Cognitive Radio Networks With Heterogeneous Users: How to Procure and Price the Spectrum?

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***Abstract*—In this paper, we investigate the optimal spectrum procurement and pricing from the perspective of a cognitive mobile virtual network operator (C-MVNO), which is a second market between the spectrum owner and the secondary users (SUs). The spectrum procurement consists of spectrum leasing and spectrum sensing, where the latter has an uncertain outcome. The SUs are assumed to be heterogeneous in their valuations and demands of the spectrum, which is generally the case in reality. Hence, we use differentiated pricing among the heterogeneous SUs to improve the profit of the C-MVNO and allow the C-MVNO to perform necessary admission control. Modeling the spectrum procurement and trading procedure as a five-stage Stackelberg game, we analyze the optimal decisions for the C-MVNO by using backward induction. The optimal decisions of spectrum sensing, spectrum leasing, admission control, and differentiated pricing are derived, and an algorithm is proposed to compute those optimal decisions efficiently. Our theoretical results are also corroborated by numerical experiments, and a threshold structure of the solution is observed.**

***Index Terms*—Differentiated pricing, heterogeneous SUs, spectrum trading, Stackelberg game.**

I. Introduction

Wireless spectrum is becoming more and more scarce nowadays due to the fast growing demand of wireless services. The traditional "Command-and-Control" regulation based spectrum allocation is considered as an inefficient way of exploiting the spectrum resource since much idle spectrum is wasted. Confronting with this problem, the Federal Communications Commission (FCC) proposes to use "Exclusive Use" and "Commons" models to enhance the spectrum efficiency [1]. In the "Exclusive use" model, the licensed users [primary users (PUs)] can use or transfer the spectrum exclusively. In the "Commons" model, unlicensed users [secondary users (SUs)] can share the spectrum according to some standards. This leads to the advance of the cognitive radio (CR) technology, which is regarded as a promising paradigm of spectrum utilization.

To access and utilize the spectrum economically and efficiently, many game theoretic schemes have been proposed in the literature [2], [3]. Auction based spectrum access mechanisms are proposed in [4]–[7]. Some researchers have studied the pricing interactions between the network operator and SUs to maximize either the social welfare or the operator's profit [8]–[12]. A contract formulation of spectrum trading in CR networks (CRNs) is investigated in [13] to model the scenario where the primary owner does not know the feature (e.g., channel condition) of each individual SU and only has the knowledge of the statistical distribution of the overall features. Evolutionary game theory is invoked to investigate the spectrum sensing and access problem in [14], [15]. An indirect reciprocity game modeling approach is studied in [16], [17]. In addition, learning and negative network externality are considered in [18] while renewal-theoretical spectrum access is investigated in [19].

In general, a cognitive mobile virtual network operator (C-MVNO) will serve as a second market between the spectrum owner and the SUs. From a network operator's perspective, it not only needs to sell spectrum to the SUs to make profit but also need to purchase spectrum from the spectrum owner or sense the idle spectrum unused by the PUs so as to get enough amount of spectrum. Generally, the spectrum sensing is much cheaper than directly leasing spectrum from the spectrum owner. Nonetheless, the amount of the sensed spectrum is uncertain due to the stochastic behaviors of the PUs while the amount of spectrum obtained by directly leasing is deterministic and guaranteed. Thus, there is a certainty-cost tradeoff in the spectrum procurement.

So far, few papers have jointly studied the problem of spectrum procurement and pricing from the operator's perspective, e.g., [9] and [20]. However, they only consider the single pricing scheme in the homogeneous case, i.e., all the SUs have the same valuation of the spectrum and the C-MVNO sets a single price for all the homogeneous SUs. This may turn out to be an oversimplified model and pricing scheme for today's mobile networks where the users are highly heterogeneous in their demands and valuations of the spectrum. For example, some SUs may have more demands (or higher valuations) on the spectrum (e.g., they are watching videos) and prefer to pay more money to gain better QoS, while some other SUs have less demands (or lower valuations) on the spectrum (e.g., they are just phoning) and do not need very high data rate.

In this paper, for a CRN with heterogeneous SUs, we use differentiated pricing to maximize the profit of the C-MVNO, i.e., we set different prices for SUs with different valuations of the spectrum. Confronting with the heterogeneous users, differentiated pricing can potentially increase the operator's profit significantly as opposed to single pricing. User heterogeneity and differentiated pricing have been studied in existing literature [23], [24]. In [23], Li and Huang apply differentiated

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pricing for the heterogeneous users to enhance the profit of the operator while in [24], Shakkottai *et al.* study the performance loss incurred by simple single pricing schemes when the users are heterogeneous. However, neither of them considers the procurement of the resources to be allocated. In this paper, we study the scenario where the C-MVNO jointly considers the spectrum procurement and pricing when the users are heterogeneous. To the best of our knowledge, such a scenario has never been investigated by any previous work. Formulating the spectrum procurement and trading as a five-stage Stackelberg game, we jointly optimize the spectrum sensing, leasing, admission control and pricing decisions from a C-MVNO's perspective. The spectrum procurement consists of two parts: spectrum sensing and spectrum leasing. The former is cheaper but subject to uncertainty while the latter is deterministic but generally more expensive and we want to find the best tradeoff between the uncertainty and the costs. After spectrum procurement, we allow the C-MVNO to perform admission control to select a subset of SUs to serve. By doing so, we assure that the heterogeneous demands of all the SUs admitted by the C-MVNO are satisfied. Then, the C-MVNO sets the optimal differentiated prices for the heterogeneous SUs to maximize its profit. At last, based on the prices announced by the C-MVNO, each admitted SU purchases an appropriate amount of spectrum so as to maximize its own utility. The main contributions of this paper are summarized as follows.

- We model the spectrum procurement and trading process as a five-stage Stackelberg game. Due to the heterogeneity of the SUs, price differentiation is introduced to improve the profit of C-MVNO as opposed to the single pricing scheme for the homogeneous user case. Admission control is also allowed to balance the spectrum supply and demand.
- Using backward induction, we derive the optimal decisions of spectrum sensing, spectrum leasing, admission control and differentiated pricing of the C-MVNO as the equilibrium of the formulated Stackelberg game. When the SUs are heterogeneous, no closed-form solution of the pricing scheme is available, which makes our analysis more challenging compared to the homogeneous-SU case. Fortunately, we can still design a simple low-complexity algorithm to calculate the optimal decisions efficiently.
- The optimality of the proposed algorithm is validated through numerical simulations. Several threshold structures of the obtained optimal solution are observed. Simulations indicate that, when the SUs are heterogeneous, our proposed differentiated pricing based scheme significantly outperforms the single pricing based scheme of prior works.

The rest of this paper is organized as follows. In Section II, we introduce the system model and formulate the problem as a Stackelberg game. In Section III, we analyze the game model using backward induction and derives the optimal decisions of the C-MVNO. In Section IV, simulation results are presented and we conclude this paper in Section V.

II. System Model

As shown in Fig. 1, we consider a system with one C-MVNO and multiple heterogeneous secondary users (SUs).

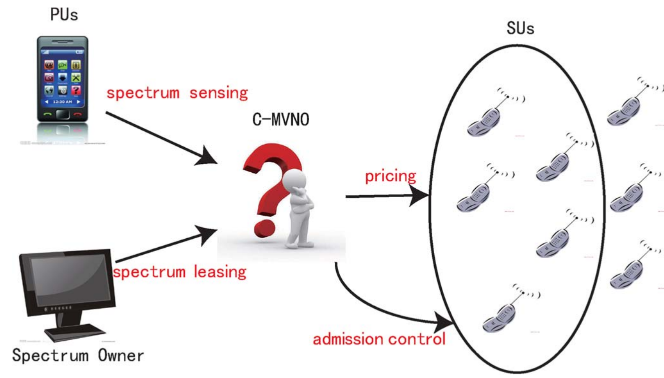

Fig. 1. Illustration of the system model.

TABLE I
Key Notations

Notations	Definitions
B_s, B_l	Sensing bandwidth and leasing bandwidth
C_s, C_l	sensing cost and leasing cost per bandwidth
α	Sensing realization factor
I	Number of different values for the willingness-to-pay θ
$\mathcal{S}_i$	The index set of the set of SUs with willingness-to-pay θ_i
$\widetilde{\mathcal{S}_i}$	The admitted SUs in $\mathcal{S}_i$
p_i	The differentiated price for SUs in $\mathcal{S}_i$
w_i	The bandwidth allocation of each SU in $\mathcal{S}_i$
g_{ij}	The wireless characteristic of the j-th SU in $\mathcal{S}_i$
G_i ($\widetilde{G_i}$)	The aggregate wireless characteristics of the SUs in $\mathcal{S}_i$ ($\widetilde{\mathcal{S}_i}$)

The objective of the C-MVNO is to collect spectrum and sell the spectrum to SUs to maximize its profit. Specifically, the C-MVNO collects spectrum through performing spectrum sensing for unused primary spectrum and leasing spectrum from spectrum owners. Since there exists uncertainty in spectrum sensing, the amount of leased spectrum depends on the outcome of spectrum sensing. After collecting the spectrum, the C-MVNO can choose SUs for selling spectrum by admission control. Since the SUs are heterogeneous, which means that they have different demands of the spectrum, the prices to different SUs are different, i.e., differentiated pricing is used. We assume that all SUs are rational and thus naturally selfish, due to which they will purchase the optimal amount of spectrum from the C-MVNO to maximize their own utility function based on the differentiated price announced by the C-MVNO. The challenge of this problem, which is the focus on this paper, is how the C-MVNO makes the optimal decisions including spectrum sensing, spectrum leasing, admission control, and differentiated pricing. The key notations of this paper are listed in Table I. In what follows, we present our system model in detail.

