Spectrum Trading in Heterogeneous Cognitive Radio Networks

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Abstract—**In this paper, we investigate the optimal spectrum procurement and pricing from the perspective of a cognitive mobile virtual network operator (C-MVNO), which is a second market between the spectrum owner and the secondary users (SUs). The spectrum procurement consists of spectrum leasing and spectrum sensing, where the latter has uncertain outcome. The SUs are assumed to be heterogeneous in their valuations and demands of the spectrum, which is generally the case in reality. Hence, we use differentiated pricing among the heterogeneous SUs to improve the profit of the C-MVNO and allow the C-MVNO to perform necessary admission control. Modeling the spectrum procurement and trading procedure as a five-stage Stackelberg game, we analyze the optimal decisions for the C-MVNO by using backward induction and propose a computationally efficient method to find them. In simulations, a threshold structure of the solution is observed and our proposed scheme outperforms the single pricing based scheme in prior works.**

I. INTRODUCTION

Wireless spectrum is becoming more and more scarce nowadays due to the fast growing demand of wireless services. This leads to the advance of the cognitive radio (CR) technology, which is regarded as a promising paradigm of efficient spectrum utilization. To access and utilize the spectrum economically and efficiently, many game theoretic schemes have been proposed in the literature [1], [2]. Auction based spectrum access mechanisms are proposed in [3]–[5]. Some researchers have studied the pricing interactions between the network operator and SUs to maximize either the social welfare or the operator's profit [6]–[10]. A contract formulation of spectrum trading in CR networks (CRNs) is investigated in [11] to model the scenario where the primary owner does not know the feature (e.g. channel condition) of each individual SU and only has the knowledge of the statistical distribution of the overall features. Evolutionary game theory is invoked to investigate the spectrum sensing and access problem in [12], [13]. An indirect reciprocity game modeling approach is studied in [14], [15]. In addition, learning and negative network externality are considered in [16].

In general, a cognitive mobile virtual network operator (C-MVNO) will serve as a second market between the spectrum owner and the SUs. It needs to first procure the spectrum from the spectrum owner and then sells it to the SUs with certain prices. So far, few papers have jointly studied the problem of spectrum procurement and pricing from the operator's perspective, e.g. [7] and [17]. However, they only consider the single pricing scheme in the homogeneous case, i.e., all the SUs have the same valuation of the spectrum and the C-MVNO sets a single price for all the homogeneous SUs. This may turn out to be an oversimplified model for today's mobile networks where the users are highly heterogeneous in their demands and valuations of the spectrum.

In this paper, for a CRN with heterogeneous SUs, we use differentiated pricing to maximize the profit of the C-MVNO, i.e., we set different prices for SUs with different valuations of the spectrum. Formulating the spectrum procurement and trading as a five-stage Stackelberg game, we jointly optimize the spectrum sensing, leasing, admission control and pricing decisions from a C-MVNO's perspective. The main contributions of this paper are summarized as follows.

- We model the spectrum procurement and trading process as a five-stage Stackelberg game. Due to the heterogeneity of the SUs, price differentiation is introduced to improve the profit of C-MVNO as opposed to the single pricing scheme for the homogeneous user case. Admission control is also allowed to balance the spectrum supply and demand.
- Using backward induction, we derive the optimal decisions of spectrum sensing, spectrum leasing, admission control and differentiated pricing of the C-MVNO as the equilibrium of the formulated Stackelberg game. The results suggest a simple method to compute these optimal decisions.
- In simulations, a threshold structures of the obtained optimal solution is observed. We also see that, when the SUs are heterogeneous, our proposed differentiated pricing based scheme outperforms the single pricing based scheme of prior works.

The rest of this paper is organized as follows. In Section II, we introduce the system model and formulate the problem as a Stackelberg game. In Section III, we analyze the game model using backward induction and derives the optimal decisions of the C-MVNO. In Section IV, simulation results are presented and we conclude this paper in Section V.

