoff of A should depend on the type of B in this social interaction. Specifically, if both A and B adopt $\mathcal{S}_f$, A gets a payoff $u_{ff}(i,j)$. If A, B adopt strategy $\mathcal{S}_f$ and $\mathcal{S}_n$ respectively, then the payoff of A is $u_{fn}(i,j)$. Similarly, we can define $u_{nf}(i,j)$ and $u_{nn}(i,j)$.

Take the baseline fitness to be 1 in Eq. (1) and thus the fitness of a user with strategy $\mathcal{S}_f$ or $\mathcal{S}_n$ is respectively given by:

$$\pi_f(i) = 1 - \alpha + \alpha \sum_{j=1}^{M} [k_f(j) u_{ff}(i,j) + k_n(j) u_{fn}(i,j)], \quad (4)$$

$$\pi_n(i) = 1 - \alpha + \alpha \sum_{j=1}^{M} [k_f(j) u_{nf}(i,j) + k_n(j) u_{nn}(i,j)], \quad (5)$$

where $k_f(j)$ ($k_n(j)$) denotes the number of type-j neighbors with strategy $\mathcal{S}_f$ ($\mathcal{S}_n$). The update rule is still the death-birth (DB), as described previously for the unknown type model. The difference is that now the player knows the types of his neighbors, hence can learn strategies only from those neighbors with the same type as his. The notations of this paper are summarized in Table I, in which some of the notations will be introduced in Section IV.

TABLE I
NOTATIONS

N	Number of nodes in the network
k	Degree of a given node
M	Number of user types in the network
$q(i)$	The proportion of Type-i users in the network
$p_f(i)$	Proportion of users adopting $\mathcal{S}_f$ among all the type-i users
p_f	Proportion of users adopting $\mathcal{S}_f$ among users of all types
$u_{ff}(i), u_{fn}(i), u_{nn}(i)$	Payoffs of Type-i users in the unknown user type model. For details, see Subsection II-B.
$\pi_f(i), \pi_n(i)$	Fitness of a Type-i user with strategy $\mathcal{S}_f$ or $\mathcal{S}_n$, respectively
k_f	Number of neighbors (of a given user) adopting strategy $\mathcal{S}_f$
$\pi_f(i,k_f), \pi_n(i,k_f)$	Fitness of a Type-i with k_f neighbors adopting strategy $\mathcal{S}_f$ while itself adopts strategy $\mathcal{S}_f$ or $\mathcal{S}_n$, respectively.
$p_{ff}(i,j), p_{fn}(i,j), p_{nn}(i,j)$	Relationship states of Type-i users in the known user type model. For details, see Section IV.
$p_{f\|f}(i,j), p_{f\|n}(i,j), p_{n\|f}(i,j), p_{n\|n}(i,j)$	Influence states of Type-i users in the known user type model. For details, see Section IV.
$u_{ff}(i,j), u_{fn}(i,j), u_{nf}(i,j), u_{nn}(i,j)$	Payoffs of Type-i users in the known user type model. For details, see Subsection II-C.
$k_f(j)$	Number of neighbors (of a given Type-j user) adopting strategy $\mathcal{S}_f$

III. Theoretical Analysis for the Unknown User Type Model

In this section, we derive the evolutionary dynamics of the network states $p_f(i), p_f$ and the corresponding evolutionarily stable states (ESSs) for the unknown user type model. The derived dynamics and ESSs connect the information diffusion process and the final steady states with the heterogeneous users' payoff matrices explicitly. We are able to give simple explanations on the ESSs of the information diffusion from the perspective of the payoff matrix.

Let's consider a type-i user with strategy $\mathcal{S}_f$ (in the following, we will call this user as the center user). Suppose among its k neighbors, there are k_f users adopting strategy $\mathcal{S}_f$ and $(k-k_f)$ users adopting strategy $\mathcal{S}_n$. The fitness $\pi_f(i,k_f)$ of the center user is given in Eq. (2). If the center user changes its strategy to $\mathcal{S}_n$, its fitness $\pi_n(i,k_f)$ becomes Eq. (3). From the perspective of the center user, a neighbor adopting strategy $\mathcal{S}_f$ (or $\mathcal{S}_n$) has fitness $\pi_f(i,k_f)$ (or $\pi_n(i,k_f)$, respectively). According to the DB update rule, the center user will adopt one of its neighbors' strategy with probability proportional to their fitness. Hence, the probability that the center user changes its strategy from $\mathcal{S}_f$ to $\mathcal{S}_n$ is given by: Eq. (7)-(9) as shown at the bottom of the next page

$$\mathbb{P}_{f\to n}(i,k_f) = \frac{(k-k_f)\pi_n(i,k_f)}{k_f\pi_f(i,k_f) + (k-k_f)\pi_n(i,k_f)}. \quad (6)$$

Substituting the expressions of $\pi_f(i,k_f)$ and $\pi_n(i,k_f)$ in Eq. (2) and Eq. (3) into Eq. (6) yields Eq. (9): Eq. (7)–(9) shown at the bottom of next page, where $\Delta(i) := 2u_{fn}(i) - u_{ff}(i) - u_{nn}(i)$, $\Delta_n(i) := u_{nn}(i) - u_{fn}(i)$ and in the last equation we invoke the fact that $\frac{1+ax}{1+bx} = 1 + (a-b)x + O(x^2)$ for small x. Because α is a small quantity, we will omit the $O(\alpha^2)$ term in the following. Since the proportion of users with strategy $\mathcal{S}_f$ is

p_f over the entire network, each neighbor has probability p_f of adopting strategy $\mathcal{S}_f$. Thus k_f is binomially distributed random variable with probability mass function:

$$\theta(k, k_f) = \begin{pmatrix} k \\ k_f \end{pmatrix} p_f^{k_f} (1-p_f)^{k-k_f}. \tag{10}$$

Hence, taking expectation of Eq. (9) (note that k is also a r.v. and we need to take expectation of it further) gives:

$$\begin{aligned}
&\mathbb{E}[\mathbb{P}_{f\to n}(i, k_f)] \\
&= 1 - p_f + \alpha\Delta(i)\Big[\left(-\overline{k} + 3 - 2\overline{k^{-1}}\right)p_f^3 \\
&\quad + \left(\overline{k} - 4 + 3\overline{k^{-1}}\right)p_f^2 + \left(1 - \overline{k^{-1}}\right)p_f\Big] \\
&\quad + \alpha\Delta_n(i)\left[-\left(\overline{k}-1\right)p_f^2 + (\overline{k}-1)p_f\right],
\end{aligned} \tag{11}$$

where $\overline{k}$ and $\overline{k^{-1}}$ denote the expectation of k and k^{-1}, respectively. In the derivation of Eq. (11), we utilize the moments of binomial distribution: $\mathbb{E}[k_f|k] = kp_f$, $\mathbb{E}[k_f^2|k] = k^2p_f^2 - kp_f^2 + kp_f$, $\mathbb{E}[k_f^3|k] = k(k-1)(k-2)p_f^3 + 2(k-1)kp_f^2 + kp_f$. In each round of the DB update, one of the N users will be selected to update its strategy randomly. The proportion of type-i users with strategy $\mathcal{S}_f$ among all the users is $p_f(i)q(i)$. According to DB update rule, in order to have one Type-i user changes its strategy from $\mathcal{S}_f$ to $\mathcal{S}_n$, i.e., for $p_f(i)$ to decrease by $\frac{1}{Nq(i)}$, the chosen user in the death process should be a Type-i user with strategy $\mathcal{S}_f$, which happens with probability $q(i)p_f(i)$. After that, the user needs to change its strategy from $\mathcal{S}_f$ to $\mathcal{S}_n$, which happens with probability $\mathbb{E}[\mathbb{P}_{f\to n}(i, k_f)]$, where the expectation is with respect to the node degree k. Thus, we have:

$$\mathbb{P}\left(\delta p_f(i) = -\frac{1}{Nq(i)}\right) = p_f(i)q(i)\mathbb{E}[\mathbb{P}_{f\to n}(i, k_f)], \tag{12}$$

where δ denotes increment. With a similar argument as above, one can compute the probability that a type-i user changes its strategy from $\mathcal{S}_n$ to $\mathcal{S}_f$. We thus obtain:

$$\mathbb{P}\left(\delta p_f(i) = \frac{1}{Nq(i)}\right) = p_n(i)q(i)(1 - \mathbb{E}[\mathbb{P}_{f\to n}(i, k_f)]). \tag{13}$$

Combining Eq. (11), Eq. (12) and Eq. (13), we deduce the expected change of $p_f(i)$:

$$\begin{aligned}
\dot{p}_f(i) &= -\frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = -\frac{1}{Nq(i)}\right) \\
&\quad + \frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = \frac{1}{Nq(i)}\right) \\
&= \frac{1}{N}p_f - \frac{1}{N}p_f(i) + \frac{\alpha}{N}p_f(p_f - 1) \\
&\quad \times \Big[\Delta(i)\left(\left(\overline{k} - 3 + 2\overline{k^{-1}}\right)p_f + 1 - \overline{k^{-1}}\right) \\
&\quad + \Delta_n(i)(\overline{k} - 1)\Big],
\end{aligned} \tag{14}$$

which is the dynamic of $p_f(i)$. Hence, from Eq. (14), the dynamic of p_f can be written as:

$$\begin{aligned}
\dot{p}_f &= \sum_{i=1}^{M} q(i)\dot{p}_f(i) \\
&= \frac{\alpha}{N}p_f(p_f - 1)\Big[\overline{\Delta}\left(\left(\overline{k} - 3 + 2\overline{k^{-1}}\right)p_f + 1 - \overline{k^{-1}}\right) \\
&\quad + \overline{\Delta}_n(\overline{k} - 1)\Big],
\end{aligned} \tag{15}$$

where $\overline{\Delta} := \sum_{i=1}^{M} q(i)\Delta(i)$ and $\overline{\Delta}_n := \sum_{i=1}^{M} q(i)\Delta_n(i)$. We summarize the theoretical evolutionary dynamics results as the following theorem, Theorem 1.