A. SU's Model

We assume that each SU has its **willingness-to-pay** parameter θ. This positive parameter is used to model the data rate valuation/demand of a SU: the larger the θ, the higher the valuation/demand of the SU. For instance, a SU who is watching video needs more data rate and its valuation of the data rate is higher (because higher data rate can improve the resolution or speed of the video a lot), as compared to a SU who is phoning. Hence, the willingness-to-pay of a video watching SU is larger than that of a phoning SU.

Consider a SU with a willingness-to-pay parameter θ. Let w be the bandwidth allocated to the SU and p be the unit price of the bandwidth. Then, the utility function of the SU can be written as:

$$u(p,w)=\theta w\ln\left(1+\frac{P^{\max}h}{n_0 w}\right)-pw=\theta w\ln\left(1+\frac{g}{w}\right)-pw, \quad (1)$$

where $P^{\max}$ is the maximal transmission power, h is the channel gain, n_0 is the noise power density, $g=\frac{P^{\max}h}{n_0}$ is the received SNR (when the bandwidth is one unit), which can be treated as the **wireless characteristic** of the SU, and $w\ln(1+g/w)$ is the achievable rate of the SU [2].

From (1), we can see that the two parameters (θ, g) can fully characterize a SU. We assume that each SU only knows its own (θ, g) and has no knowledge about others'. Similar to [20], we focus on the high SNR regime where $\text{SNR}=g/w\gg 1$.[1] In such a case, the utility function in (1) can be approximated as

$$u(p,w)=\theta w\ln\left(\frac{g}{w}\right)-pw. \quad (2)$$

In this paper, we assume that there are I possible willingness-to-pay parameters, i.e., $\theta\in\{\theta_1,\theta_2,\ldots,\theta_I\}$, where each θ_i represents a different wireless services such as video streaming and website browsing. Let $\mathcal{S}_i$ be the index set of the set of SUs with the same θ_i, and g_{ij} be the wireless characteristic of j-th SU in $\mathcal{S}_i$.

We note that the model for the SUs is somewhat simplified. In practice, due to location limitations, SUs generally can only access the spectrum in a certain band range but not any spectrum provided by the operator. In addition, since spatial reuse of the spectrum is feasible in many scenarios, the C-MVNO may sell the same spectrum to multiple SUs as long as they are sufficiently distant away from each other. However, our model still captures many essential aspects of the spectrum trading process such as users' utility function, spectrum leasing and sensing of the operator. More importantly, its mathematical tractability allows us to perform theoretical analysis and thus to gain some insights of the spectrum trading procedure, which can be applied to the practice in turn.

B. C-MVNO's Model

As discussed above, the decisions of the C-MVNO include spectrum sensing, spectrum leasing, admission control and differentiated pricing. In the following, we discuss them in details one by one.

1) Spectrum Sensing: Let B_s be the bandwidth that the C-MVNO senses. Due to the stochastic nature of PUs' behaviors, the amount of unused primary spectrum that is available for the C-MVNO is uncertain. Let $\alpha\in[0,1]$ be the random variable standing for the portion of unused primary spectrum. Then, the amount of spectrum C-MVNO can obtain through spectrum sensing is αB_s. In this paper, we assume that α is uniformly distributed within the interval [0, 1]. Nevertheless, similar analysis can be conducted with other distributions. Note that there is a certain cost for the C-MVNO to perform sensing. Let C_s be the sensing cost per unit bandwidth. Then, by sensing bandwidth B_s, the C-MVNO can obtain unused spectrum αB_s at the cost of $C_s B_s$.

2) Spectrum Leasing: Since the spectrum obtained through sensing may not be enough, the C-MVNO may need to lease more bandwidth from the spectrum owner. Let B_l be the amount of leased spectrum, and C_l be the unit leasing cost. Then the total leasing cost is $C_l B_l$. In general, the leasing cost C_l is larger than the sensing cost C_s. Here, we have implicitly assumed that the C-MVNO first performs spectrum sensing and then performs spectrum leasing whenever needed. This assumption is reasonable. Actually, the practical spectrum owner often has some "Transference Band" which it does not use temporarily. The spectrum owner is thus willing to lease this band to other operators for temporary usage. As for the C-MVNO, the realized sensing spectrum may not be sufficient, so it may need to lease some more spectrum to satisfy the SUs' demands after observing the spectrum sensing result. The spectrum leasing we consider here is short-term, in accordance with the changing cognitive networks: SUs may arrive and leave. When the cognitive network changes or the leased/sensed spectrum expires, the C-MVNO needs to remodel the network again.

We remark that, for ease of analysis, we assume that there are no upper bounds for B_s and B_l, i.e., the C-MVNO can lease or sense any amount of spectrum as long as it pays the costs. This is reasonable since the amount of available spectrum is generally very large in practice.

3) SU Admission Control: To achieve the best profit, the C-MVNO may perform admission control on SUs, i.e., the C-MVNO can select only a subset of the SUs to serve. Specifically, for each set $\mathcal{S}_i$, suppose the C-MVNO only serves a subset $\widetilde{\mathcal{S}}_i$. We note that the C-MVNO decides which SUs to serve before it announces prices to the SUs. When the C-MVNO is doing admission control, there is no service agreement between the C-MVNO and the SUs. Thus, the C-MVNO has the freedom to perform admission control without worrying about breaking its agreement with the SUs.

4) Differentiated Pricing: We assume that the C-MVNO knows the willingness-to-pay θ and the wireless characteristic g of each SU. The justification of this assumption is as follows. Consider an uplink system where the base station (BS) represents the operator. Then, $P^{\max}h$ is the received power at the BS when certain SU is transmitting and n_0 is the noise density. Both of these two quantities are known to the BS. Hence, the wireless characteristic $g=P^{\max}h/n_0$ of that SU is also known to the BS. Recall that the θ of a SU is related to its required wireless service type, which the C-MVNO could manage to know (e.g., by observing the data rate of the SU). Thus, the C-MVNO can give an estimation of the θ of that SU accordingly.

With the knowledge of (g, θ) for each SU, the C-MVNO can use differentiated pricing to maximize its profit. Specifically, we assume that the spectrum price is the same for SUs with the same willingness-to-pay, and generally different for SUs with different willingness-to-pay parameters. Denote p_i as the price for SUs in $\widetilde{\mathcal{S}}_i$. One may raise the following question: if the prices for different SUs with the same willingness-to-pay, i.e., SUs

[1] In real-world wireless communications, even the SNR lower bound for the "very low signal" is 10 dB, i.e., $\text{SNR}=10\gg 1$. A typical SNR with medium signal is around $1000\gg 1$.

in the same set $\widetilde{\mathcal{S}}_i$, are also different, is the optimal algorithm proposed in this paper still optimal? The answer is yes. In other words, allowing differentiated prices among SUs in the same $\widetilde{\mathcal{S}}_i$ will not change the result of this paper. Later in Remark 2, we will see this more clearly.

C. Problem Formulation

Let $w_{ij}^*(p_i)$ be the best response of the j-th SU in $\widetilde{\mathcal{S}}_i$ when the C-MVNO announces price p_i to the set $\widetilde{\mathcal{S}}_i$. Then, the optimal strategies of the C-MVNO can be derived by the following optimization problem:

$$\max_{B_s \geq 0} \quad \mathbb{E}_{\alpha \in [0,1]} F(\alpha, B_s), \tag{3}$$

where $F(\alpha, B_s)$ is defined as:

$$F(\alpha, B_s) = \max_{\forall i, p_i \geq 0, \widetilde{\mathcal{S}}_i \subseteq \mathcal{S}_i, B_l \geq 0} \left[\sum_{i=1}^{I} p_i \sum_{j \in \widetilde{\mathcal{S}}_i} w_{ij}^*(p_i) - B_s C_s - B_l C_l \right]$$
$$\text{s.t.} \quad \sum_{i=1}^{I} \sum_{j \in \widetilde{\mathcal{S}}_i} w_{ij}^*(p_i) \leq B_s \alpha + B_l. \tag{4}$$

The problem in (4) can be solved by the following three steps.

1) For fixed $B_s, \alpha, B_l, \{\widetilde{\mathcal{S}}_i\}$, find the best differentiated pricing vector $\{p_i^*\}$.
2) Substituting the expression of $\{p_i^*\}$ back into (4) and fixing B_s, α, B_l, find the best admission control scheme $\{\widetilde{\mathcal{S}}_i\}$.
3) Substituting the optimal pricing and admission control back into (4) and fixing B_s, α, find the best leasing bandwidth B_l.