II. SYSTEM MODEL

As shown in Figure 1, we consider a system with one C-MVNO and multiple heterogeneous secondary users (SUs). The objective of the C-MVNO is to collect spectrum and sell the spectrum to SUs to maximize its overall profit. Specifically, the C-MVNO collects spectrum through performing spectrum sensing for unused primary spectrum and leasing

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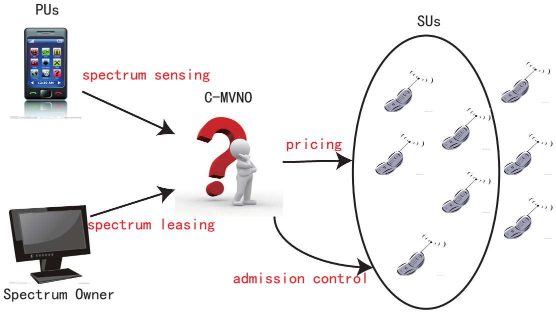

Fig. 1. Illustration of the system model

spectrum from spectrum owners. Since there exists uncertainty in spectrum sensing, the amount of leased spectrum depends on the outcome of spectrum sensing. After collecting the spectrum, the C-MVNO can choose SUs for selling spectrum by admission control. Since the SUs are heterogeneous, which means that they have different demands of the spectrum, the prices to different SUs are different, i.e., differentiated pricing is used. We assume that all SUs are rational and thus naturally selfish, due to which they will purchase the optimal amount of spectrum from the C-MVNO to maximize their own utility function based on the differentiated price announced by the C-MVNO. In what follows, we present our system model in detail.

A. SU's Model

We assume that each SU has its **willingness-to-pay** parameter θ. This positive parameter is used to model the quality-of-service (QoS) requirement of a SU: the larger the θ, the higher the requirement of the SU. For instance, a SU who is watching a video requires much more data rate and thus has a larger θ, compared with a SU who is just phoning.

Consider a SU with a willingness-to-pay parameter θ. Let w be the bandwidth allocated to the SU and p be the unit price of the bandwidth. Then, the utility function of the SU can be written as:

$$\begin{aligned} u(p,w) &= \theta w \ln\left(1+\frac{P^{\max}h}{n_0 w}\right) - pw \\ &= \theta w \ln\left(1+\frac{g}{w}\right) - pw, \end{aligned} \tag{1}$$

where $P^{\max}$ is the maximal transmission power, h is the channel gain, n_0 is the noise power density, $g = \frac{P^{\max}h}{n_0}$ is the received SNR (when the bandwidth is one unit), which can be treated as the **wireless characteristic** of the SU, and $w\ln(1+g/w)$ is the achievable rate of the SU [1].

From (1), we can see that the two parameters (θ, g) can fully characterize a SU. In this paper, we focus on the high SNR regime where SNR $= g/w \gg 1$. In such a case, the utility function in (1) can be approximated as

$$u(p,w) = \theta w \ln\left(\frac{g}{w}\right) - pw. \tag{2}$$

In this paper, we assume that there are I possible willingness-to-pay parameters, i.e., $\theta \in \{\theta_1, \theta_2, ..., \theta_I\}$, where each θ_i represents a different wireless services such as video streaming and website browsing. Let $\mathcal{S}_i$ be the index set of the set of SUs with the same θ_i, and g_{ij} be the wireless characteristic of j-th SU in $\mathcal{S}_i$.

B. C-MVNO's Model

As discussed above, the decisions of the C-MVNO include spectrum sensing, spectrum leasing, admission control and differentiated pricing. In the following, we discuss them in details one by one.

1) Spectrum Sensing: Let B_s be the bandwidth that the C-MVNO senses. Due to the stochastic nature of PUs' behaviors, the amount of unused primary spectrum that is available for the C-MVNO is uncertain. Let $\alpha \in [0,1]$ be the random variable standing for the portion of unused primary spectrum. Then, the amount of spectrum C-MVNO can obtain through spectrum sensing is αB_s. In this paper, we assume that α is uniformly distributed within the interval $[0,1]$. Nevertheless, similar analysis can be conducted with other distributions. Note that there is a certain cost for the C-MVNO to perform sensing. Let C_s be the sensing cost per unit bandwidth. Then, by sensing bandwidth B_s, the C-MVNO can obtain unused spectrum αB_s at the cost of $C_s B_s$.

2) Spectrum Leasing: Since the spectrum obtained through sensing may not be enough, the C-MVNO may need to lease more bandwidth from the spectrum owner after the sensing outcome is given. Let B_l be the amount of leased spectrum, and C_l be the unit leasing cost. Then the total leasing cost is $C_l B_l$. In general, the leasing cost C_l is much larger than the sensing cost C_s, but we do not make this assumption in the following.

3) SU Admission Control: To achieve the best profit, the C-MVNO may perform admission control on SUs, i.e., the C-MVNO can select only a subset of the SUs to serve. Specifically, for each set $\mathcal{S}_i$, suppose the C-MVNO only serves a subset $\widetilde{\mathcal{S}}_i$.