Theorem 1: (Evolutionary Dynamics) In the unknown user type model, the evolutionary dynamics for the network states $p_f(i)$ and p_f are given in Eqs. (14) and (15), respectively.

From Theorem 1, we observe that the population dynamics $p_f(i)$ in Eq. (14) depend on both the global population dynamics p_f and the type-specific utility-related parameters $\Delta(i), \Delta_n(i)$. Consequently, a connection between the heterogeneous type-specific payoff matrix and the heterogeneous information diffusion dynamics of each time is established explicitly. Additionally, comparing Eq. (15) with the evolutionary population dynamics of a homogeneous social network given in [26] and [27], we note that the global population dynamics p_f evolve as if the network is homogeneous with corresponding payoff matrix being the weighted average (with weights $q(i)$) of those among all the types.

Given the dynamical system described in Theorem 1, we want to identify its ESSs. This is accomplished by the following theorem, Theorem 2.

$$\mathbb{P}_{f\to n}(i, k_f) \tag{7}$$

$$= \frac{k-k_f}{k}\frac{1 + \alpha[k_f u_{fn}(i) + (k-k_f)u_{nn}(i) - 1]}{1 + \alpha\left[\frac{k_f}{k}(k_f u_{ff}(i) + (k-k_f)u_{fn}(i) - 1) + (1 - \frac{k_f}{k})(k_f u_{ff}(i) + (k-k_f)u_{fn}(i) - 1)\right]} \tag{8}$$

$$= \frac{k-k_f}{k} + \alpha(k-k_f)\left[\frac{k_f^2}{k^2}\Delta(i) + \frac{k_f}{k}\Delta_n(i)\right] + O(\alpha^2), \tag{9}$$

Theorem 2: (ESSs) In the unknown user type model, the ESSs of the network are as follows:

$$p_f^* = \begin{cases} 0, & \text{if } \overline{u}_{nn} > \overline{u}_{fn}, \\ 1, & \text{if } \overline{u}_{ff} > \overline{u}_{fn}, \\ \dfrac{\overline{\Delta}_n(1-\overline{k}) + \overline{\Delta}(\overline{k^{-1}}-1)}{\overline{\Delta}(\overline{k}-3+2\overline{k^{-1}})}, & \text{if } \max\{\overline{u}_{ff}, \overline{u}_{nn}\} \\ & < \overline{u}_{fn}, \end{cases} \tag{16}$$

$$p_f^*(i) = p_f^* + \alpha p_f^*(p_f^*-1)\Big[\Delta(i)\Big(\Big(\overline{k}-3+2\overline{k^{-1}}\Big)p_f^* + 1 - \overline{k^{-1}}\Big) + \Delta_n(i)(\overline{k}-1)\Big], \tag{17}$$

where $\overline{u}_{ff} = \sum_{i=1}^{M} q(i)u_{ff}(i)$ and $\overline{u}_{fn}, \overline{u}_{nn}$ are similarly defined. Recall that $\Delta(i) = 2u_{fn}(i) - u_{ff}(i) - u_{nn}(i), \Delta_n(i) = u_{nn}(i) - u_{fn}(i)$ and $\overline{\Delta} = \sum_{i=1}^{M} q(i)\Delta(i)$, $\overline{\Delta}_n = \sum_{i=1}^{M} q(i)\Delta_n(i)$. Note that it is possible that the system has more than one ESS.

Proof: Letting the R.H.S. of Eq. (14) be zero, we obtain the three equilibrium points for the dynamic of p_f:

$$p_f^* = 0,\ 1,\ \frac{\overline{\Delta}_n(1-\overline{k}) + \overline{\Delta}(\overline{k^{-1}}-1)}{\overline{\Delta}(\overline{k}-3+2\overline{k^{-1}})}. \tag{18}$$

Given p_f^*, the equilibrium state of $p_f(i)$ can be derived from Eq. (14) as stated in Eq. (17).

For an equilibrium point to be an ESS, it needs to be locally asymptotically stable for the underlying dynamical system. Note that for each i, $p_f(i)$ and p_f can be regarded as a dynamical system consisting of two states as indicated by Eq. (14) and Eq. (15). The Jacobian matrix of the system is given by:

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial \dot{p}_f(i)}{\partial p_f(i)} & \dfrac{\partial \dot{p}_f(i)}{\partial p_f} \\ \dfrac{\partial \dot{p_f}}{\partial p_f(i)} & \dfrac{\partial \dot{p_f}}{\partial p_f} \end{bmatrix}, \tag{19}$$

where

$$\begin{aligned} \frac{\partial \dot{p}_f(i)}{\partial p_f(i)} &= -\frac{1}{N}, \\ \frac{\partial \dot{p}_f(i)}{\partial p_f} &= \frac{1}{N} + \frac{\alpha}{N}(2p_f-1)\Big[\Delta(i)\left(\overline{k}-3+2\overline{k^{-1}}\right)p_f \\ &\quad + \Delta(i)(1-\overline{k^{-1}}) + \Delta_n(i)(\overline{k}-1)\Big] \\ &\quad + \frac{\alpha\Delta(i)}{N}(p_f^2-p_f)(\overline{k}-3+2\overline{k^{-1}}), \ \frac{\partial \dot{p_f}}{\partial p_f(i)} = 0, \\ \frac{\partial \dot{p_f}}{\partial p_f} &= \frac{\alpha}{N}(2p_f-1)\Big[\overline{\Delta}\left(\overline{k}-3+2\overline{k^{-1}}\right)p_f + \overline{\Delta}(1-\overline{k^{-1}}) \\ &\quad + \overline{\Delta}_n(\overline{k}-1)\Big] + \frac{\alpha\Delta}{N}\left(p_f^2-p_f\right)\left(\overline{k}-3+2\overline{k^{-1}}\right). \end{aligned} \tag{20}$$

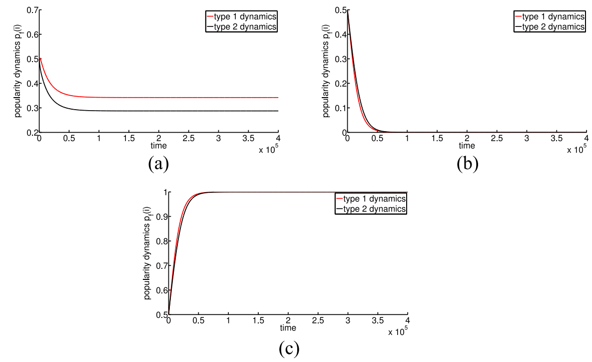

Fig. 1. Evolutionary dynamics under different parameter setups. Parameter setup 1: $u_{ff}(1) = 0.4, u_{ff}(2) = 0.2, u_{fn} = 0.6, u_{fn}(2) = 0.4, u_{nn}(1) = 0.3, u_{nn}(2) = 0.5$; Parameter setup 2: $u_{ff}(1) = 0.4, u_{ff}(2) = 0.2, u_{fn} = 0.3, u_{fn}(2) = 0.5, u_{nn}(1) = 0.6, u_{nn}(2) = 0.4$; $u_{ff}(1) = 0.6, u_{ff}(2) = 0.4, u_{fn} = 0.3, u_{fn}(2) = 0.5, u_{nn}(1) = 0.4, u_{nn}(2) = 0.2$. In every setup, we have $q(1) = q(2) = 0.5, N = 1000, k = 20$. The ESSs match the assertions in Theorem 2: some dynamics decrease to 0 (subfigure b) or increase to 1 (subfigure c) while some will stay at some stable state between 0 and 1 (subfigure a). (a) Parameter setup 1. (b) Parameter setup 2. (c) Parameter setup 3.

Since $\mathbf{J}$ is an upper triangular matrix and $\frac{\partial \dot{p}_f(i)}{\partial p_f(i)}$ is always negative, the condition for stability is simply $\frac{\partial \dot{p_f}}{\partial p_f} < 0$. Substituting the three equilibrium points in Eq. (18) into it yields the conditions for the three possible ESSs given in Eq. (16), where we make use of the fact that the node degree k is generally much larger than 1 in practice. ■

The ESS results Eq. (16) in Theorem 2 can be interpreted easily as follows. If $\overline{u}_{ff}$ is large enough (larger than $\overline{u}_{fn}$), i.e., on average the players favor forwarding the information, then $p_f^* = 1$ is an ESS of the network. The ESS $p_f^* = 0$ can be similarly interpreted. On the contrary, if neither $\overline{u}_{ff}$ nor $\overline{u}_{nn}$ is not large enough (both smaller than $\overline{u}_{fn}$), an ESS between 0 and 1 is in presence. As shown in Fig. 1, for different parameter setups, we have different evolutionary dynamics. Some dynamics decrease to 0 (Fig. 1-b) or increase to 1 (Fig. 1-c) while some will stay at some stable state between 0 and 1 (Fig. 1-a). The corresponding ESSs are correctly predicted by Theorem 2. We observe that the population dynamics $p_f(i)$ always vary quickly at first and gradually slow down the varying speed until finally converge to a stable state. This can be explained by Eq. (15). As p_f gets closer and closer to the ESS (be it 0, 1, or some number between 0 and 1), the absolute value of R.H.S. of Eq. (15) gets smaller and hence the varying speed of p_f slows down until it finally equals to the ESS. Meanwhile, when p_f is stable, according to Eq. (14), all the type specific population dynamics $p_f(i)$ will also converge to their respective ESSs.