Finally, with the optimal $B_l, \{\widetilde{\mathcal{S}}_i\}, \{p_i\}$, i.e., after the problem (4) is solved, we optimize the sensing bandwidth B_s in problem (3) to completely solve the profit maximization problem.

Actually, the interaction between the C-MVNO and the SUs (shown in Fig. 1) can be formulated as a five-stage Stackelberg game as illustrated in Fig. 2. The Stackelberg leader is the C-MVNO and the followers are the SUs. In the first stage, the C-MVNO determines the sensing bandwidth B_s and then realizes the available sensing result αB_s. In the second stage, based on the sensing result αB_s, the C-MVNO determines the leasing bandwidth B_l. In the third stage, the C-MVNO performs admission control to serve a subset of SUs. In the fourth stage, the C-MVNO sets the differentiated price p_i for each $\widetilde{\mathcal{S}}_i$, where $i \in \{1, \ldots, I\}$. Finally, in the fifth stage, given the prices announced by the C-MVNO, each SU buys an optimal amount of bandwidth so as to maximize its own utility. Notice that the middle three stages can be merged into one single stage without influencing the problem essentially. The first stage, spectrum sensing, cannot be merged with them since the sensing result is uncertain and the decisions in the middle three stages are made after observing the realized sensing result.

III. Backward Induction Analysis of the Game

A general technique to find the solution (equilibrium) to the Stackelberg game is backward induction. Note that the solution to the game is the optimal choice of sensing bandwidth B_s, leasing bandwidth B_l, admission control and differentiated pricing p_i for the C-MVNO and the optimal bandwidth demand w_{ij} for each SU.

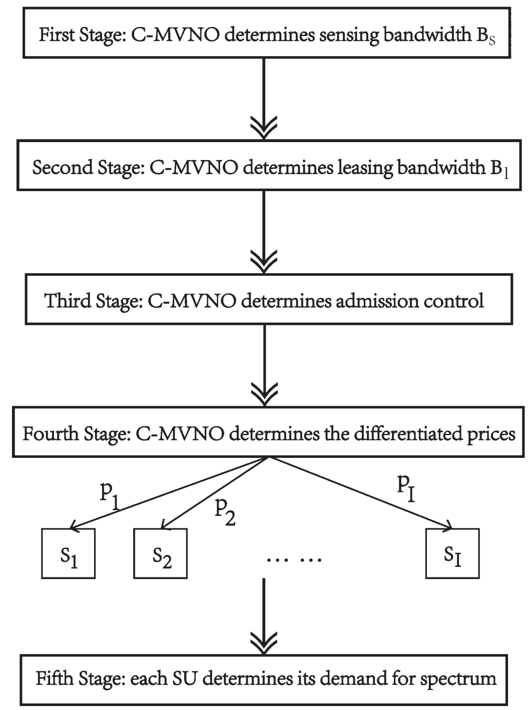

Fig. 2. A Stackelberg game formulation.

A. Spectrum Allocation in the Fifth Stage

After the C-MVNO announces its price $\{p_i\}_{1 \leq i \leq I}$ to the SUs, each SU determines its spectrum demand by maximizing its utility defined in (2). Considering the j-th SU in $\widetilde{\mathcal{S}}_i$ (recall that $\widetilde{\mathcal{S}}_i$ is a subset of $\mathcal{S}_i$ after admission control), we write its utility maximization problem as:

$$\max_{w_{ij} \geq 0} \quad u(w_{ij}) = \theta_i w_{ij} \ln\left(\frac{g_{ij}}{w_{ij}}\right) - p_i w_{ij}. \tag{5}$$

Taking derivative of (5) and setting it to be zero, we get the optimal value of w_{ij} for SU j as:

$$w_{ij}^*(p_i) = g_{ij} \exp\left\{-1 - \frac{p_i}{\theta_i}\right\}. \tag{6}$$

From (1) and (6), we can see that all the SUs in $\widetilde{\mathcal{S}}_i$ has the same SNR ($\text{SNR} = g_{ij}/w_{ij} = \exp(1 + p_i/\theta_i)$) regardless of their different wireless characteristics g. This essentially states that SUs using the same wireless service have the same SNR regardless of their different channel conditions and transmission power. Furthermore, the term $\exp\{-1 - p_i/\theta_i\}$ is the same for all SUs in the same set $\widetilde{\mathcal{S}}_i$. Hence, the aggregate bandwidth demand of SUs in $\widetilde{\mathcal{S}}_i$ is $\sum_{j \in \widetilde{\mathcal{S}}_i} w_{ij}^*(p_i) = \widetilde{G}_i \exp\{-1 - p_i/\theta_i\}$, where $\widetilde{G}_i$ is the aggregate wireless characteristics for the set $\widetilde{\mathcal{S}}_i$ defined as follows $\widetilde{G}_i = \sum_{j \in \widetilde{\mathcal{S}}_i} g_{ij}$. Similarly, we can define the aggregate wireless characteristics for the set $\mathcal{S}_i$ as $G_i = \sum_{j \in \mathcal{S}_i} g_{ij}$. Note that $\widetilde{G}_i \leq G_i$, which is a constraint for admission control.

B. Differentiated Pricing in the Fourth Stage

Based on the best response of the heterogeneous SUs in the fifth stage, the aim of the fourth stage is to maximize the C-MVNO's revenue by selling spectrum to the SUs. The differentiated pricing problem (P1) can be formulated as follows.

$$\text{(P1)} \quad \max_{\overrightarrow{p} \succeq \overrightarrow{0}} \sum_{i=1}^{I} p_i \widetilde{G}_i \exp\left\{-1 - \frac{p_i}{\theta_i}\right\} \tag{7}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-1 - \frac{p_i}{\theta_i}\right\} \leq B, \tag{8}$$

where B denotes the total available bandwidth consisting of sensing spectrum and leasing spectrum and $\overrightarrow{p}$ is the vector of $\{p_i\}_{1\leq i\leq I}$. The solution to the optimization problem (P1) is summarized in the following lemma.

Lemma 1: The solution to the optimal differentiated pricing problem (P1) is as follows.

1) If $\sum_{i=1}^{I} \widetilde{G}_i e^{-2} \leq B$, then $p_i^* = \theta_i, \forall i$ and the optimal value of (P1) is $\sum_{i=1}^{I} \theta_i \widetilde{G}_i e^{-2}$.
2) Otherwise, $p_i^* = \lambda^* + \theta_i$ and the optimal value of (P1) is:

$$\lambda^* B + \sum_{i=1}^{I} \theta_i \widetilde{G}_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\}, \tag{9}$$

where λ^* is determined as the unique solution to the following equation:

$$\sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\} = B. \tag{10}$$

Proof: See Appendix A. ■

Several remarks are in order.

Remark 1: The optimal value of the problem (P1) is the maximum revenue gained by selling the spectrum to the SUs. It is not the final profit of the C-MVNO since it does not take into account the spectrum procurement cost incurred by spectrum sensing and leasing.

Remark 2: As we have stated in Section II, in this paper, we assume that the price for different SUs with the same willingness-to-pay θ, i.e., SUs in the same set $\widetilde{\mathcal{S}}_i$, is the same. In other words, the prices are different only for SUs with different willingness-to-pay parameters. Nevertheless, the results in this paper are still valid even without this assumption, as explained below. Suppose the prices for different SUs could be different regardless of the willingness-to-pay parameters. In such a case, the optimal differentiated pricing problem (P1) can be simply modified by replacing the aggregate wireless characteristic $\widetilde{G}_i$ with individual characteristic g_{ij}, due to which the corresponding optimal pricing structure is the same as that in Lemma 1. From Lemma 1, we can see that SUs with the same willingness-to-pay θ will be given the same price. Therefore, the assumption that the price for different SUs with the same θ is the same is implicitly guaranteed.

Remark 3: From Remark 2, the optimal differentiated pricing depends only on the willingness-to-pay but not the wireless characteristics. The essential reason of this phenomenon is that the best demand response of each SU is a linear function of the wireless characteristic g_{ij} in (6). Hence, the SU can be treated as g_{ij} number of virtual SUs. Each of them has willingness-to-pay θ_i and unit wireless characteristic. In such a way, all the SUs with willingness-to-pay θ_i can be regarded as $\widetilde{G}_i$ number of virtual SUs with the same willingness-to-pay and unit wireless characteristic. Since all these virtual SUs have the same parameters, the optimal pricing for them is naturally the same. Thus, back to the real SUs, the SUs with the same willingness-to-pay parameter will receive the same optimal price, though they may have different wireless characteristics.

