Understanding Sequential User Behavior in Social Computing: To Answer or to Vote?

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Abstract—Understanding how users participate is of key importance to social computing systems since their value is created from user contributions. In many social computing systems, users decide sequentially whether to participate or not and, if participate, whether to create a piece of content directly, i.e., answering, or to rate existing content, i.e., voting. Moreover, there exists an answering-voting externality as a user's utility for answering depends on votes received in the future. We present in this paper a game-theoretic model that formulates the sequential decision making of strategic users under the presence of such an answering-voting externality. We prove theoretically the existence and uniqueness of a pure strategy equilibrium. To further understand the equilibrium participation of users, we show that there exist advantages for users with higher abilities and for answering earlier. Therefore, the equilibrium has a threshold structure and the threshold for answering gradually increases as answers accumulate. We further extend our results to a more general setting where users can choose endogenously their efforts for answering. To show the validness of our model, we analyze user behavior data collected from a popular Q&A site Stack Overflow and show that the main qualitative predictions of our model match up with observations made from the data. Finally, we formulate the system designer's problem and abstract from numerical simulations several design principles that could potentially guide the design of incentive mechanisms for social computing systems in practice.

Index Terms—Game theory, incentives, social computing, social networks, user generated content (UGC)

1 Introduction

Social computing systems refer to online applications where value is created by voluntary user contributions. Recently, with rapid development of social media, the barrier for people to participate in online activities and create online content has been greatly reduced, which leads to a proliferation of social computing systems on the web. Until now, successful examples can be found in a wide range of domains, from question and answering (Q&A) sites like Yahoo! Answers, Stack Overflow or Quora where users answer questions asked by other users; to online reviews like product reviews on Amazon, restaurant reviews on Yelp or movie reviews on Rotten Tomatoes; to social news sites like Digg or Reddit where users post and promote stories under various categories. These applications help to make the Web useful by enabling large-scale high quality user generated content (UGC) and by allowing easy access to UGC. As social computing systems derive almost all their values from user contributions, it is of key importance for designers of social computing systems to understand how users participate and interact on their sites.

User participation in social computing systems can take multiple forms. In addition to creating UGC directly like answering a question on Stack Overflow or writing a product review on Amazon, an increasingly large fraction of social computing systems now allow users to participate by rating existing contributions on the site. For example, instead of answering the question, users on Stack Overflow can choose to either vote up or vote down answers posted by other users. Similarly, users on Amazon have the option to mark other users' reviews as useful or not. Such an indirect form of user participation plays multiple roles in social computing systems. First, voting provides important information regarding the quality and popularity of contributions from users. Many social computing systems like Stack Overflow, Quora and Reddit rank and display user contributions according to their received votes. More importantly, the mechanism of voting creates a strong incentive for users to participate directly and contribute high quality UGC. Users are motivated by not only the desire for peer recognition but also virtual points rewarded by the system for every positive vote they receive. For example, it has been shown that most users on Stack Overflow gain a significant portion of their reputation points through received votes [1]. It is this incentive effect of voting on user contributions the focus of this paper. In particular, we are interested in how the voting behavior of users may affect the amount and quality of UGC in social computing systems. Without loss of generality, we will adopt Q&A terminologies and refer the action of creating UGC as answering henceforth.

A key aspect of modeling and analyzing the close interaction between answering and voting is to recognize that users participate in social computing systems sequentially rather than simultaneously. Let us consider, for example, a question to be answered on a Q&A site. Potential contributors view the question sequentially and decide whether

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to participate based on observations of the history of the question. If users decide to participate, they can further choose to answer the question directly with possibly different efforts or to vote on existing answers contributed by previous users. Moreover, actions from future users have a great impact on a current user's utility since the utility of answering a question depends on the future votes received by the answer. What can we understand in such a sequential setting about the externality created by future users' voting choices on the current user's answering action? And given the presence of such an externality, how can we model and analyze sequential user behavior for social computing systems? Finally, how should designers of social computing systems adjust their incentive mechanisms to steer user behavior to achieve various system objectives?