IV. Theoretical Analysis for Known User Type Model

In this section, the evolutionary dynamics for the known user type model are derived. It is observed that the influence states (which we will define later) always keep track of the corresponding population states, which can be exploited to further simplify the dynamics.

Since a user's type and strategy affect its neighbors' payoffs, they may also influence the neighbors' strategies. Thus, the edge information is also required to fully characterize the network state. Specifically, we define network edge states as $p_{ff}(i,j), p_{fn}(i,j), p_{nn}(i,j)$, where $p_{ff}(i,j)$ ($p_{nn}(i,j)$) denotes the proportion of edges connecting a type-i user with strategy $\mathcal{S}_f$ ($\mathcal{S}_n$) and a type-j user with strategy $\mathcal{S}_f$ ($\mathcal{S}_n$), and $p_{fn}(i,j)$ denotes the proportion of edges connecting a type-i user with strategy $\mathcal{S}_f$ and a type-j user with strategy $\mathcal{S}_n$. Moreover, we denote $p_{f|f}(i,j)$ the percentage of type-i neighbors adopting strategy $\mathcal{S}_f$, given a center type-j user using strategy $\mathcal{S}_f$. Similarly, we can define $p_{f|n}(i,j), p_{n|f}(i,j), p_{n|n}(i,j)$. In summary, we have *population states* (e.g. $p_f(i)$), *relationship states* (e.g. $p_{ff}(i,j)$) and *influence states* (e.g. $p_{f|f}(i,j)$) as the network states. Because these states are related to each other, we only need a subset of them to characterize the entire network state. For example, we can use $p_f(i), 1 \le i \le M$ and $p_{ff}(i,j), 1 \le i \le j \le M$ to compute all the other states.

Consider a type-i user using strategy $\mathcal{S}_f$. Rigorously speaking, $k_f(j)$ and $k_n(j)$ are random variables with expectation $kq(j)p_{f|f}(j,i)$ and $kq(j)p_{n|f}(j,i)$ respectively. Since in real world social networks, k is relatively large (more than 100 for typical online social networks such as Facebook) and a small number of types (i.e., M) is enough to capture the user behaviors, we approximate $k_f(j), k_n(j)$ with their expectations for ease of analysis in the following. This approximation can be justified as follows. Recall the Chernoff bound: Suppose $X_1, X_2, \ldots, X_n$ are independent random variables taking values in $[0,1]$, $X = \sum_{i=1}^n X_i$ and $\mu = \mathbb{E}(X)$. Then, for any $0 < \delta < 1$, we have: (i) $\mathbb{P}(X \ge (1+\delta)\mu) \le \exp(-\frac{\delta^2\mu}{3})$; (ii) $\mathbb{P}(X \le (1-\delta)\mu) \le \exp(-\frac{\delta^2\mu}{2})$. In our case, for a Type-i user with strategy $\mathcal{S}_f$ and k neighbors, each one of its neighbors is a Type-j user with strategy $\mathcal{S}_f$ with probability $q(j)p_{f|f}(j,i)$ independently. Let the random variable $X_l (l = 1, \ldots, k)$ be 1 if the l-th neighbor is a Type-j with strategy $\mathcal{S}_f$ and be 0 otherwise. Thus, X_l's are i.i.d. random variables. Denote $X = \sum_{l=1}^k X_i$ the total number of Type-j neighbors with strategy $\mathcal{S}_f$, which is $k_f(j)$ in our context. Because M is small, usually each $q(j), j = 1, 2, \ldots, M$ (altogether sum to 1) is not too small. Furthermore k is large and $p_{f|f}(j,i)$ is generally not too small. Hence, $\mu = \mathbb{E}(X) = kq(j)p_{f|f}(j,i)$ is large. Applying the multiplicative form of Chernoff bound, we can assert that X is close to its expectation with high probability. Thus, it is reasonable to replace $k_f(j)$ with its expectation. Similar arguments hold for $k_n(j)$. With this approximation, Eq. (4) becomes

$$\pi_f(i) = 1 - \alpha + \alpha k \sum_{j=1}^{M} q(j)[p_{f|f}(j,i)u_{ff}(i,j) + p_{n|f}(j,i)u_{fn}(i,j)]. \tag{21}$$

Similarly, if a type-i user is adopting strategy $\mathcal{S}_n$, its fitness Eq. (5) can be approximated as:

$$\pi_n(i) = 1 - \alpha + \alpha k \sum_{j=1}^{M} q(j)[p_{f|n}(j,i)u_{nf}(i,j) + p_{n|n}(j,i)u_{nn}(i,j)]. \tag{22}$$

Now, consider a type-i center user using strategy $\mathcal{S}_f$, who is selected to update its strategy. On average, there are $kp_{f|f}(i,i)$ type-i neighbors using strategy $\mathcal{S}_f$ and $kp_{n|f}(i,i)$ type-i neighbors using strategy $\mathcal{S}_n$. Thereby, according to the DB update rule, the probability that the center user will update its strategy to be $\mathcal{S}_n$ is:

$$\mathbb{P}_{f\to n}(i) = \frac{\pi_n(i)p_{n|f}(i,i)}{\pi_f(i)p_{f|f}(i,i) + \pi_n(i)p_{n|f}(i,i)}. \tag{23}$$

The probability that a type-i user with strategy $\mathcal{S}_f$ is chosen to update its strategy is $q(i)p_f(i)$. Hence, we have:

$$\mathbb{P}\left(\delta p_f(i) = -\frac{1}{Nq(i)}\right) = q(i)p_f(i)\mathbb{E}[\mathbb{P}_{f\to n}(i)]. \tag{24}$$

Similarly, we can analyze the situation where a type-i user with strategy $\mathcal{S}_n$ is selected to update its strategy. And we obtain:

$$\mathbb{P}_{n\to f}(i) = \frac{\pi_f(i)p_{f|n}(i,i)}{p_{f|n}(i,i)\pi_f(i) + p_{n|n}(i,i)\pi_n(i)}. \tag{25}$$

$$\mathbb{P}\left(\delta p_f(i) = \frac{1}{Nq(i)}\right) = q(i)p_n(i)\mathbb{E}[\mathbb{P}_{n\to f}(i)]. \tag{26}$$

We know that:

$$\dot{p}_f(i) = -\frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = -\frac{1}{Nq(i)}\right) + \frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = \frac{1}{Nq(i)}\right). \tag{27}$$

For ease of notation, we temporarily denote that $a = k\sum_{j=1}^M q(j)[p_{f|n}(j,i)u_{nf}(i,j) + p_{n|n}(j,i)u_{nn}(i,j)]$ and $b = k\sum_{j=1}^M q(j)[p_{f|f}(j,i)u_{ff}(i,j) + p_{n|f}(j,i)u_{fn}(i,j)]$. Thus, the first term in Eq. (27) can be rewritten as:

$$-\frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = -\frac{1}{Nq(i)}\right) \tag{28}$$

$$= -\frac{p_f(i)p_{n|f}(i,i)}{N} \tag{29}$$

$$\times\ \mathbb{E}\left\{\frac{1+\alpha(a-1)}{1+\alpha[(b-1)p_{f|f}(i,i) + (a-1)p_{n|f}(i,i)]}\right\} \tag{30}$$

$$= -\frac{p_f(i)p_{n|f}(i,i)}{N}\mathbb{E}[1 + p_{f|f}(i,i)(a-b)\alpha] + O(\alpha^2), \tag{31}$$

where we make use of the fact that $p_{f|f}(i,i) + p_{n|f}(i,i) = 1$, which can be easily seen from the definition. The expectation is taken over k. Similarly, we can derive the second term in Eq. (27) as:

$$\frac{1}{Nq(i)}\mathbb{P}\left(\delta p_f(i) = \frac{1}{Nq(i)}\right) = \frac{p_n(i)p_{f|n}(i,i)}{N}\mathbb{E}[1 + \alpha p_{n|n}(i,i)(b-a)] + O(\alpha^2). \tag{32}$$