Remark 4: When the total available spectrum B is large, i.e., in case (1) of the lemma, there is a gap between the total demand of the users and B. In other words, in such a case, the operator will not sell out all the spectrum and some spectrum is wasted. The reason that the operator will do so is that to sell out all the spectrum, the price has to be very low which hurts the profit. This large B scenario may happen when the sensing cost C_s is small, in which case the operator may sense "redundant" spectrum to ensure that with very high probability (i.e., α is not very small), the operator does not need to lease spectrum. But when α turns out to be relatively large, the operator may have a large B even $B_l = 0$. Alternatively, if B is small, i.e., in case (2) of the lemma, all the available spectrum will be sold out.

C. Admission Control in the Third Stage

In this part, based on the results of the fourth stage and the fifth stage, we analyze the admission control decision of the C-MVNO, and the results are stated in the following lemma.

Lemma 2: The optimal admission control decision is to admit all the SUs, i.e., $\widetilde{\mathcal{S}}_i = \mathcal{S}_i, \widetilde{G}_i = G_i, \forall i$, and the optimal revenue[2] is shown as follows.

1) If $\sum_{i=1}^{I} G_i e^{-2} \leq B$, the optimal revenue is $\sum_{i=1}^{I} \theta_i G_i e^{-2}$.
2) Otherwise, the optimal revenue is given by:

$$\lambda^* B + \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\}, \tag{11}$$

where λ^* is determined by the unique solution to:

$$\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\} = B. \tag{12}$$

Proof: We prove the lemma by considering two cases.

Case 1: $\sum_{i=1}^{I} G_i e^{-2} \leq B$: In this case, for any admission control decisions we always have $\sum_{i=1}^{I} \widetilde{G}_i e^{-2} \leq B$ since $\widetilde{G}_i \leq G_i, \forall i$. Thus, according to Lemma 1, the revenue by selling the spectrum to the SUs is always $\sum_{i=1}^{I} \theta_i \widetilde{G}_i e^{-2}$, which is an increasing function in $\widetilde{G}_i, \forall i$. Hence, the optimal admission control decision should be $\widetilde{\mathcal{S}}_i = \mathcal{S}_i, \widetilde{G}_i = G_i, \forall i$. And the optimal revenue by selling spectrum in this case is: $\sum_{i=1}^{I} \theta_i G_i e^{-2}$.

Case 2: $\sum_{i=1}^{I} G_i e^{-2} > B$: We consider an arbitrary admission control decision $\{\widetilde{\mathcal{S}}_i\}_{1\leq i\leq I}$. Though the values of $\{\widetilde{G}_i\}_{1\leq i\leq I}$ are indeed discrete since each of them is the aggregate of a finite number of wireless characteristics, we still treat $\{\widetilde{G}_i\}_{1\leq i\leq I}$ as

[2] Here, by revenue, we mean the revenue gained by selling the spectrum to the SUs. It is not the overall profit which should include the spectrum procurement costs.

continuous variables here and this will not hurt the rigidity of our analysis. If $\sum_{i=1}^{I}\widetilde{G}_i e^{-2} \le B$, we can increase the revenue by increasing some $\widetilde{G}_i$ until $\sum_{i=1}^{I}\widetilde{G}_i e^{-2} = B$. Thus, we can just focus on the situation $\sum_{i=1}^{I}\widetilde{G}_i e^{-2} \ge B$. In this case, the revenue is given by (9). Note that λ^* in Lemma 1 can be viewed as an implicit function of $\{\widetilde{G}_i\}_{1\le i\le I}$. By taking the derivative of (9) with respect to $\widetilde{G}_k$, we have:

$$\frac{\partial}{\partial\widetilde{G}_k}\left(\lambda^* B+\sum_{i=1}^{I}\theta_i\widetilde{G}_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}\right)=\theta_k\exp\left\{-2-\frac{\lambda^*}{\theta_k}\right\}>0.$$

Therefore the revenue is still an increasing function of $\widetilde{G}_k, \forall 1 \le k \le I$. Therefore, the optimal admission control is still performing no admission control, i.e., $\widetilde{\mathcal{S}}_i = \mathcal{S}_i, \widetilde{G}_i = G_i, \forall i$. The optimal revenue in this case is:

$$\lambda^* B+\sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}, \tag{13}$$

where λ^* is determined by:

$$\sum_{i=1}^{I}G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}=B. \tag{14}$$

■

Remark 5: Lemma 2 essentially claims that explicit admission control is not necessary at the optimum.

D. Spectrum Leasing in the Second Stage

Denote R_2 the partial profit which is defined as the income from selling the spectrum to the SUs minus the leasing cost. We further define the following five frequently used constants:

$$A \triangleq \sum_{i=1}^{I}G_i\exp\left\{-2-\frac{C_l}{\theta_i}\right\}, D \triangleq \sum_{i=1}^{I}G_i e^{-2},$$
$$E \triangleq \sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{C_l}{\theta_i}\right\}, F \triangleq \sum_{i=1}^{I}\theta_i G_i e^{-2}$$
$$H \triangleq e^{-4}\sum_{i,j=1}^{I}\frac{G_iG_j\theta_i}{\theta_i+\theta_j}\left(\frac{\theta_i\theta_j}{\theta_i+\theta_j}-C_l\exp\left\{-C_l\frac{\theta_i+\theta_j}{\theta_i\theta_j}\right\}-\frac{\theta_i\theta_j}{\theta_i+\theta_j}\exp\left\{-C_l\frac{\theta_i+\theta_j}{\theta_i\theta_j}\right\}\right). \tag{15}$$

The physical meaning of these constants will be explained later. At this moment, one can just treat them as five constants related to the given system parameters. Thus, based on the optimal decisions in the fifth stage, fourth stage and third stage, the optimal spectrum leasing strategy in the second stage and the corresponding optimal partial profit are specified in the following lemma.

Lemma 3: The optimal leasing strategy and the corresponding optimal partial profit is specified as follows.

1) If $\alpha B_s > D$, then the optimal partial profit is $R_2^* = F$ and the optimal leasing bandwidth is $B_l^* = 0$.
2) If $D > \alpha B_s \ge A$, then the optimal partial profit is given by:

$$R_2^* = \lambda^*\alpha B_s+\sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}, \tag{16}$$

where the λ^* is determined as the unique solution to:

$$\sum_{i=1}^{I}G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}=\alpha B_s. \tag{17}$$

The optimal leasing bandwidth is $B_l^* = 0$.
3) If $A > \alpha B_s \ge 0$, then the optimal partial profit is given by: $R_2^* = E + C_l\alpha B_s$. The optimal leasing bandwidth is $B_l^* = A - \alpha B_s$.

Proof: Obviously, the total available bandwidth B consists of the leasing bandwidth and the realized sensing bandwidth: $B = B_l + \alpha B_s$. If $\alpha B_s \ge D$, any further investment in leasing spectrum is meaningless since, according to 2, this will not further increase the income of selling spectrum to SUs. Thus, if $\alpha B_s \ge D$, the optimal leasing bandwidth is $B_l^* = 0$ and the optimal partial profit is: $R_2 = \sum_{i=1}^{I}\theta_i G_i e^{-2} = F$.

In the following, we focus on the case $\alpha B_s < D$. Notice that the optimal B_l must satisfy $\alpha B_s + B_l \le D$ since any further investment on B_l will not increase the income of selling spectrum to SUs. According to the second part of Lemma 2, the partial profit is:

$$R_2=\lambda^*(B_l+\alpha B_s)+\sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}-C_lB_l, \tag{18}$$

where λ^* is determined by:

$$\sum_{i=1}^{I}G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}=B_l+\alpha B_s. \tag{19}$$

Viewing λ^* as an implicit function of B_l, we take derivative of R_2 with respect to B_l as follows:

$$\frac{\partial R_2}{\partial B_l}=\frac{\partial\lambda^*}{\partial B_l}(B_l+\alpha B_s)+\lambda^*+\sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}\left(-\frac{1}{\theta_i}\right)\frac{\partial\lambda^*}{\partial B_l}-C_l=\lambda^*-C_l,$$

where we have used (19) in the last step. Now, we discuss two cases.

Case 1: $\alpha B_s < \sum_{i=1}^{I}G_i\exp\{-2-\frac{C_l}{\theta_i}\} = A$*:* In this case, when $B_l = 0$, we have $\lambda^* > C_l$. Hence, $\frac{\partial R_2}{\partial B_l}\big|_{B_l=0} > 0$. But when $B_l = D - \alpha B_s$, we have $\lambda^* = 0$. Hence, $\frac{\partial R_2}{\partial B_l}\big|_{B_l=D-\alpha B_s} < 0$. From (19), λ^* is a decreasing function of B_l. Therefore, the optimal B_l^* must lead to $\lambda^* = C_l$, i.e., $B_l^* = A - \alpha B_s$, and the optimal partial profit is: $R_2^* = E + C_l\alpha B_s$.