1.1 Our Contributions

We address the above questions from a game-theoretic perspective. Our first contribution is a sequential game model that captures the strategic decision making of sequentially arrived users who choose endogenously whether to participate or not and, if participate, whether to answer the question or to vote on existing answers. Users who choose voting can either vote up or vote down an answer based on the quality of the answer. Users who answer the question will receive certain amount of virtual points for each upvote their answers receive and lose virtual points for each downvote, which creates a form of externality among users. We call this the answering-voting externality. We further incorporate into our model two typical scenarios in social computing. In the first scenario, inspired by questions on focused Q&A sites like Stack Overflow, the quality of an answer is determined primarily by the domain knowledge and the level of expertise of a user. Therefore, we consider a homogenous effort model where the quality of answer is a function of a user's ability and the cost incurred by answering is uniform among users. The second scenario corresponds to a more general setting where users can greatly improve the quality of answers by increasing effort. In this case, we assume that users who decide to answer the question can also choose endogenously the amount of effort to put. Therefore, the quality of answer becomes a function of a user's ability and the effort he exerts; the cost incurred by answering is modeled as a function of a user's effort. We refer to this model as the endogenous effort model. We will discuss the proposed sequential game in details in Section 3.

Next, we analyze the sequential user behavior through equilibrium analysis of the proposed game. We begin with the homogenous effort model in Section 4. The solution concept of symmetric Nash equilibrium (SNE) is adopted and we show that there always exists a unique pure strategy SNE for the proposed game. To further investigate the equilibrium user behavior, the key is to understand the answering-voting externality, which is expressed by the long-term expected reward for answering. We show that such a reward is increasing with respect to answer quality and as a direct result, there exists a threshold structure of the equilibrium. Such a threshold structure greatly reduces the action space of users at the equilibrium and enables us to develop a dynamic programming algorithm to efficiently calculate the equilibrium. Moreover, we find that the reward for answering is decreasing in terms of the number of previous answers which indicates an advantage for answering earlier. As a result, as answers accumulate, the threshold of user ability for answering increases, showing that it becomes more and more competitive to answer the question. We then turn our attention to the endogenous effort model in Section 5, where we show that results obtained for the homogenous effort model capture the essence of the game and can be extended naturally to incorporate the more general setting.

Thirdly, after developing a sequential game-theoretic model and analyzing user behavior through equilibrium analysis, we investigate how qualitative predictions derived from our model compare with aggregated user behavior on a large-scale social computing site. Towards this end, we use user behavior data from one of the most popular Q&A site Stack Overflow to evaluate our model in Section 6. We find that the main qualitative predictions of our model are consistent with observations made from the real-world data, which validates our model.

Finally, in Section 7, we study how system designers can use our model to aid their design of incentive mechanisms, i.e., the allocation of virtual points, in practice. We formalize the system designer's problem by proposing a utility function that can be designed to incorporate several typical use case scenarios. We abstract through numerical simulations several design principles that could guide system designers on how to steer user behavior to achieve a wide range of system objectives. The impact of other factors such as user distributions on system designer's utility is also studied.

2 Related Work

There is a growing body of literature that studies user contributions on social computing sites using game-theoretic approaches. Different forms of incentives have been considered to stimulate user participation, including badges [2], [3], monetary rewards or virtual points [4], [5], [6], [7] and attention [8]. Badges are employed by social computing sites to recognize users for various types and degrees of overall contributions to the site. In [2], Anderson et al. proposed a model for user behavior on social media sites in the presence of badges. Through analyzing the best strategy for users, they find that users are influenced by badges, which is consistent with aggregated user behavior they observed from Stack Overflow. In [3], Easley and Ghosh analyzed equilibrium existence and equilibrium user participation for two widely adopted badge mechanisms: badges with absolute standards and badges with relative standards. Our work is different from [2] and [3] mainly in that we consider virtual points as the motivating factor for users and study user behavior within a single task such as a question rather than overall contributions of users on a site.