Noticing the fact that $p_f(i)p_{n|f}(i,i) = p_n(i)p_{f|n}(i,i)$, we obtain:

$$\begin{aligned}\dot{p}_f(i) \approx \frac{\alpha\overline{k}}{N} p_f(i)p_{n|f}(i,i)(p_{n|n}(i,i) + p_{f|f}(i,i)) \\ \times \sum_{j=1}^{M} q(j)[p_{f|f}(j,i)u_{ff}(i,j) + p_{n|f}(j,i)u_{fn}(i,j) \\ - p_{f|n}(j,i)u_{nf}(i,j) - p_{n|n}(j,i)u_{nn}(i,j)], \end{aligned} \tag{33}$$

where $\overline{k}$ denotes the average degree of the network and we omit the $O(\alpha^2)$ terms. Next, we compute the dynamics of $p_{ff}(i,l)$ (or equivalently, $p_{f|f}(i,l)$). To change the value of $p_{ff}(i,l)$, either a type-i user or a type-l user changes its strategy. If $i \neq l$, there are totally four situations: i) a type-i user changes its strategy from $\mathcal{S}_f$ to $\mathcal{S}_n$; ii) a type-i user changes its strategy from $\mathcal{S}_n$ to $\mathcal{S}_f$; iii) a type-l user changes its strategy from $\mathcal{S}_f$ to $\mathcal{S}_n$; iv) a type-l user changes its strategy from $\mathcal{S}_n$ to $\mathcal{S}_f$. They correspond to the following four equations:

$$\begin{aligned}&\mathbb{P}\left(\delta p_{ff}(i,l) = -\frac{2}{N}q(l)p_{f|f}(l,i)\right) \\ &= q(i)p_f(i)\mathbb{P}_{f\to n}(i) \approx q(i)p_f(i)p_{n|f}(i,i), \\ &\mathbb{P}\left(\delta p_{ff}(i,l) = -\frac{2}{N}q(i)p_{f|f}(i,l)\right) \\ &= q(l)p_f(l)\mathbb{P}_{f\to n}(l) \approx q(l)p_f(l)p_{n|f}(l,l), \\ &\mathbb{P}\left(\delta p_{ff}(i,l) = \frac{2}{N}q(l)p_{f|n}(l,i)\right) \\ &= q(i)p_n(i)\mathbb{P}_{n\to f}(i) \approx q(i)p_n(i)p_{f|n}(i,i), \\ &\mathbb{P}\left(\delta p_{ff}(i,l) = \frac{2}{N}q(i)p_{f|n}(i,l)\right) \\ &= q(l)p_n(l)\mathbb{P}_{n\to f}(l) \approx q(l)p_n(l)p_{f|n}(l,l),\end{aligned} \tag{34}$$

where in the last step we omit $O(\alpha)$ terms, i.e., treating α as 0. The reason that we omit $O(\alpha)$ terms instead of $O(\alpha^2)$ terms as before is that we have nonzero $O(1)$ terms here. Combining the four equations in Eq. (34), we get (for $i \neq l$):

$$\begin{aligned}&\dot{p}_{ff}(i,l) \\ &= -\frac{2}{N}q(l)p_{f|f}(l,i)\mathbb{P}\left(\delta p_{ff}(i,l) = -\frac{2}{N}q(l)p_{f|f}(l,i)\right) \\ &\quad - \frac{2}{N}q(i)p_{f|f}(i,l)\mathbb{P}\left(\delta p_{ff}(i,l) = -\frac{2}{N}q(i)p_{f|f}(i,l)\right) \\ &\quad + \frac{2}{N}q(l)p_{f|n}(l,i)\mathbb{P}\left(\delta p_{ff}(i,l) = \frac{2}{N}q(l)p_{f|n}(l,i)\right) \\ &\quad + \frac{2}{N}q(i)p_{f|n}(i,l)\mathbb{P}\left(\delta p_{ff}(i,l) = \frac{2}{N}q(i)p_{f|n}(i,l)\right) \\ &= \frac{2}{N}q(i)q(l)p_f(i)p_{n|f}(i,i)(p_{f|n}(l,i) - p_{f|f}(l,i)) \\ &\quad + \frac{2}{N}q(i)q(l)p_f(l)p_{n|f}(l,l)(p_{f|n}(i,l) - p_{f|f}(i,l)) \\ &= \frac{2}{N}q(i)q(l)p_f(i)(1 - p_{f|f}(i,i)) \\ &\quad \times \left[\frac{p_f(l)}{p_n(i)}(1 - p_{f|f}(i,l)) - p_{f|f}(l,i)\right] \\ &\quad + \frac{2}{N}q(i)q(l)p_f(l)(1 - p_{f|f}(l,l)) \\ &\quad \times \left[\frac{p_f(i)}{p_n(l)}(1 - p_{f|f}(l,i)) - p_{f|f}(i,l)\right],\end{aligned} \tag{35}$$

where we have used the equalities $p_{n|f}(i,i) = 1 - p_{f|f}(i,i)$ and $p_{f|n}(l,i) = \frac{p_f(l)}{p_n(i)}(1 - p_{f|f}(i,l))$ in the last step so as to substitute all the influence states by $p_{f|f}(\cdot,\cdot)$. Similarly we can derive the dynamics of $p_{ff}(i,i)$ as follows:

$$\begin{aligned}\dot{p}_{ff}(i,i) = \frac{2}{Np_n(i)}q^2(i)p_f(i)(1 - p_{f|f}(i,i))(p_f(i) \\ - p_{f|f}(i,i)).\end{aligned} \tag{36}$$

Recall Eq. (33), where we note that the population dynamics $p_f(\cdot)$ evolves at the speed of $O(\alpha)$. From Eq. (35) and Eq. (36), we observe that the relationship dynamics $p_{ff}(\cdot,\cdot)$ (hence the influence dynamics $p_{f|f}(\cdot,\cdot)$) evolve at the speed of $O(1)$. Due to the assumption that α is very small, the relationship dynamics and influence dynamics change at a much faster speed than population dynamics do. This implies that we can select a time window with an appropriate length such that the population dynamics $p_f(\cdot)$ basically remain unchanged while the relationship dynamics $p_{ff}(\cdot,\cdot)$ and influence dynamics $p_{f|f}(\cdot,\cdot)$ vary a lot. In the following, we focus on such a time period in which the population dynamics $p_f(\cdot)$ remains a constant and only relationship dynamics and influence dynamics vary with time. Taking derivative w.r.t time on both sides of the equation $p_{ff}(i,l) = 2q(i)q(l)p_f(i)p_{f|f}(l,i), i \neq l$, we obtain:

$$\dot{p}_{ff}(i,l) = 2q(i)q(l)p_f(i)\dot{p}_{f|f}(l,i). \tag{37}$$

Combining Eq. (35) and Eq. (37) yields the dynamics of $p_{f|f}(l,i), l \neq i$:

$$\begin{aligned}&\dot{p}_{f|f}(l,i) \\ &= \frac{1}{N}(1 - p_{f|f}(i,i))\left[\frac{p_f(l)}{p_n(i)}(1 - p_{f|f}(i,l)) - p_{f|f}(l,i)\right] \\ &\quad + \frac{1}{N}(1 - p_{f|f}(l,l)) \\ &\quad \times \left[\frac{p_f(l)}{p_n(l)}(1 - p_{f|f}(l,i)) - \frac{p_f(l)}{p_f(i)}p_{f|f}(i,l)\right].\end{aligned} \tag{38}$$

Leveraging the equation $p_f(i)p_{f|f}(l,i) = p_f(l)p_{f|f}(i,l)$, we can further simplify Eq. (38) as follows:

$$\dot{p}_{f|f}(l,i) = \frac{1}{N}(p_f(l) - p_{f|f}(l,i)) \times \left[\frac{1-p_{f|f}(i,i)}{p_n(i)} + \frac{1-p_{f|f}(l,l)}{p_n(l)}\right], \forall l \neq i. \quad (39)$$

On the other hand, if $l = i$, then $\dot{p}_{ff}(i,i) = q^2(i)p_f(i)\dot{p}_{f|f}(i,i)$. Thus, from Eq. (36), we obtain:

$$\dot{p}_{f|f}(i,i) = \frac{2}{Np_n(i)}(1 - p_{f|f}(i,i))(p_f(i) - p_{f|f}(i,i)), \forall i. \quad (40)$$

Since Eq. (40) is equivalent to letting $i = l$ in Eq. (39), we know that Eq. (39) applies to any i, l (not necessarily unequal). Recall that in Eq. (39), we treat the population dynamics $p_f(i), p_n(i)$ as constants. In other words, we are considering a small time period where the population dynamics do not vary with time while the influence dynamics $p_{f|f}(\cdot,\cdot)$ vary according to the deduced dynamics Eq. (39). Next, we show that in this small time period, the influence dynamics $p_{f|f}(\cdot,\cdot)$ will converge to the corresponding population dynamics $p_f(\cdot)$.