Case 2: $A \le \alpha B_s < D$*:* In this case, we have $\frac{\partial R_2}{\partial B_l}\big|_{B_l=0} \le 0$. Thus, R_2 always decreases with B_l and the optimal leasing bandwidth is $B_l^* = 0$. And the optimal partial profit is given by:

$$R_2^* = \lambda^*\alpha B_s+\sum_{i=1}^{I}\theta_i G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}, \tag{20}$$

where λ^* is determined by:

$$\sum_{i=1}^{I}G_i\exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}=\alpha B_s. \tag{21}$$

■

Remark 6: From Lemma 3, we observe a threshold structure of the optimal leasing bandwidth: $B_l = (A - \alpha B_s)^+$. This essentially says that the C-MVNO lease spectrum only when the realized sensing spectrum is below the threshold A. In particular, if unfortunately $\alpha = 0$, i.e., the realized sensing spectrum is 0, then the corresponding optimal leasing spectrum is $B_l^* = A$ and the optimal partial profit R_2^* is E. The above are the physical meanings of the constants A and E.

Remark 7: Moreover, comparing case 1 and case 2 of Lemma (3), we find that the optimal partial profit R_2^* varies in different manners, though the optimal leasing bandwidth is always zero in these two cases. Specifically, if the realized sensing spectrum αB_s is larger than a threshold D (case 1), then R_2^* remains a constant F independent of αB_s, i.e., further increase in αB_s will not enhance the partial profit any more. However, if $A < \alpha B_s < D$ (case 2), then R_2^* depends on the value of αB_s (actually as αB_s decreases, R_2^* also decreases), though the optimal decision is still to lease no spectrum. The above are the physical meanings of the constants D and F.

E. Spectrum Sensing in the First Stage

Denote R the overall profit of the C-MVNO. Based on the results of the previous subsections, we are now ready to derive the optimal sensing bandwidth B_s^* which maximizes the expected profit $\mathbb{E}(R)$ of the C-MVNO. We first give an important property of the relation between $\mathbb{E}(R)$ and B_s, based on which we propose a method to compute the optimal sensing bandwidth B_s^* afterwards.

Proposition 3.1: As a function of B_s, $\mathbb{E}(R)$ has the following properties:

1) When $B_s < D$, the expression of $\mathbb{E}(R)$ is given by:

$$\mathbb{E}(R) = F - C_s B_s - \frac{C_l A^2}{2B_s} - \frac{H}{B_s}, \tag{22}$$

which is a concave function of B_s on the interval $[D, \infty)$;

2) When $D \geq B_s > A$, the expression of $\mathbb{E}(R)$ is given by:

$$\mathbb{E}(R) = AC_l + E - \frac{A^2 C_l}{2B_s} - C_s B_s - \frac{1}{B_s}\int_A^{B_s} t\lambda^*(t)\,\mathrm{d}t + \int_A^{B_s} \lambda^*(t)\,\mathrm{d}t, \tag{23}$$

which is a concave function of B_s on the interval $[A, D]$. Here, $\lambda^*(t)$ is defined as the unique solution to the following equation:

$$\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda^*(t)}{\theta_i}\right\} = t, \tag{24}$$

where $0 < t \leq D$;

3) When $B_s \leq A$, the expression of $\mathbb{E}(R)$ is given by:

$$\mathbb{E}(R) = E + B_s\left(\frac{C_l}{2} - C_s\right), \tag{25}$$

which is a linear function of B_s on the interval $[0, A]$.

Proof: See Appendix B. ■

Proposition 3.1 guilds us to identify three sensing cost regimes based on the sign of $\frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s}$ at the two cut-off points of the three cases in Proposition 3.1. Accordingly, the method for computing the optimal B_s^* is presented as follows.

Lemma 4: The optimal sensing bandwidth B_s^* can be obtained as follows:

Low Sensing Cost Regime: When $0 < C_s \leq \frac{H}{D^2} + \frac{C_l A^2}{2D^2}$, the optimal B_s^* is given by:

$$B_s^* = \sqrt{\frac{1}{C_s}\left(H + \frac{1}{2}C_l A^2\right)}; \tag{26}$$

Medium Sensing Cost Regime: When $\frac{H}{D^2} + \frac{C_l A^2}{2D^2} < C_s \leq \frac{C_l}{2}$, the optimal B_s^* is given by

$$B_s^* = \sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\mu^*}{\theta_i}\right\}, \tag{27}$$

where μ^* is determined as the unique solution to the following equation on the interval $[0, C_l]$:

$$e^{-4}\sum_{i,j=1}^{I} \frac{G_i G_j \theta_i}{\theta_i + \theta_j}\left[-\left(C_l + \frac{\theta_i\theta_j}{\theta_i + \theta_j}\right)\exp\left\{-\frac{\theta_i + \theta_j}{\theta_i\theta_j}C_l\right\} + \left(\mu + \frac{\theta_i\theta_j}{\theta_i + \theta_j}\right)\exp\left\{-\frac{\theta_i + \theta_j}{\theta_i\theta_j}\mu\right\}\right] - C_s\left(\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\mu}{\theta_i}\right\}\right)^2 + \frac{A^2 C_l}{2} = 0; \tag{28}$$

High Sensing Cost Regime: When $C_s > \frac{C_l}{2}$, the optimal $B_s^* = 0$.

Proof:

Low Sensing Cost Regime: $0 < C_s \leq \frac{H}{D^2} + \frac{C_l A^2}{2D^2}$ In this regime, the optimal B_s^* is achieved in the interval $[D, \infty)$. By taking the derivative of $\mathbb{E}(R)$ in (22) with respect to B_s, we have:

$$\frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s} = -C_s + \frac{C_l A^2}{2B_s^2} + \frac{H}{B_s^2}. \tag{29}$$

Hence, $\frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s}\big|_{B_s = D} \geq 0$ and $\lim_{B_s \to \infty} \frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s} = -C_s < 0$. Moreover, since $\mathbb{E}(R)$ is concave in the interval $[D, \infty)$ according to Proposition 3.1, the maximum point for $\mathbb{E}(R)$ should satisfy $\frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s} = 0$ where $B_s \in [D, \infty)$. Therefore, the optimal sensing bandwidth B_s^* in the low sensing cost regime is:

$$B_s^* = \sqrt{\frac{1}{C_s}\left(H + \frac{1}{2}C_l A^2\right)}. \tag{30}$$

Medium Sensing Cost Regime: $\frac{H}{D^2} + \frac{C_l A^2}{2D^2} < C_s \leq \frac{C_l}{2}$ In this regime, the optimal B_s^* is achieved in the interval $[A, D]$. Similar to the analysis in the low sensing cost regime, because of the concavity of $\mathbb{E}(R)$, the optimal B_s must satisfy $\frac{\mathrm{d}\mathbb{E}(R)}{\mathrm{d}B_s} = 0$, where $B_s \in [A, D]$. According to (23), this is equivalent to:

$$\int_A^{B_s^*} t\lambda^*(t)\,\mathrm{d}t - C_s (B_s^*)^2 = -\frac{A^2 C_l}{2}. \tag{31}$$

It can be shown that the L.H.S. of (31) first increases and then decreases when B_s increases from A to D and (31) has one unique solution, which is just the optimal sensing bandwidth.

Performing variable transformation $\mu = \lambda^*(B_s) \in [0, C_l]$, we note that the first term of (31) can be rewritten as:

$$\int_A^{B_s} t\lambda^*(t)\,\mathrm{d}t = \int_{C_l}^{\mu} \lambda \left(\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda}{\theta_i}\right\} \right) \times \mathrm{d}\left(\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda}{\theta_i}\right\} \right). \quad (32)$$

Calculating the integral in (32) and substituting it back into (31) yields (28). Similar to that of (31), it can also be shown that the L.H.S. of (28) first increases then decreases and (28) has a unique solution μ^* in the interval $[0, C_l]$. Hence, we can use simple bisection method to find μ^*. After obtaining μ^*, the optimal sensing bandwidth can be calculated by:

$$B_s^* = \sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\mu^*}{\theta_i}\right\}. \quad (33)$$

High Sensing Cost Regime: $C_s > \frac{C_l}{2}$ In this regime, according to the third case in Proposition 3.1, the optimal sensing bandwidth is simply $B_s^* = 0$. ■

Remark 8: From Lemma 4, we can see that in the Low Sensing Cost Regime and High Sensing Cost Regime, the optimal B_s^* can be obtained using closed-form expressions, while in the Medium Sensing Cost Regime, the optimal B_s^* can be found by solving (28) using simple bisection methods. Therefore, the computational complexity for finding the optimal B_s^* is very low in the proposed scheme.

F. Summary

Based on our discussion in the previous five subsections, the equilibrium of the proposed Stackelberg game is: optimal sensing bandwidth B_s^* in Lemma 4, optimal leasing bandwidth B_l^* in Lemma 3, optimal admission control in Lemma 2, optimal differentiated pricing p_i^* in Lemma 1 as well as the SUs' best responses w_{ij}^* in (6). In Algorithm 1, we summarize how to compute the optimal decisions of the C-MVNO and the SUs.

Algorithm 1 Finding the optimal decisions for the C-MVNO and the SUs

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Inputs:
The sensing cost C_s, the leasing cost C_l, the willingness-to-pay θ_i of each $\mathcal{S}_i$, the aggregate wireless characteristics G_i of each $\mathcal{S}_i$, and the realization of the RV α after B_s is determined.