Our work relates more closely to studies that use monetary rewards or virtual points as means to incentivize user contributions [4], [5], [6], [7]. Gao et al. studied cost effective incentive mechanisms for microtask crowdsourcing in [5], where a novel mechanism for quality-aware worker training is proposed to reduce the requester's cost in stimulating high quality solutions from self-interested workers. In [6], Ghosh and Hummel studied the issue of whether, in the presence of strategic users, the optimal outcome can be implemented through a set of mechanisms that are based on virtual points. Our model shares several common features with [6], including the assumption of voluntary participation and the consideration of both homogenous effort and endogenous effort. The incentive mechanism design problem for online Q&A sites has been studied in [7], where the objective is to incentivize users to contribute their answers more quickly. Similar as in [7], we use in our model the number of answers to summarize the history of a question without further distinguishing between answer qualities.

Our work differs from studies in [4], [5], [6], [7], [8] mainly in the following two respects. First, we consider that users participate sequentially rather than simultaneously. In many social computing systems, users act sequentially and will make different decisions under different situations. For example, a user may choose to answer a question if there are few answers or not to participate if the question has already received a large number of answers. The sequential game model enables us to study the strategic decision making of users under different states and thus provides a better characterization of the dynamics of user behavior on social computing sites. Second, we explicitly consider the answering-voting externality in our model, whereas in prior studies the voting action either is not considered [4], [5], [7], or has no impact on other users' utility [6] or is assumed to be performed by another group of non-strategic users [8]. To the best of our knowledge, this is the first work that studies the sequential user behavior in the presence of answering-voting externality for social computing systems.

Another active line of research on user participation in the context of social computing is based on analysis of empirical data from social-psychological perspectives [9], [10], [11], [12]. User behavior on a crowdsourcing site Taskcn has been investigated in [11]. The authors show that users who remain on the site learn to behave more strategically to optimize their expected payoffs. In [12], Tausczik and Pennebaker find through surveys and user behavior data that building reputation, i.e., accumulating virtual reputation points, serves as an important incentive for users on Q&A sites. These studies provide evidence for strategic user behavior in social computing systems and thus motivate the search for a more systematic understanding of user participation in these environments using game-theoretic models.

Our work is also related to the economics literature on contests. Contests model situations where agents contribute strategically to a common task in order to win rewards. The focus is to design mechanisms that can elicit the most desirable contributions from participants through the allocation of rewards [13], [14]. The contest model has been applied to many scenarios, such as research tournament [15] and crowdsourcing [16]. The model in this paper is different from a contest in that agents bid their efforts simultaneously in a contest while we assume users take actions sequentially. Moreover, the answering-voting externality has not been considered in contest models.

3 The Model

Let us consider a single task on a social computing site that solicits contributions from users. Such a task can be either a question from an online Q&A forum, a product/restaurant on Amazon/Yelp for users to review, or a tourist site on Tripadviser where users can report their experience. In the remaining of this paper, we will use terminologies from Q&A scenarios such as questions and answers for the ease of discussion, while our results apply equally to other social computing systems as well.

We assume that there is a countably infinite set of users, denoted by $\mathcal{N} = \{1, 2, 3, \ldots\}$, who view and may contribute to the question. We divide time into time slots and assume that users arrive sequentially one at a time slot. Once arrived, users choose strategically to either answer the question, vote on an existing solution, or do not participate. Denote by $\Theta = \{A, V, N\}$ the action set where A represents to answer, V to vote and N not to participate.

Different users have different types, which influence their choices of actions. We represent the type of a user as a tuple of two elements: $\sigma = (\sigma_A, \sigma_V)$. The first element, $\sigma_A \in [0, 1]$, represents the ability of a user in answering the question. A user with a higher value of σ_A is more capable of answering the question than a user who has a lower value. The second element, $\sigma_V \in [V_{min}, V_{max}]$, models the degree to which a user would like to express his opinions through voting, which we refer to as the voting preference. The σ_V can have either positive or negative values; the larger value of σ_V a user has, the more he favors voting. Denote by $\Omega = [0, 1] \times [V_{min}, V_{max}]$ the set of user types.