We first solve the ODE Eq. (40) with single variable $p_{f|f}(i,i)$. Without loss of generality, we assume the initial value of $p_{f|f}(i,i)$ is less than $p_f(i)$. Thus, by solving Eq. (40), we have:

$$p_{f|f}(i,i) = p_f(i) - \frac{p_n(i)}{e^{\frac{4t}{N}+C_i} - 1}, \quad (41)$$

where $C_i := \ln(1-p_{f|f}(i,i)|_{t=0}) - \ln(p_f(i)-p_{f|f}(i,i)|_{t=0})$ is a constant. From Eq. (41), we see that $\lim_{t\to+\infty} p_{f|f}(i,i) = p_f(i)$. Substituting Eq. (41) into Eq. (39), we obtain:

$$\dot{p}_{f|f}(l,i) = \frac{1}{N}(p_f(l) - p_{f|f}(l,i)) \left[\frac{e^{\frac{4t}{N}+C_i}}{e^{\frac{4t}{N}+C_i} - 1} + \frac{e^{\frac{4t}{N}+C_l}}{e^{\frac{4t}{N}+C_l} - 1}\right]. \quad (42)$$

Hence, by solving for $p_{f|f}(l,i)$, we have:

$$\ln\left|p_f(l) - p_{f|f}(l,i)\right| - \ln\left|p_f(l) - p_{f|f}(l,i)\big|_{t=0}\right| + \frac{2t}{N} = -\frac{1}{N}\int_0^t \left(\frac{1}{e^{\frac{4\sigma}{N}+C_i} - 1} + \frac{1}{e^{\frac{4\sigma}{N}+C_l} - 1}\right) d\sigma. \quad (43)$$

The R.H.S. of Eq. (43) is clearly a bounded quantity as t goes to infinity. Hence, from the L.H.S., we observe that $\ln\left|p_f(l) - p_{f|f}(l,i)\right| \to -\infty$ as $t \to +\infty$. In other words, $\lim_{t\to+\infty} p_{f|f}(l,i) = p_f(l), \forall l \neq i$. We summarize the results obtained for the evolutionary dynamics in the known user type model as the following theorem, Theorem 3.

Theorem 3: In the known user type model, the population dynamics $p_f(i)$ are given in Eq. (33) while the relationship dynamics $p_{ff}(i,l)$ are given in Eq. (35) (for $i \neq l$) and Eq. (36) (for $i = l$).

The population dynamics evolve at a much slower speed than the influence dynamics and the relationship dynamics. In a small time period such that the population states $p_f(\cdot)$ remain constants, the influence dynamics $p_{f|f}(l,i)$ are given by Eq. (39) (for any l, i). In such a small time period, each influence state $p_{f|f}(l,i)$ will converge to the corresponding fixed population state $p_f(l)$.

According to Theorem 3, since the influence state will keep track of the corresponding population state, we can make the approximation that $p_{f|f}(l,i) = p_f(l), \forall l, i$. Thus, the population dynamics can be further simplified into the following form.

Corollary 1: In the known user type model, the population dynamics $p_f(i)$ for each type $i = 1, 2$, are (approximately) given by:

$$\dot{p}_f(i) = \frac{\alpha\overline{k}}{N}p_f(i)p_n(i)\sum_{j=1}^{M} q(j)[p_f(j)(u_{ff}(i,j) - u_{nf}(i,j)) + p_n(j)(u_{fn}(i,j) - u_{nn}(i,j))]. \quad (44)$$

V. Experiments

In this section, we implement synthetic data as well as real data experiments to verify the theoretical results on information diffusion dynamics and ESSs. First, using synthetic data, we show that the simulations match the theoretical findings well. Then, using real data, we find that the theoretical dynamics also fit the real-world information diffusion dynamics well and can even make predictions for the future diffusion dynamics.

A. Synthetic Data Experiments

In this subsection, we conduct simulations to validate the theoretical evolutionary dynamics and ESSs. We set $M = 2$, i.e., the network consists of two types of users. We synthesize a constant degree network, i.e., all the nodes have the same degree (k is a deterministic constant). We first consider the unknown user type model. The payoff parameters of the two types of players are set as following: $u_{ff}(1) = 0.4, u_{ff}(2) = 0.2, u_{fn}(1) = 0.6, u_{fn}(2) = 0.4, u_{nn}(1) = 0.3, u_{nn}(2) = 0.5$. Other parameters are $N = 1000, k = 20, q(1) = q(2) = 0.5, \alpha = 0.05$. The result is reported in Fig 2. The theoretical dynamics match the simulation dynamics well and the theoretical ESSs are near the simulated ESSs with average relative ESS error[2] 3.54%. If we model the heterogeneous network as a homogeneous one like in [26], [27], i.e., all the payoffs are set to be the average over all types, then the average relative ESS error is 6.83%, indicating the advantage of the proposed heterogeneous model. In addition, we simulate the evolutionary dynamics under another utility parameter setup in Fig. 3 and observe that the simulated dynamics still match well with the theoretical ones. Furthermore, to manifest the extreme ESSs highlighted in Theorem 2, i.e., ESSs of 0 and 1, we alter the utility parameters to simulate and the results are shown in Fig. 4, where population dynamics

[2]The average relative ESS error is calculated as follows. We denote these two simulated ESSs (for two different types, respectively) as x_1 and x_2. We denote the two theoretical ESSs as y_1 and y_2. Then the average relative ESS error is $\frac{1}{2}(|y_1 - x_1|/x_1 + |y_2 - x_2|/x_2)$. If we use homogeneous network to model, we only have one global theoretical ESS z. In such a case, the average relative ESS error is computed as $\frac{1}{2}(|z - x_1|/x_1 + |z - x_2|/x_2)$.

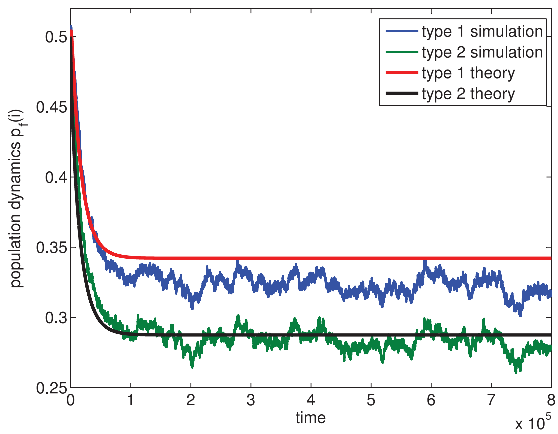

Fig. 2. Simulation results of the evolution dynamics for the unknown user type model. The theoretical dynamics fit the simulation dynamics well and the ESSs are predicted accurately. The average relative ESS error of the heterogeneous model is 3.54%. If we model the entire network as a homogeneous one as in [26], [27], the average relative ESS error becomes 6.83%, indicating the advantage of the heterogeneous model in this paper.

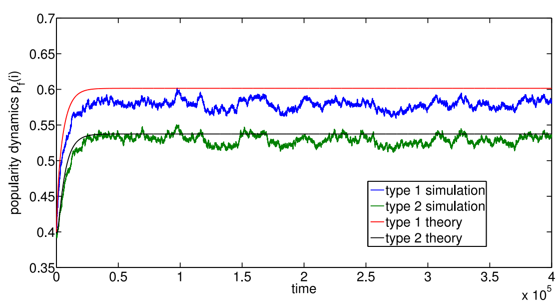

Fig. 3. Simulation results of evolution dynamics for the unknown user type model with another utility parameter setup: $u_{ff}(1) = 0.5$, $u_{ff}(2) = 0.1$, $u_{fn}(1) = 0.8, u_{fn}(2) = 0.5, u_{nn}(1) = 0.1, u_{nn}(2) = 0.3$. We observe that the simulated dynamics still match well with the theoretical ones.

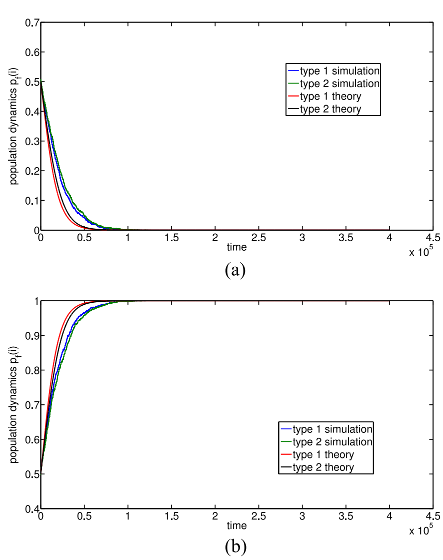

Fig. 4. Simulations for unknown user type model: population dynamics wit ESSs of 0 and 1, respectively. In (a), the utility parameters are: $u_{ff}(1) = 0.4$, $u_{ff}(2) = 0.2, u_{fn}(1) = 0.3, u_{fn}(2) = 0.5, u_{nn}(1) = 0.6, u_{nn}(2) = 0.4$. In (b), the utility parameters are: $u_{ff}(1) = 0.6, u_{ff}(2) = 0.4, u_{fn}(1) = 0.3$, $u_{fn}(2) = 0.5$, $u_{nn}(1) = 0.4$, $u_{nn}(2) = 0.2$.

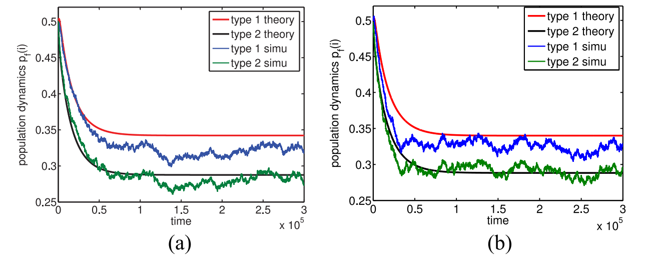

Fig. 5. More simulations of the evolutionary dynamics for the unknown user type model with different networks. (a) Erdos-Renyi network. (b) Barabasi-Albert network.