Outputs:
The optimal sensing bandwidth B_s^*, the optimal leasing bandwidth B_l^*, the optimal differentiated pricing $\{p_i^*\}_{1\le i\le I}$, and the optimal spectrum allocation w_{ij}^*.

Compute the constants A, D, E, F, H according to (15).
if $C_s \le \frac{H}{D^2} + \frac{C_l A^2}{2D^2}$ **then**
  Set B_s^* as in (26).
else if $\frac{H}{D^2} + \frac{C_l A^2}{2D^2} < C_s \le \frac{C_l}{2}$ **then**
  Solve (28) for μ^* using the bisection method. Compute B_s^* as in (27).
else
  Set $B_s^* = 0$.
end if
Return B_s^* at the C-MVNO's side. Perform spectrum sensing and thus sensing realization factor α is realized.
Set $B_l^* = (A - \alpha B_s^*)^+$ and return it at C-MVNO's side. Skip admission control and serve all the SUs.
if $\alpha B_s + B_l \ge D$ **then**
  Set $p_i^* = \theta_i, \forall i$.
else
  In (10), set $\widetilde{G}_i = G_i, \forall i$ and $B = \alpha B_s + B_l$. Solve (10) using similar bisection method for λ^* and then set $p_i^* = \lambda^* + \theta_i, \forall i$.
end if
Return $\{p_i\}_{1\le i\le I}$ at the C-MVNO's side.
At the SUs' sides, compute the w_{ij}^* according to (6) with price $p_i = p_i^*$. Return w_{ij}^* at the j-th SU of $\mathcal{S}_i$.

In practice, when the C-MVNO is serving a cognitive network, it does the following things repeatedly. The C-MVNO observes the temporary spectrum leasing price C_l announced by the spectrum owners. Since the spectrum sensing price C_s is determined by the sensing technology of the C-MVNO, the C-MVNO is always aware of it. Then, the C-MVNO estimates the wireless characteristic g and the willingness-to-pay θ of each SU based on its channel condition and service type (watching video or phoning etc.), respectively. With these system parameters, the C-MVNO can run Algorithm 1 to compute the optimal decisions. Since the computational complexity of Algorithm 1 is very low, it can be implemented at C-MVNO easily. The C-MVNO may repeat the aforementioned steps once the temporarily leased spectrum expires or the SU network changes. We remark that the optimal profit of the C-MVNO is hard to obtain in closed-form analytically in our heterogeneous setting. However, through numerical simulations in Section IV, we could see that the profit is improved significantly compared to the single pricing scheme in [20].

IV. Simulation Results

In this section, we use simulations to evaluate the viability and optimality of the proposed Algorithm 1 and illustrate several threshold structures of the equilibrium of the formulated Stackelberg game. The threshold structures rely on the last part of Lemma 4, where we show that as long as $C_s < C_l/2$, the C-MVNO should not perform any spectrum sensing.

In all simulations, other than specifically mentioned, the parameters are set to be: $I = 20, |\mathcal{S}_i| = 20, \theta_i = i, g_{ij} = 50$ for $i \in [1, 9]$ and $g_{ij} = 100$ for $i \in [10, 20]$. In such a case we have $G_i = 1000$ for $i \in [1, 9]$ and $G_i = 2000$ for $i \in [10, 20]$. The selection of these parameters is just for demonstration purpose. Other parameters will give similar results, i.e., our analysis is not restricted by the selection of the parameters.

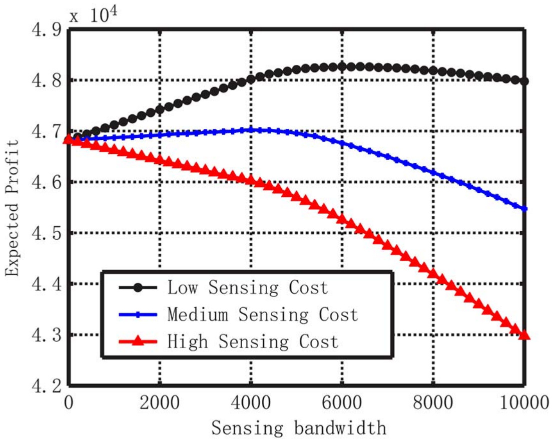
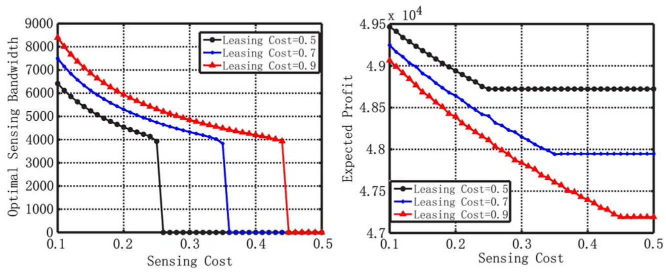


Fig. 3. Verifying the optimality of Algorithm 1: $C_l = 1$ is fixed. The black curve, blue curve, and red curve correspond to $C_s = 0.2$, $C_s = 0.45$, and $C_s = 0.7$, respectively.

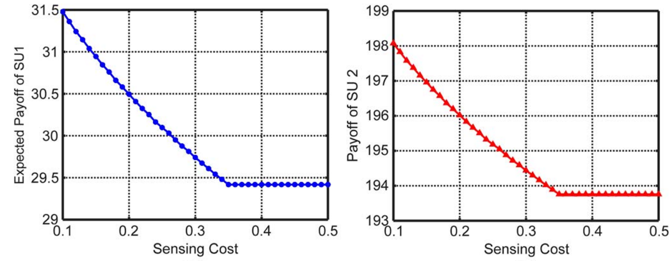


(a) (b)

Fig. 4. (a) Impact of C_s and C_l on the optimal sensing bandwidth B_s^*: the black curve, blue curve, red curve represents the case $C_l = 0.5$, 0.7, 0.9 respectively. (b) Impact of C_s and C_l on the optimal expected profit of the C-MVNO.

In the first simulation, we verify the optimality of sensing bandwidth B_s^* derived by Algorithm 1. To do so, we compute the profit of C-MVNO by varying B_s while keeping the best response of B_l and p_i obtained in Algorithm 1. We test three different cost parameters for (C_s, C_l): (0.2,1) for the Low Sensing Cost Regime, (0.45,1) for the Medium Sensing Cost Regime, and (0.7,1) for the High Sensing Cost Regime. The results are shown in Fig. 3 where we average over 10^5 independent runs. With these parameters, the optimal sensing bandwidth B_s^* derived by Algorithm 1 are 6220.4, 4144.9, and 0 respectively. From Fig. 3, we can see that the profit of C-MVNO indeed achieves the maximum with these optimal B_s^*. We can also see that the less the sensing cost C_s, the larger the expected profit of the C-MVNO.

In the second simulation, we study the impact of sensing cost C_s and leasing cost C_l on the optimal sensing bandwidth B_s^*. The results are shown in Fig. 4(a), where the horizontal axis represents the sensing cost C_s and the vertical axis represents the optimal sensing bandwidth B_s^*. We observe that the optimal sensing bandwidth B_s^* exhibits certain threshold structure. Specifically, B_s^* first decreases smoothly with C_s and then suddenly drops to zero and keeps to be zero when C_s increases. This phenomenon can be explained by the solution structure of Algorithm 1 as follows. When C_s is relatively small, the system is in the Low Sensing Cost Regime or Medium Sensing Cost Regime, so the value of B_s^* decreases smoothly with C_s according to the first two parts of Lemma 4. However, when C_s further increases and the system enters into the High Sensing Cost Regime, the value of B_s^* will drop to zero immediately according to the last part of Lemma 4. From Fig. 4(a), we also observe that generally B_s^* is large when C_l is large. This is reasonable since the C-MVNO tends to sense more bandwidth if the leasing cost is high.



(a) (b)

Fig. 5. Expected payoffs of two SUs at the equilibrium of the Stackelberg game. (a) SU1, $\theta = 5$, $g = 50$. (b) SU2, $\theta = 15$, $g = 100$.

We then investigate the impact of sensing cost C_s and leasing cost C_l on the optimal expected profit of the C-MVNO. The results are illustrated in Fig. 4(b), from which we can also observe the threshold structure. We can see that the profit first decreases with C_s and finally remains a constant when C_s is large enough. This is natural because when the system is in the High Sensing Cost Regime, the optimal sensing bandwidth B_s^* is always zero, i.e., the C-MVNO senses no spectrum and thus further increase in C_s will not affect the profit.

We also study the impact of C_s on the expected payoff of SUs at the equilibrium of the Stackelberg game, which is specified in (5) and (6). We consider two different SUs, denoted as SU1 and SU2. The willingness-to-pay θ of SU1 and SU2 are 5 and 15, respectively, while the wireless characteristic g of SU1 and SU2 are 50 and 100, respectively. We set $C_l = 0.7$ in this simulation. The results are shown in Fig. 5, which again exhibit threshold structure. We can see that the payoffs of the SUs first decrease with C_s and finally remain a constant. The reason is that once C_s enters into the High Sensing Cost Regime, the realized sensing bandwidth maintains to be zero and thus further increase in C_s will not influence the price of the spectrum for SUs. Therefore, the payoffs of the SUs remain unchanged. A comparison between SU1 and SU2 also shows that the SU with larger willingness-to-pay and wireless characteristic (e.g., SU2 here) enjoys larger payoff.