The user type σ is independent and identically distributed according to a distribution with cumulative distribution function (CDF) $F(\sigma_A, \sigma_V)$. Such a distribution is assumed to be public knowledge while the instantiation of type is known only to a user himself.

Among the three possible actions, action N is the most straightforward one. A user who chooses action N will simply leave the question quietly without making any impact on the state of the question. Users incur no cost by choosing action N and will not receive any reward from the system as well. We now describe in details the other two actions.

3.1 The Answering Action

Users who choose action A will submit answers of various qualities. We denote by $q \in [0, 1]$ the quality of an answer, which represents the probability of being favored by a future user.

For the answering action, we consider two typical scenarios in social computing. In the first scenario such as questions on focused Q&A sites like Stack Overflow, the quality of an answer is determined primarily by the domain knowledge and the level of expertise of a user. The cost of creating an answer is incurred mostly by transcribing a user's knowledge and thus is uniform among users. On the other hand, in the second scenario, users can greatly improve the quality of answer by increasing their effort, which incurs a higher cost. For example, by putting a considerable amount of effort, users can write good reviews on Amazon or

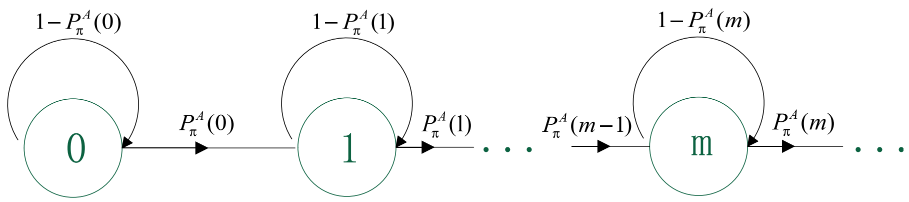


Fig. 1. The state transition of the proposed game.

interesting travel notes on TripAdvisor. We formally capture these two scenarios by the homogeneous effort model and the endogenous effort model.

1) *Homogenous effort model:* In the homogenous effort model, the quality of answer is determined purely by a user's ability σ_A. Without loss of generality, we assume that $q = \sigma_A$. The cost to answering is uniform among all users but may depend on the number of existing answers m. We use $c(m)$ to represent the cost and assume
   a) $c(m)$ is non-decreasing in m, i.e., it may be harder to provide a novel answer to a question that has more answers than the one that has fewer answers; and
   b) $c(0) > 0$, i.e., answering a question, even when there are no existing answers, incurs some cost.

   A simple example is $c(m) = c > 0$, i.e., there is a constant cost for answering the question.
2) *Endogenous effort model:* In the endogenous effort model, conditioned on choosing action A, a user will also decide the amount of effort $e \in [0,1]$ in creating the answer. The quality of answer becomes a function of not only a user's ability σ_A but also his effort e, which we write as $q = \phi(\sigma_A, e)$. We assume ϕ is monotonically increasing in both σ_A and e. The cost incurred by answering is denoted by $c(m, e)$, which we assume is strictly greater than 0 and non-decreasing in m and e.

In the following, we will first focus on the homogenous effort model, which helps to understand the essence of the game. That is, we assume $q = \sigma_A$ and adopt $c(m)$ as the cost for answering. Then in Section 5, we extend our results to the endogenous effort case. The gain from answering a question depends on voting actions of future users and will be discussed later in this section.