4) Differentiated Pricing: We assume that the C-MVNO knows the willingness-to-pay θ and the wireless characteristic g of each SU. With the knowledge of (g, θ) for each SU, the C-MVNO can use differentiated pricing to maximize its profit. Specifically, the C-MVNO sets different prices for SUs with different willingness-to-pay parameter θ (i.e., in different sets $\widetilde{\mathcal{S}}_i$). Denote p_i as the price for SUs in $\widetilde{\mathcal{S}}_i$. Thus, the C-MVNO should determine I different prices for the corresponding I different values of willingness-to-pay parameters.

C. Stackelberg Game Formulation

The interaction between the C-MVNO and the SUs can be formulated as a five-stage Stackelberg game. The Stackelberg leader is the C-MVNO and the followers are the SUs. In the first stage, the C-MVNO determines the sensing bandwidth B_s and then realizes the available sensing result αB_s. In the second stage, based on the sensing result αB_s, the C-MVNO determines the leasing bandwidth B_l. In the third stage, the C-MVNO performs admission control to serve a subset of SUs. In the fourth stage, the C-MVNO sets the differentiated price

p_i for each $\widetilde{\mathcal{S}}_i$, where $i \in \{1, ..., I\}$. Finally, in the fifth stage, given the prices announced by the C-MVNO, each SU buys an optimal amount of bandwidth so as to maximize its own utility. Notice that the middle three stages can be merged into one single stage without influencing the problem essentially.

III. BACKWARD INDUCTION ANALYSIS

In this section, we use backward induction to find the solution (equilibrium) to the formulated Stackelberg game, i.e., the optimal decisions of spectrum sensing, spectrum leasing, admission control, pricing of the C-MVNO and the best demand response of the SUs. All the proofs are omitted due to the space limitation.

A. Spectrum Allocation in the Fifth Stage

After the C-MVNO announces its price $\{p_i\}_{1 \le i \le I}$ to the SUs, each SU determines its spectrum demand by maximizing its utility defined in (2). Considering the j-th SU in $\widetilde{\mathcal{S}}_i$ (recall that $\widetilde{\mathcal{S}}_i$ is a subset of $\mathcal{S}_i$ after admission control), we write its utility maximization problem as:

$$\max_{w_{ij} \ge 0} \quad u(w_{ij}) = \theta_i w_{ij} \ln\left(\frac{g_{ij}}{w_{ij}}\right) - p_i w_{ij}. \tag{3}$$

Taking derivative of (3) and setting it to be zero, we get the optimal value of w_{ij} for SU j as:

$$w_{ij}^*(p_i) = g_{ij} \exp\left\{-1 - \frac{p_i}{\theta_i}\right\}. \tag{4}$$

B. Differentiated Pricing in the Fourth Stage

Based on the best response of the heterogeneous SUs in the fifth stage, the aim of the fourth stage is to maximize the C-MVNO's revenue by selling spectrum to the SUs. The differentiated pricing problem (P1) can be formulated as follows:

$$\begin{aligned} \text{(P1)} \quad \max_{\overrightarrow{p} \succeq \overrightarrow{0}} \quad & \sum_{i=1}^{I} p_i \widetilde{G}_i \exp\left\{-1 - \frac{p_i}{\theta_i}\right\} \\ \text{s.t.} \quad & \sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-1 - \frac{p_i}{\theta_i}\right\} \le B, \end{aligned}$$

where B denotes the total available bandwidth consisting of sensing spectrum and leasing spectrum and $\overrightarrow{p}$ is the vector of $\{p_i\}_{1 \le i \le I}$. The solution to the optimization problem (P1) is summarized in the following lemma.

Lemma 1: The solution to the optimal differentiated pricing problem (P1) is as follows.

1) If $\sum_{i=1}^{I} \widetilde{G}_i e^{-2} \le B$, then $p_i^* = \theta_i, \forall i$ and the optimal value of (P1) is $\sum_{i=1}^{I} \theta_i \widetilde{G}_i e^{-2}$.
2) Otherwise, $p_i^* = \lambda^* + \theta_i$ and the optimal value of (P1) is:

$$\lambda^* B + \sum_{i=1}^{I} \theta_i \widetilde{G}_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\}, \tag{5}$$

where λ^* is determined as the unique solution to the following equation:

$$\sum_{i=1}^{I} \widetilde{G}_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\} = B. \tag{6}$$

C. Admission Control in the Third Stage

In this part, based on the results of the fourth stage and the fifth stage, we analyze the admission control decision of the C-MVNO, and it turns out that the optimal admission control scheme is to admit all the SUs. This is formally stated in the following lemma.