Next, we study the influence of the sensing realization factor α on the profit of the C-MVNO, the optimal leasing bandwidth, the price of SU and the payoff of SU, where we set $C_l = 1$ and C_s to be 0.2,0.45, and 0.7 to represent the Low, Medium, and High Sensing Cost Regimes, respectively. The willingness-to-pay parameter θ and the wireless characteristic g of the SU whose price and payoff are shown are 5 and 50, respectively. The results are shown in Fig. 6. We can see that all results exhibit threshold structure and the sensing realization factor α has no influence on any network behavior in the High Sensing Cost Regime since the corresponding B_s^* is zero. From Fig. 6(a), we can see that the profit of the C-MVNO increases as α increases in the Medium Sensing Cost Regime, but saturates in the Low Sensing Cost Regime. This is because, with large enough α, we

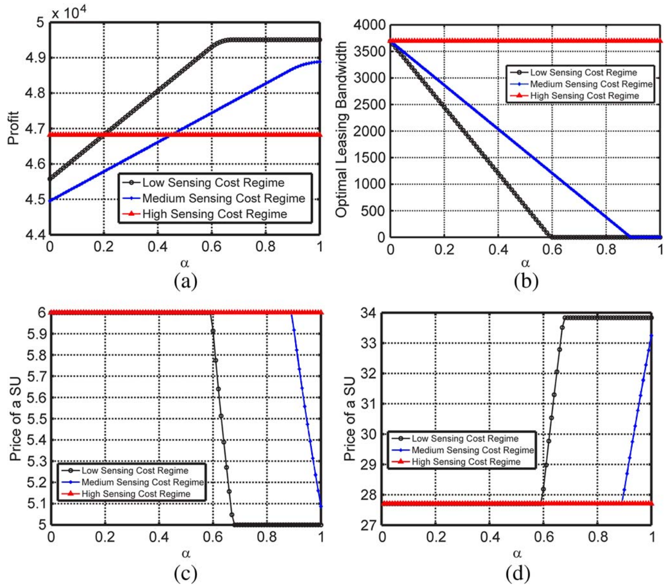


Fig. 6. Impact of the sensing realization factor α. (a) Impact of α on the realized profit of the C-MVNO. (b) Impact of α on the optimal leasing bandwidth. (c) Impact of α on the realized price of a SU. (d) Impact of α on the realized payoff of a SU.

have $\alpha B_s \geq D$, and according to Lemma 2, the profit becomes independent with α. Similar arguments apply to Fig. 6(b)–(d).

Furthermore, we compare the heterogeneous scheme proposed in this paper with the homogeneous scheme proposed in [20]. In order to have a fair comparison, we change the simulation parameters to: $I = 19, \theta_i = 0.1i, |\mathcal{S}_i| = 20$ and $g_{ij} = 100, \forall i, j, G_i = 2000, \forall 1 \leq i \leq I$. Hence, the average willingness-to-pay is $\overline{\theta} = 1$. The results are shown in Fig. 7, from which we can see that the proposed algorithm achieves much better profit for C-MVNO than the scheme in [20]. The reason can be explained as follows. Since the homogeneous scheme in [20] can only handle the homogeneous case, it will treat all heterogeneous SUs as homogeneous SUs with average willingness-to-pay parameter $\theta = 1$. In such a case, the total demand of the SUs will not match with the bandwidth procured by the C-MVNO, due to which the procured spectrum may be wasted or may not be enough to serve all SUs, and thus the profit of C-MVNO is degraded. Therefore, when the SUs are heterogeneous, which is generally the case in reality, differentiated pricing should be used to achieve the optimal profit for C-MVNO.

Finally, we consider the case where the sensing realization factor α is not uniformly distributed. We note that, regardless of the distribution of α, the best responses of the leasing bandwidth, pricing and admission control remain the same, i.e., Lemmas 1, 2, and 3 always hold. Compared to the uniformly distributed α case, the only difference lies in the spectrum sensing decision. In general, when α is not uniformly distributed, we cannot find simple solution structure for the optimal sensing bandwidth B_s^* as in Lemma 4. However, we could find a simple upper bound for the optimal B_s^* as follows. According to Lemma 1, the C-MVNO should not procure spectrum more than $\sum_{i=1}^{I} G_i e^{-2}$ since the C-MVNO will never sell more spectrum in the optimal pricing. Hence, the optimal B_s^* should be no more than $\frac{C_l}{C_s}\sum_{i=1}^{I} G_i e^{-2}$. Otherwise, an obviously better choice

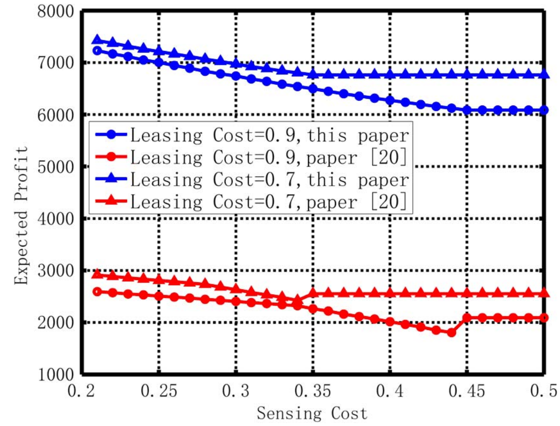


Fig. 7. A comparison between the heterogeneous scheme proposed in this paper and the homogeneous scheme proposed in [20].

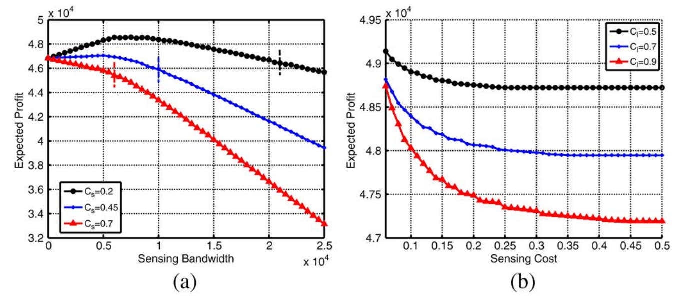


Fig. 8. The α is distributed according to truncated Gaussian. (a) Finding the optimal sensing bandwidth. (b) The optimal profit.

would be $B_s = 0, B_l = \sum_{i=1}^{I} G_i e^{-2}$. With this upper bound, to find the optimal B_s^*, we perform an exhaustive search on the interval $[0, \sum_{i=1}^{I} G_i e^{-2}]$. Though termed exhaustive search, the process has very low computational overhead and we generally only need to evaluate dozens of (e.g., 30) values of the expected profit. This procedure is illustrated in Fig. 8(a), where we only need to evaluate the expected profit from $B_s = 0$ to the dashed lines, i.e., the upper bound mentioned previously. The α follows a truncated Gaussian distribution with mean 0.5 and variance 0.04 over the interval [0,1]. We further plot the relationship between the optimal profit and the sensing/leasing costs C_s, C_l in Fig. 8(b). Similar to the uniformly distributed α scenario, the optimal profit decreases with C_s and C_l and saturates when C_s is large enough in which case the C-MVNO will sense no spectrum, i.e., $B_s^* = 0$.

V. Conclusion and Future Work

In this paper, we study the optimal spectrum sensing, spectrum leasing, admission control as well as differentiated pricing decisions from a C-MVNO's perspective. The SUs are heterogeneous in their demands of spectrum and this heterogeneity is modeled as different willingness-to-pay parameters. Knowing the characteristic of each SU, we invoke differentiated pricing instead of single pricing to improve the profit of the C-MVNO. Formulating the problem as a Stackelberg game, we use backward induction to analyze the optimal decisions of the C-MVNO as the equilibrium of the game. A simple algorithm of computing the optimal decisions of the C-MVNO is explicitly derived and presented. At last, numerical experiments are implemented to confirm the optimality of the proposed algorithm as well as to explore the structure of solution.

There are two directions for future work. First, in practice, there may be more than one C-MVNO and there may exist competition between the multiple C-MVNOs [9]. Unlike the monopoly case studied in this paper, the case of multiple non-cooperative C-MVNOs needs a new dynamic game formulation and analysis such as in [22]. Intuitively, competition between the C-MVNOs may degrade their profits and enhance the utilities of the SUs. It is interesting to see how the utility improvement differs among the heterogeneous SUs. Second, the C-MVNO may not have the complete information of all the SUs in the cognitive network. For instance, the C-MVNO may not estimate the willingness-to-pay and the wireless characteristics of the users accurately. In such a case, differentiated pricing scheme is inviable and a contract-theoretic formulation is necessary [13], [21]. This may degrade the profit of the C-MVNO. It is interesting to quantify this profit loss due to the lack of user information.