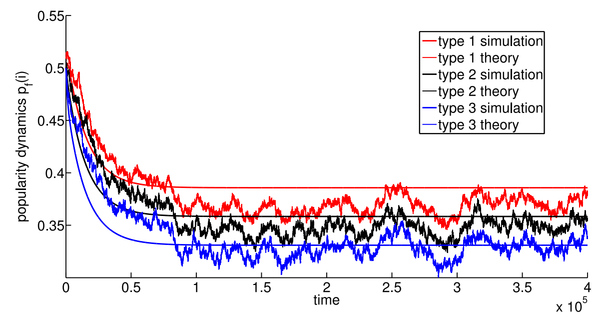

Fig. 6. Simulation results for unknown user type model with three types of users. We observe that the theoretical dynamics still match well with the simulated ones.

with ESSs of 0 and 1 are exhibited, respectively. We observe that the theoretical dynamics again match well with the simulated ones. Simulation results for Erdos-Renyi network [36] and Barabasi-Albert network [37] with the same parameter setup are shown in Fig. 5-(a), (b) respectively. The population dynamics is very similar to that of the constant degree network, and the theoretical dynamics still fit the simulated one well. In Fig. 6, we simulate the information diffusion of a heterogeneous network with three types of users. We observe that the theoretical dynamics still match well with the simulated ones. All of the above results demonstrate the effectiveness and accuracy of the proposed heterogeneous network theory.

Next, we implement a simulation for the known user type model with payoff parameters randomly chosen as follows:

$$u_{ff} = \begin{bmatrix} 0.5882 & 0.0116 \\ 0.8688 & 0.1590 \end{bmatrix}, u_{fn} = \begin{bmatrix} 0.9619 & 0.7370 \\ 0.5595 & 0.7180 \end{bmatrix},$$
$$u_{nf} = \begin{bmatrix} 0.9339 & 0.9864 \\ 0.3288 & 0.4593 \end{bmatrix}, u_{nn} = \begin{bmatrix} 0.2479 & 0.3385 \\ 0.6570 & 0.2437 \end{bmatrix}. \tag{45}$$

The other parameters are $N = 1000, k = 20, q(1) = 0.5518, q(2) = 0.4482, \alpha = 0.05$. The simulated and theoretical population dynamics are shown in Fig. 7, where the known user type model based theoretical dynamics and the simulated dynamics match well. In Fig. 7, we also plot the evolutionary dynamics given by the theory of the unknown user type model. This does not match the simulated evolutionary dynamics under the known user type model, indicating the necessity of the theory of the known user type model. Simulations under two different

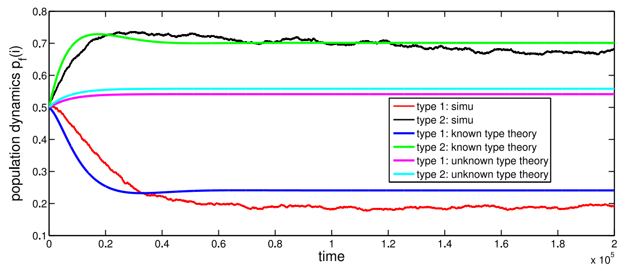

Fig. 7. Simulation of evolutionary dynamics: the known user type model.

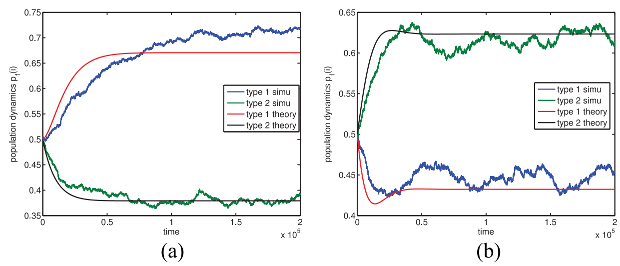

Fig. 8. Known user type model: more simulations of the evolutionary dynamics with different parameter setups. (a) Parameter setup 1. (b) Parameter setup 2.

parameter setups are shown in Fig. 8, where the theoretical dynamics and the simulated dynamics match. In Fig. 8-(a), the utility parameters are set as follows:

$$u_{ff} = \begin{bmatrix} 0.4228 & 0.1052 \\ 0.9184 & 0.5182 \end{bmatrix}, u_{fn} = \begin{bmatrix} 0.9641 & 0.9865 \\ 0.3008 & 0.7058 \end{bmatrix},$$
$$u_{nf} = \begin{bmatrix} 0.7453 & 0.7104 \\ 0.8943 & 0.9505 \end{bmatrix}, u_{nn} = \begin{bmatrix} 0.3199 & 0.6119 \\ 0.3162 & 0.4556 \end{bmatrix}. \tag{46}$$

And in Fig. 8-(b), the utility parameters are set as follows:

$$u_{ff} = \begin{bmatrix} 0.6673 & 0.1855 \\ 0.0703 & 0.2549 \end{bmatrix}, u_{fn} = \begin{bmatrix} 0.7964 & 0.1144 \\ 0.9288 & 0.9262 \end{bmatrix},$$
$$u_{nf} = \begin{bmatrix} 0.7979 & 0.1071 \\ 0.8047 & 0.4854 \end{bmatrix}, u_{nn} = \begin{bmatrix} 0.2721 & 0.7794 \\ 0.7564 & 0.0574 \end{bmatrix}. \tag{47}$$

In Fig. 8-(b), we observe some oscillations of the simulated dynamics. The reason may be that the number of parameters in the known user type model is relatively large and the strategy update rule is more complicated than the unknown user type model, which may lead to unstable behaviors of the users.

B. Real Data Experiments

In this subsection, we use the Twitter hashtag dataset in [7] to validate the theory. The dataset, comprising sequences of adopters and timestamps for the observed hashtags, is based on sampled public tweets from March 24, 2012 to April 25, 2012. To characterize the heterogeneity of the users, we classify the users into two types. The classification is based on the users' activity. Specifically, we compute the number of hashtags each user has mentioned. Then, the top 10% users with highest number of hashtag mentioning are categorized as Type-1 users while the remaining users are categorized as Type-2 ones. After classification, the number of type-1 users is 62757 while that of type-2 users is 533262. We set k to be 100, a typical number of neighbors/friends in social networks. Since the dataset does not contain the network structure of the users, we postulate the network to be a constant degree network where each user has the same degree $k = 100$. The selection strength α is not important in the curve fitting/prediction process, since it can be absorbed into the payoff parameters as it always multiplies with all the payoff parameters. In our dataset, the physical unit of time indices is not specified. In the following experiments, we choose appropriate time slot length so that (i) the data dynamics are smooth (so the time slot length cannot be too small), (ii) the data dynamics vary continuously and can correctly reflect the variation of the diffusion dynamics of real data (so the time slot length cannot be too large).

We first fit the theoretical dynamics for the unknown user type model in Eq. (14) and Eq. (15) with the real data. We use the real data to estimate the parameters (i.e., $\Delta(i)$ and $\Delta_n(i)$) in Eq. (14) and Eq. (15), and then calculate the theoretical dynamics based on the estimated parameters. We invoke the Matlab function `lsqcurvefit` to implement the curve fitting, or in other words, to estimate the payoff parameters. The parameter estimation process is built inside this MATLAB function. Given data and a function to be fit, `lsqcurvefit` selects the optimal parameters in order to minimize the squared fitting error. The fitting results for four popular hashtags are reported in Fig. 9. Type-1 users are more active than type-2 users since the population state $p_f(1)$ is always larger than $p_f(2)$. We observe that the proposed theoretical dynamics fit the real-world information diffusion dynamics well, indicating the effectiveness of taking the heterogeneous users' interactions and decision making into account. In the curve fitting of the dynamics of the hashtag #ThoughtsDuringSchool, the utility parameters are estimated to satisfy: $u_{ff}(1) - u_{fn}(1) = -3.32$, $u_{nn}(1) - u_{fn}(1) = -0.578$, $u_{ff}(2) - u_{fn}(2) = -0.64, u_{nn}(2) - u_{fn}(2) = -0.004$. From these relationships, we see that for real-world information diffusion data, the estimated utility parameters satisfy the condition $\bar{u}_{fn} > \max\{\bar{u}_{ff}, \bar{u}_{nn}\}$. From Theorem 2, we see that this condition leads to an ESS between 0 and 1, which is clearly the case in most real-world applications. In the previous subsection on simulations, the utility parameters are also chosen in compliance with this condition (e.g., Fig. 3 and Fig. 4) and are hence justified by the real data. Furthermore, we see that $u_{nn}(1)$ is much smaller than $u_{fn}(1)$ while $u_{nn}(2)$ is basically the same as $u_{fn}(2)$. To some extent, this explains why Type-1 users are more active than Type-2 users. Furthermore, we fit two less popular hashtags #ididnttextback and #imhappywhen (with peak mention counts about 1/6 of that of the hashtag #ThougtsDuringSchool). The results are reported in Fig. 10 from which we observe that the fitting is still accurate though the data become more noisy as these two hashtags are less popular, indicating the robustness of our approach.