Lemma 2: The optimal admission control decision is to admit all the SUs, i.e., $\widetilde{\mathcal{S}}_i = \mathcal{S}_i, \widetilde{G}_i = G_i, \forall i$, and the optimal revenue[1] is shown as follows.

1) If $\sum_{i=1}^{I} G_i e^{-2} \le B$, the optimal revenue is $\sum_{i=1}^{I} \theta_i G_i e^{-2}$.
2) Otherwise, the optimal revenue is given by:

$$\lambda^* B + \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\}, \tag{7}$$

where λ^* is determined by the unique solution to:

$$\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\} = B. \tag{8}$$

D. Spectrum Leasing in the Second Stage

Denote R_2 the partial profit which is defined as the income from selling the spectrum to the SUs minus the leasing cost. We further define the following five frequently used constants:

$$\begin{aligned} & A \triangleq \sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{C_l}{\theta_i}\right\}, D \triangleq \sum_{i=1}^{I} G_i e^{-2}, \\ & E \triangleq \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2 - \frac{C_l}{\theta_i}\right\}, F \triangleq \sum_{i=1}^{I} \theta_i G_i e^{-2} \\ & H \triangleq e^{-4} \sum_{i,j=1}^{I} \frac{G_i G_j \theta_i}{\theta_i + \theta_j} \Bigg(\frac{\theta_i \theta_j}{\theta_i + \theta_j} - C_l \exp\left\{-C_l \frac{\theta_i + \theta_j}{\theta_i \theta_j}\right\} \\ & \qquad - \frac{\theta_i \theta_j}{\theta_i + \theta_j} \exp\left\{-C_l \frac{\theta_i + \theta_j}{\theta_i \theta_j}\right\} \Bigg) \end{aligned} \tag{9}$$

Based on the optimal decisions in the fifth stage, the fourth stage and the third stage, the optimal spectrum leasing strategy in the second stage and the corresponding optimal partial profit are specified in the following lemma.

Lemma 3: The optimal leasing strategy and the corresponding optimal partial profit is specified as follows.

1) If $\alpha B_s > D$, then the optimal partial profit is $R_2^* = F$ and the optimal leasing bandwidth is $B_l^* = 0$.
2) If $D > \alpha B_s \ge A$, then the optimal partial profit is given by:

$$R_2^* = \lambda^* \alpha B_s + \sum_{i=1}^{I} \theta_i G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\}, \tag{10}$$

where the λ^* is determined as the unique solution to:

$$\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\lambda^*}{\theta_i}\right\} = \alpha B_s. \tag{11}$$

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[1] Here, by revenue, we mean the revenue gained by selling the spectrum to the SUs. It is not the overall profit which should include the spectrum procurement costs.

The optimal leasing bandwidth is $B_l^* = 0$.

3) If $A > \alpha B_s \geq 0$, then the optimal partial profit is given by: $R_2^* = E + C_l \alpha B_s$. The optimal leasing bandwidth is $B_l^* = A - \alpha B_s$.

E. Spectrum Sensing in the First Stage

Denote R the overall profit of the C-MVNO. Based on the results of the previous subsections, we are now ready to derive the optimal sensing bandwidth B_s^* which maximizes the expected profit $\mathbb{E}(R)$ of the C-MVNO. The result is stated in the following lemma.

Lemma 4: The optimal sensing bandwidth B_s^* can be obtained as follows:

Low Sensing Cost Regime: When $0 < C_s \leq \frac{H}{D^2} + \frac{C_l A^2}{2D^2}$, the optimal B_s^* is given by:

$$B_s^* = \sqrt{\frac{1}{C_s}\left(H + \frac{1}{2} C_l A^2\right)}; \tag{12}$$

Medium Sensing Cost Regime: When $\frac{H}{D^2} + \frac{C_l A^2}{2D^2} < C_s \leq \frac{C_l}{2}$, the optimal B_s^* is given by

$$B_s^* = \sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\mu^*}{\theta_i}\right\}, \tag{13}$$

where μ^* is determined as the unique solution to the following equation on the interval $[0, C_l]$:

$$\begin{aligned} & e^{-4} \sum_{i,j=1}^{I} \frac{G_i G_j \theta_i}{\theta_i + \theta_j} \Bigg[-\left(C_l + \frac{\theta_i \theta_j}{\theta_i + \theta_j}\right) \exp\left\{-\frac{\theta_i + \theta_j}{\theta_i \theta_j} C_l\right\} \\ & + \left(\mu + \frac{\theta_i \theta_j}{\theta_i + \theta_j}\right) \exp\left\{-\frac{\theta_i + \theta_j}{\theta_i \theta_j} \mu\right\} \Bigg] \qquad (14) \\ & - C_s \left(\sum_{i=1}^{I} G_i \exp\left\{-2 - \frac{\mu}{\theta_i}\right\}\right)^2 + \frac{A^2 C_l}{2} = 0, \end{aligned}$$

High Sensing Cost Regime: When $C_s > \frac{C_l}{2}$, the optimal $B_s^* = 0$.

Remark 1: Lemmas 1-4 actually give us a computationally efficient algorithm to compute the optimal decisions of the C-MVNO. The algorithm operates in a reverse order to find the optimal decisions sequentially, i.e., from Lemma 4 back to Lemma 1. This enables us to study the formulated game numerically in the next section.

IV. SIMULATION RESULTS

In this section, we implement the algorithm suggested by the theoretical results in Lemmas 1-4 and test it numerically.

We first investigate the impact of sensing cost C_s and leasing cost C_l on the optimal expected profit of the C-MVNO. We set the parameters to be: $I = 20, |\mathcal{S}_i| = 20, \theta_i = i, g_{ij} = 50$ for $i \in [1, 9]$ and $g_{ij} = 100$ for $i \in [10, 20]$. In such a case we have $G_i = 1000$ for $i \in [1, 9]$ and $G_i = 2000$ for $i \in [10, 20]$. The selection of these parameters is just for demonstration purpose. Other parameters will give similar results. The results are illustrated in Figure 2, from which we can observe a threshold structure. We can see that the profit first decreases with C_s and finally remains a constant when C_s is large enough. The reason is that when the system enters into the High Sensing Cost Regime, the optimal sensing bandwidth B_s^* is always zero, i.e., the C-MVNO senses no spectrum and thus further increase in C_s will not affect the profit.

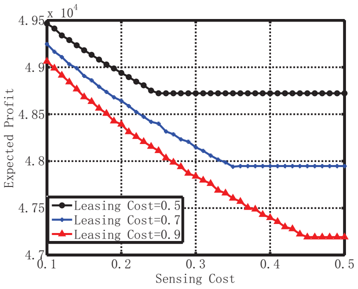

Fig. 2. Impact of C_s and C_l on the optimal expected profit of the C-MVNO.

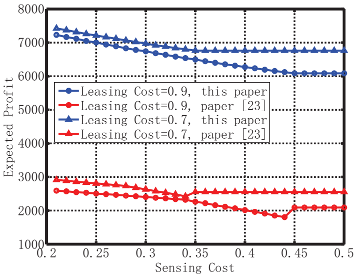

Fig. 3. A comparison between the heterogeneous scheme proposed in this paper and the homogeneous scheme proposed in [17].

Next, we compare the differentiated pricing based scheme proposed in this paper with the single pricing based scheme proposed in [17]. In order to have a fair comparison, we change the simulation parameters to be: $I = 19, \theta_i = 0.1i, |\mathcal{S}_i| = 20$ and $g_{ij} = 100, \forall i, j, G_i = 2000, \forall 1 \leq i \leq I$. Hence, the average willingness-to-pay is $\overline{\theta} = 1$. The results are shown in Figure 3. We can see that the proposed algorithm achieves much higher profit for C-MVNO than the scheme in [17], which shows the advantage of our proposed scheme in the presence of heterogeneous SUs.

V. CONCLUSION

In this paper, we study the optimal spectrum sensing, spectrum leasing, admission control as well as differentiated pricing decisions from a C-MVNO's perspective. The SUs are heterogeneous in their valuations of spectrum and this heterogeneity is modeled as different willingness-to-pay parameters. Knowing the characteristic of each SU, we invoke differentiated pricing instead of single pricing to improve the profit of the C-MVNO. Formulating the problem as a Stackelberg game, we use backward induction to analyze the optimal decisions of the C-MVNO as the equilibrium of the game. At last, in simulations, a threshold structure of the solution is observed and our proposed scheme outperforms the single pricing based scheme in previous works.

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