Appendix A
Proof of Lemma 1

By relating the constraint (8) with Lagrange multiplier $\lambda \geq 0$, we can write the dual function of (P1) as:

$$\begin{aligned} g(\lambda) = \inf_{\overrightarrow{p} \succeq \overrightarrow{0}} & \left\{ -\sum_{i=1}^{I} p_i \widetilde{G}_i \exp\left\{-1-\frac{p_i}{\theta_i}\right\} \right. \\ & \left. + \lambda \left(\sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-1-\frac{p_i}{\theta_i}\right\} - B \right) \right\} \\ = & -\lambda B - \sum_{i=1}^{I} \theta_i \widetilde{G}_i \exp\left\{-2-\frac{\lambda}{\theta_i}\right\}, \end{aligned}$$

where the minimum is obtained when $p_i = \lambda + \theta_i, 1 \leq i \leq I$. Hence, the dual problem of (P1) can be written as follows.

$$\text{(P2)} \quad \max_{\lambda \geq 0} -\lambda B - \sum_{i=1}^{I} \theta_i \widetilde{G}_i \exp\left\{-2-\frac{\lambda}{\theta_i}\right\} \tag{34}$$

The derivative of (34) with respect to λ is: $-B + \sum_{i=1}^{I} \widetilde{G}_i \exp\{-2-\frac{\lambda}{\theta_i}\}$. Hence, the optimal λ^* of (P2) can be either: 1) $\lambda^* = 0$ if $\sum_{i=1}^{I} \widetilde{G}_i e^{-2} \leq B$; or 2) the unique solution to

$$\sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\} = B, \tag{35}$$

otherwise. Then, the optimal differentiated pricing, i.e., the solution to (P1), is $p_i^* = \lambda^* + \theta_i, 1 \leq i \leq I$. If $\sum_{i=1}^{I} \widetilde{G}_i e^{-2} \leq B$, the optimal value of (P1) is $\sum_{i=1}^{I} \theta_i \widetilde{G}_i e^{-2}$. Otherwise, the optimal value is:

$$\sum_{i=1}^{I} (\lambda^* + \theta_i) \widetilde{G}_i \exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\} = \lambda^* B + \sum_{i=1}^{I} \theta_i \widetilde{G}_i \exp\left\{-2-\frac{\lambda^*}{\theta_i}\right\}, \tag{36}$$

where λ^* is the unique solution to (35).

Appendix B
Proof of Proposition 3.1

Case 1—$B_s > D$: In this case, utilizing Lemma 3, we have:

$$\begin{aligned} \mathbb{E}(R) = & (F - C_s B_s)\left(1 - \frac{D}{B_s}\right) + \int_{\frac{A}{B_s}}^{\frac{D}{B_s}} \left[\alpha B_s \lambda^*(\alpha B_s) \right. \\ & \left. + \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2-\frac{\lambda^*(\alpha B_s)}{\theta_i}\right\} - C_s B_s \right] \mathrm{d}\alpha \\ & + \int_0^A B_s (E + \alpha B_s C_l - C_s B_s) \, \mathrm{d}\alpha \end{aligned} \tag{37}$$

The first term corresponds to the scenario $\alpha B_s \geq D$, while the second term and the third term correspond to the scenarios $D > \alpha B_s \geq A$ and $A > B_s \geq 0$ respectively. Here we define the function $\lambda^*(t)$ as the unique solution to the equation: $\sum_{i=1}^{I} G_i \exp\{-2-\frac{\lambda^*(t)}{\theta_i}\} = t$, where $0 < t \leq D$ and the solution $\lambda^*(t)$ is always non-negative. Thus, the expression of $\mathbb{E}(R)$ in (37) can be simplified into the following form:

$$\begin{aligned} \mathbb{E}(R) = & F - \frac{DF}{B_s} - C_s B_s + \frac{EA}{B_s} + \frac{C_l A^2}{2B_s} + \frac{1}{B_s}\int_A^D t\lambda^*(t) \, \mathrm{d}t \\ & + \frac{1}{B_s}\int_A^D \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2-\frac{\lambda^*(t)}{\theta_i}\right\} \mathrm{d}t. \end{aligned} \tag{38}$$

Noting that $\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{I} \theta_i G_i \exp\{-2-\frac{\lambda^*(t)}{\theta_i}\} = -t\frac{\mathrm{d}\lambda^*(t)}{\mathrm{d}t}$, we simplify the last term in (37) into the following form:

$$\begin{aligned} & \int_A^D \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2-\frac{\lambda^*(t)}{\theta_i}\right\} \mathrm{d}t \\ & \qquad = DF - AE - A^2 C_l - 2\int_A^D t\lambda^*(t) \, \mathrm{d}t, \end{aligned} \tag{39}$$

where we use $\lambda^*(A) = C_l, \lambda^*(D) = 0$. We can further calculate the integral term as follows:

$$\begin{aligned} \int_A^D t\lambda^*(t) \, \mathrm{d}t = & \int_{C_l}^0 \lambda \left(\sum_{i=1}^{I} G_i \exp\left\{-2-\frac{\lambda}{\theta_i}\right\} \right) \\ & \times \mathrm{d}\left(\sum_{i=1}^{I} G_i \exp\left\{-2-\frac{\lambda}{\theta_i}\right\} \right) \\ = & H, \end{aligned} \tag{40}$$

where H is defined in (15). Substituting (39) and (40) back into (38) yields:

$$\mathbb{E}(R) = F - C_s B_s - \frac{C_l A^2}{2B_s} - \frac{H}{B_s}. \tag{41}$$

From (41), we observe that $\mathbb{E}(R)$ is a concave function of B_s on the interval $\in [D, +\infty)$.

Case 2—$D \geq B_s >; A$: In this case, we can evaluate $\mathbb{E}(R)$ as follows:

$$\begin{aligned} \mathbb{E}(R) = & \frac{EA}{B_s} + \frac{C_l A^2}{2B_s} - C_s B_s + \frac{1}{B_s}\int_A^{B_s} t\lambda^*(t) \, \mathrm{d}t \\ & + \frac{1}{B_s}\int_A^{B_s} \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2-\frac{\lambda^*(t)}{\theta_i}\right\} \mathrm{d}t. \end{aligned} \tag{42}$$

We further have:

$$\sum_{i=1}^{I} N_i\theta_i \exp\left\{-2-\frac{\lambda^*(B_s)}{\theta_i}\right\} = -B_s\lambda^*(B_s)+AC_l+E+\int_A^{B_s}\lambda^*(t)\,\mathrm{d}t. \quad (43)$$

We rewrite the last term of (42) as:

$$\int_A^{B_s}\sum_{i=1}^{I}\theta_i G_i \exp\left\{-2-\frac{\lambda^*(t)}{\theta_i}\right\}\mathrm{d}t = AC_lB_s+B_s\int_A^{B_s}\lambda^*(t)\,\mathrm{d}t+EB_s-AE-A^2C_l-2\int_A^{B_s}t\lambda^*(t)\,\mathrm{d}t. \quad (44)$$

Substituting (44) into (42) yields:

$$\mathbb{E}(R)=AC_l+E-\frac{A^2C_l}{2B_s}-C_sB_s-\frac{1}{B_s}\int_A^{B_s}t\lambda^*(t)\,\mathrm{d}t+\int_A^{B_s}\lambda^*(t)\,\mathrm{d}t. \quad (45)$$

Taking second order derivative of $\mathbb{E}(R)$ with respect to B_s yields:

$$\frac{\mathrm{d}^2\mathbb{E}(R)}{\mathrm{d}B_s^2}=-\frac{1}{B_s^3}\left[2\int_A^{B_s}t\lambda^*(t)\,\mathrm{d}t-B_s^2\lambda^*(B_s)+A^2C_l\right]. \quad (46)$$

Define the quantity inside the parentheses of (46) as $f(B_s)$, i.e.,

$$f(B_s)=2\int_A^{B_s}t\lambda^*(t)\,\mathrm{d}t-B_s^2\lambda^*(B_s)+A^2C_l. \quad (47)$$

Taking derivative of $f(B_s)$, we obtain: $\frac{\mathrm{d}f(B_s)}{\mathrm{d}B_s}=-B_s^2\frac{\mathrm{d}\lambda^*(B_s)}{\mathrm{d}B_s}\geq 0$.

Thus, $f(B_s)$ is an increasing function of when $B_s\in[A,D]$. We further note that $f(A)=0$. Hence, we have $f(B_s)\geq 0,\forall B_s\in[A,D]$. From (46), we know that $\frac{\mathrm{d}^2\mathbb{E}(R)}{\mathrm{d}B_s^2}\leq 0,\forall B_s\in[A,D]$. Hence, $\mathbb{E}(R)$ is a concave function of B_s on the interval $[A,D]$.

Case 3—$0\leq B_s\leq A$: In this case, we have:

$$\mathbb{E}(R)=\int_0^1(E+C_l\alpha B_s-C_sB_s)\,\mathrm{d}\alpha=E+B_s\left(\frac{C_l}{2}-C_s\right), \quad (48)$$

which is a linear function of B_s.

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