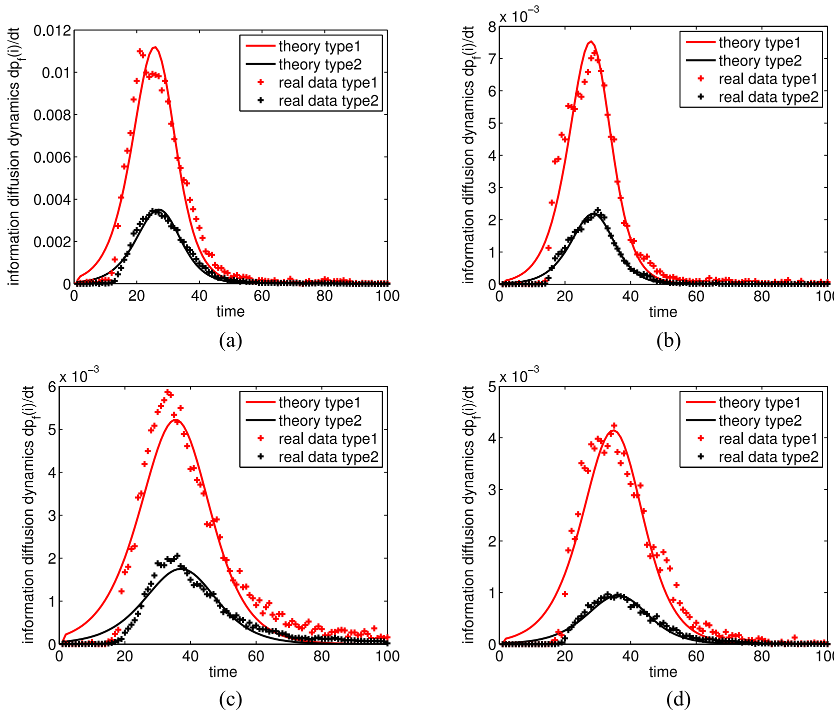

Fig. 9. Fitting results for the unknown user type model. Type-1 users are always more active than type-2 users because $p_f(1)$ is always larger than $p_f(2)$. The proposed theoretical dynamics fit the information diffusion dynamics of the real-world heterogeneous social networks well, which validates the effectiveness of considering the individuals' interactions. The theory suggests that the heterogeneous behavior dynamics of online users are consequences of their heterogeneous payoff structures. (a) #ThoughtsDuringSchool. (b) #WhenIwasLittle. (c) #DearOOMF. (d) #YouGetMajorPointsIf.

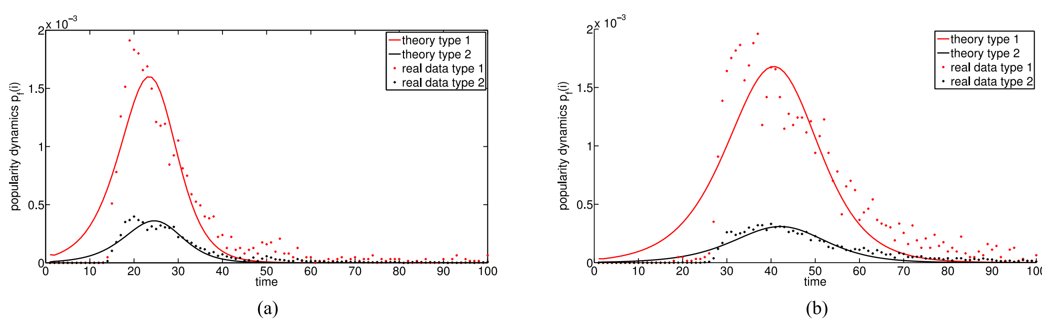

Fig. 10. Fitting results for the unknown user type model. Two less popular hashtags, #ididnttextback and #imhappywhen, are fitted. The fitting is still accurate though the data become more noisy as these two hashtags are less popular. (a) #ididnttextback. (b) #imhappywhen.

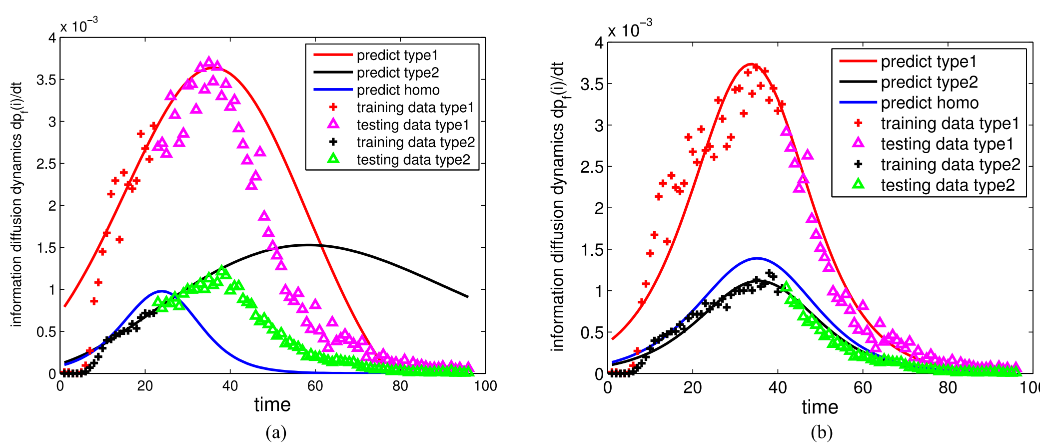

Fig. 11. Predictions. The heterogeneous game-theoretic model can predict future diffusion dynamics. The predictions made by the heterogeneous model outperforms that of the homogeneous one in [27]. (a) Using data up to time 22. (b) Using data up to time 41.

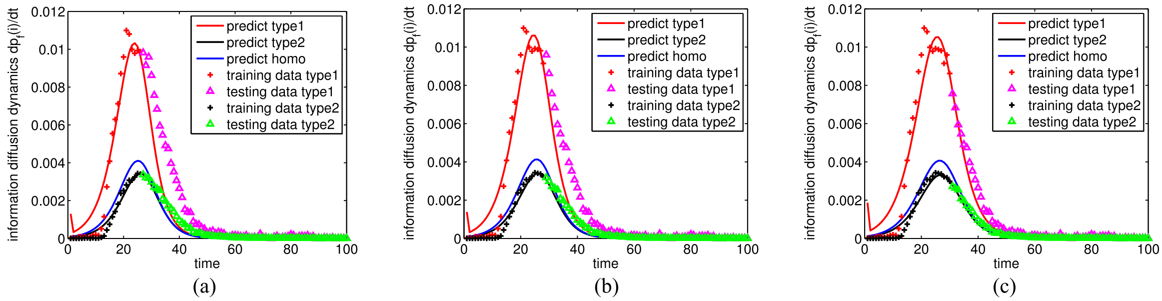

Fig. 12. Predictions for Twitter hashtag #ThoughtsDuringSchool. (a) Using data up to time 26. (b) Using data up to time 28. (c) Using data up to time 30.

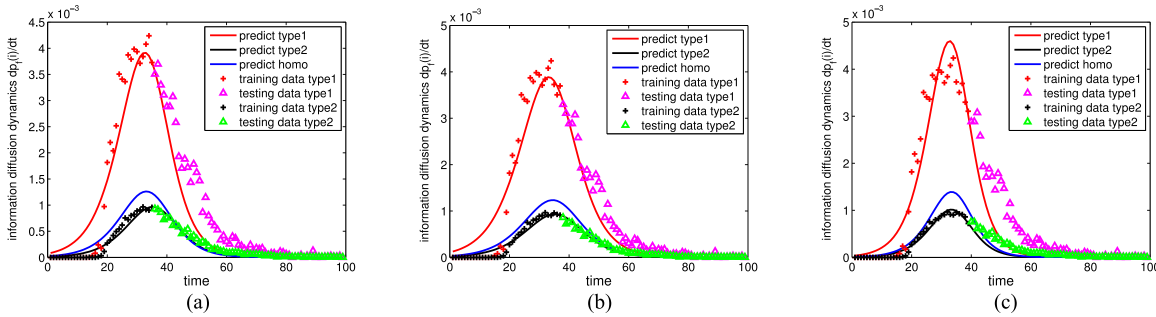

Fig. 13. Predictions for Twitter hashtag #YouGetMajorPointsIf. (a) Using data up to time 36. (b) Using data up to time 38. (c) Using data up to time 40.

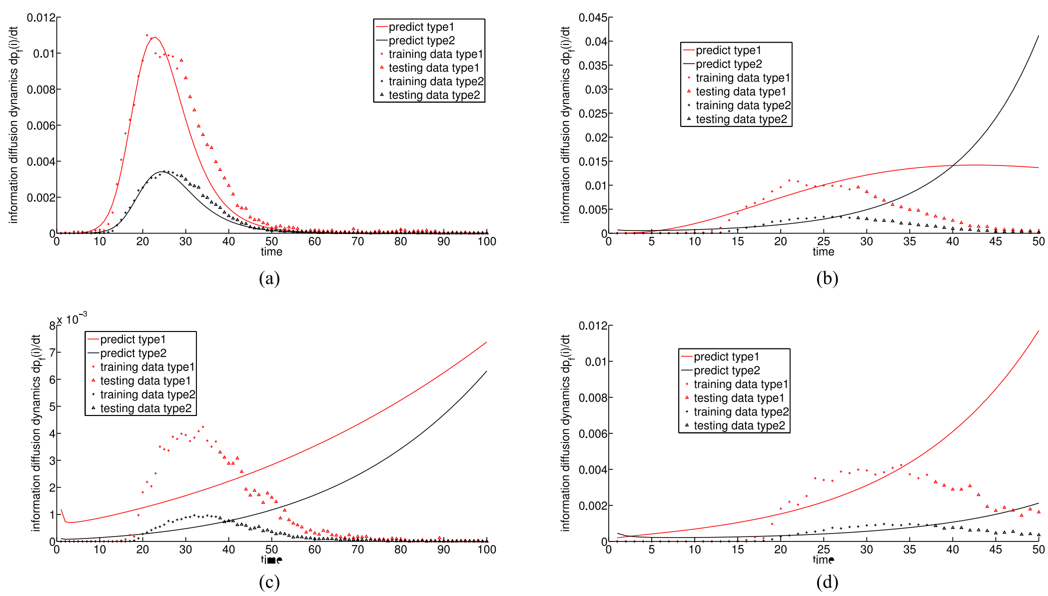

Fig. 14. Prediction results of [40] and [3]. Comparisons subfigures (a)(b) with Fig. 12-(b) and subfigures (c)(d) with Fig. 13-(b) highlight the advantage of the proposed game-theoretic approach. In particular, the results in subfigures (b)(c)(d) fail to give meaningful predictions. (a) [40], #ThoughtsDuringSchool, using data up to time 28. (b) [3], #ThoughtsDuringSchool, using data up to time 28. (c) [40], #YouGetMajorPointsIf, using data up to time 38. (d) [3], #YouGetMajorPointsIf, using data up to time 38.

In addition, we conduct experiments on the prediction of future diffusion dynamics. Specifically, we only use part of the data to train the payoff parameters in Eqs. (14), Eq. (15), and use the trained parameters to predict future diffusion dynamics. To compare with the homogeneous model in [26], [27], we also model the heterogeneous network as a homogeneous one and use the homogeneous network theory in [27] to make predictions, which serve as benchmarks. The prediction results for one popular hashtag #WhenIwasLittle are shown in Fig. 11. Two different training data lengths are investigated. The heterogeneous game-theoretic model can predict the future diffusion dynamics well. In contrast, by modeling the network as a homogeneous one, the prediction does not match the real data well, especially for type-1 users. The reason is that the prediction made by the homogeneous model can be thought of as a prediction of the overall diffusion dynamics averaged over the two types. But, type-1 users are active minority (10% of all the users). So, its diffusion dynamic is far from the average one and is poorly predicted.

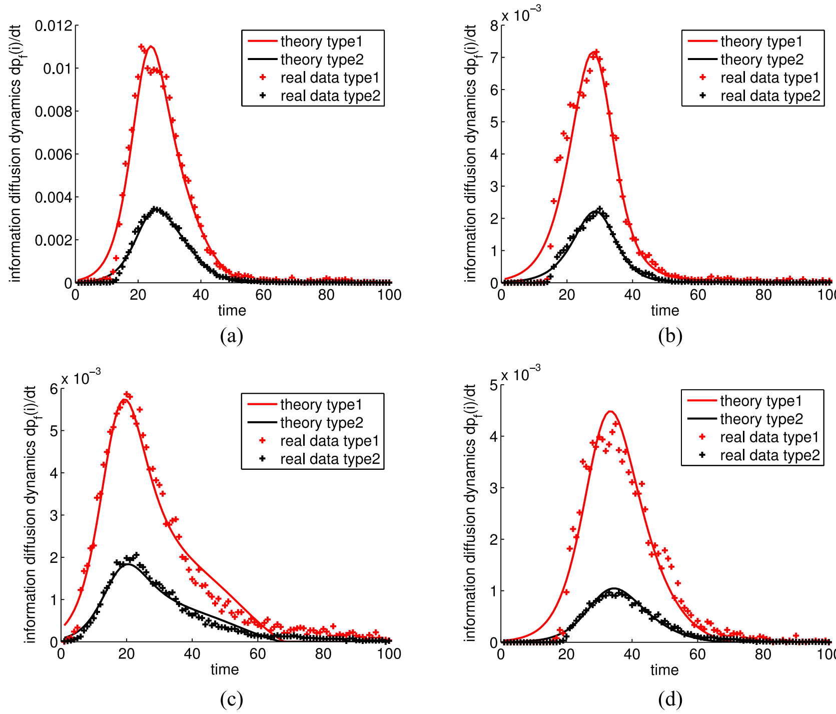

Fig. 15. Fitting results of the known user type model for the four popular Twitter hashtags. (a) #ThoughtsDuringSchool. (b) #WhenIwasLittle. (c) #DearOOMF. (d) #YouGetMajorPointsIf.

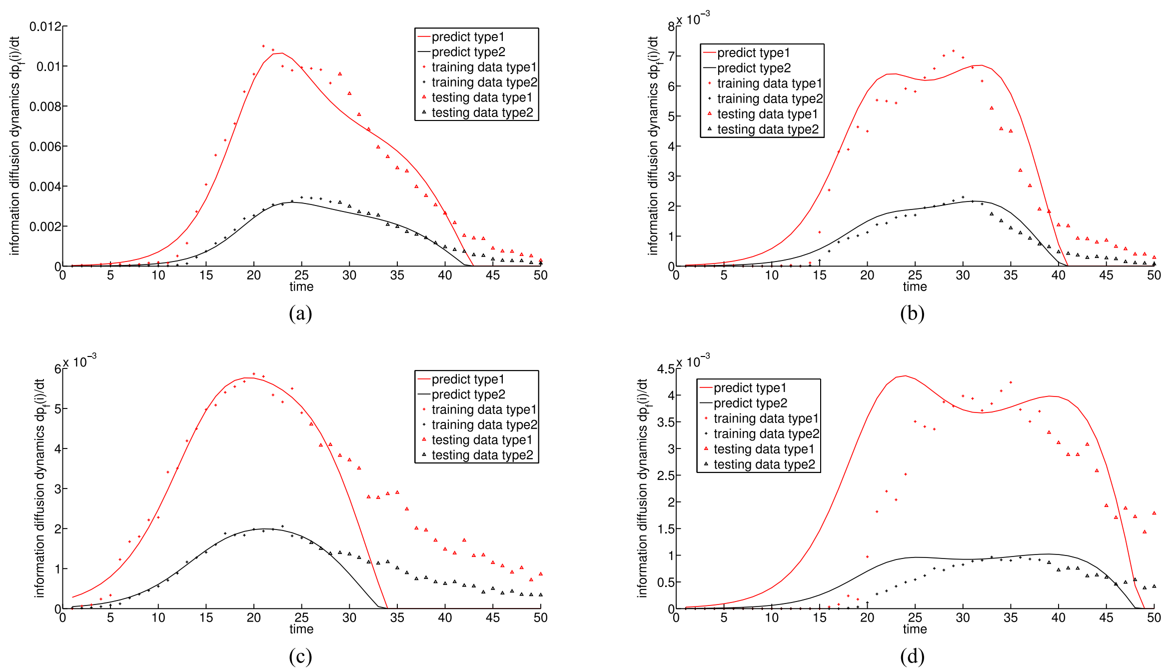

Fig. 16. Known user type model: prediction results for various Twitter hashtags. The prediction performance of the known user type model is not stable. Sometimes, it is accurate (subfigures (a) and (b)) while sometimes not (subfigures (c) and (d)). (a) #ThoughtsDuringSchool. (b) #WhenIwasLittle. (c) #DearOOMF. (d) #YouGetMajorPointsIf.

The prediction results of two other Twitter hashtags #ThoughtsDuringSchool and #YouGetMajorPointsIf are shown in Fig. 12 and Fig. 13, respectively. For both hashtags, the prediction performance of our heterogeneous model is good. In addition, we perform predictions for future 10 time slots immediately after the peak of the diffusion dynamics is observed for the 8 most popular hashtags in the dataset. The average relative error of the heterogeneous game model is 23% while that of the homogeneous game model in [27] is 47%. Furthermore, prediction results of the existing methods in [40] and [3] are reported in

Fig. 14. Comparison with the corresponding prediction results of the proposed approach in Fig. 12-(b) and Fig. 13-(b) demonstrate the advantage of the proposed game-theoretic approach.

Lastly, we fit the theoretical dynamics of the known user type model with the real data of the four popular Twitter hashtags. As shown in Fig. 15, the theoretical dynamics fit the real data well. However, the prediction performance of the known user type model is not stable, as shown in Fig. 16. The reason may be that the known user type model involves more parameters and the observed data quality is not high enough to estimate them accurately.

VI. Discussion and Conclusion

From the real data experiments, we see that sometimes the known user type model cannot predict the future dynamics of information diffusion well. We ascribe this to the quality of the data, i.e., the time resolution of the data is not good enough or equivalently the data is not smooth enough when we narrow the time window, since the known user type model involves more parameters than the unknown user type model and needs better data to estimate all the parameters accurately. Another reason is that unlike Facebook, in Twitter network (from which the data are collected), users often follow celebrities rather than acquaintances, which implies that Twitter users may not know their friends' types very well. Hence, the known user type model may not fit the Twitter network well. But, in the corresponding simulations, since the setup is just the known user type model, the theoretical dynamics still match the simulated ones well, demonstrating the theory itself is accurate.

Overall, we present an evolutionary game-theoretic framework to analyze the information diffusion over the heterogeneous social networks. The theoretical results fit and predict the information diffusion data generated by real-world social networks well, confirming the effectiveness of the heterogeneous game-theoretic modeling approach. The derived evolutionary dynamics can be absorbed to improve the state-of-art machine learning based method in the literature of information diffusion. More importantly, with a few parameters, our model gives a game-theoretic interpretation to the mechanism of the individuals' decision-making in the information diffusion process over the heterogeneous social networks.

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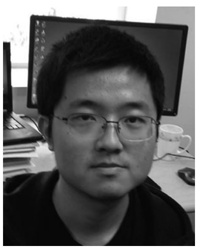

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