3.2 The Voting Action

Users can choose action V if there are existing answers to the question, i.e., $m > 0$. We assume that once decides to vote, a user will randomly choose an answer with equal probability to cast the vote. Users can either vote up or vote down an answer based on the answer quality. In particular, if the chosen answer has quality q, then the user will vote up with probability q and vote down with probability $1 - q$. The utility of a user with type σ who chooses action V can be written as $\sigma_V + R_V - C_V$. Recall that σ_V is the preference of a user towards voting. When $\sigma_V < 0$, it implies that the user dislikes voting and more incentives are needed to stimulate him to vote. The R_V represents the reward provided by the system. For more generality, we assume it is possible for R_V to have negative values, which models the case where the system discourages voting by charging users for voting. The $C_V > 0$ denotes the cost incurred by users for casting a vote, for instance the effort of evaluating the quality of answer.

The answering action and the voting action are connected through an incentive mechanism that is built with virtual points. In particular, if a user chooses action A, he will receive $R_u > 0$ points for every upvote his answer receives and loses $R_d > 0$ points for every downvote. Therefore, R_u and R_d, together with R_V, define the mechanism in our model, which connects the answering and voting actions of users, determines the equilibrium of the game, and provides a tool for the system designer to incentivize desired user behavior.

3.3 Action Rule and Utility

An action rule describes how a user will play under all possible situations in the game. We use the number of existing answers m to represent the state of the game, which summarizes the history of the question. When a user arrives to the question, he first observes the state of the question and then chooses his action based on the state as well as his own type σ. For more generality, we assume mixed actions. That is, a user will choose a probability distribution over the action set Θ instead of a single action item. Therefore, an action rule in the proposed game is a mapping from m and σ to a probability distribution over Θ. $\forall m \geq 0$ and $\sigma \in \Sigma$, we have $\pi(m, \sigma) = [\pi_A(m, \sigma), \pi_V(m, \sigma), \pi_N(m, \sigma)]$, where $\pi_\theta(m, \sigma)$ with $\theta \in \Theta$ represents the probability of choosing action θ and satisfies $\pi_A(m, \sigma) + \pi_V(m, \sigma) + \pi_N(m, \sigma) = 1$. We denote by Π the set of all action rules.

Given an action rule π, the probability of a random user choosing action A at state m can be calculated as $P_\pi^A(m) = \mathbb{E}_\sigma[\pi_A(m, \sigma)]$, where the expectation is taken over the distribution of user types. Similarly, the probability of voting can be expressed as $P_\pi^V(m) = \mathbb{E}_\sigma[\pi_V(m, \sigma)]$. To summarize, we show state transitions of the proposed game given an action rule π in Fig. 1.

We assume users are impatient and prefer to receive the reward sooner rather than later, which is modeled by a discounting factor $\delta \in (0, 1)$. In particular, the future reward of a user will be discounted by δ at each time slot. Such a modeling approach is a standard practice that is widely adopted in the economics literature [17], [18]. The reward a user can receive by answering the question comes from subsequent users' votes. Let $g_\pi(m, q)$ represent the long-term expected reward a user, who has produced an answer with quality q, will receive starting from state m given that the action rule π is adopted by subsequent users. We will refer to such a function as the reward function for answering or simply as reward function henceforth. Note that $g_\pi(m, q)$ is defined for $m \geq 1$. We can write an expression for $g_\pi(m, q)$ as follows:

$$g_\pi(m,q) = \frac{P_\pi^V(m)}{m}[(R_u + R_d)q - R_d] + \delta\big[P_\pi^A(m)g_\pi(m+1,q) + (1 - P_\pi^A(m))g_\pi(m,q)\big]. \quad (1)$$

The first term in (1) corresponds to the immediate reward received in the current time slot, where $\frac{P_\pi^V(m)}{m}$ represents the probability of receiving a vote and $(R_u + R_d)q - R_d = R_u q - R_d(1-q)$ is the expected reward brought by a vote. The second term represents the future reward, which is determined by state transitions of the game and discounted by δ.

Equation (1) illustrates that a user's reward for answering depends on the action rule of subsequent users. Such a dependence creates an answering-voting externality among users and motivates users to condition their decision makings on other users' action rules. We evaluate the utility of a user by assuming a common action rule for other users, which is sufficient for analyzing symmetric outcomes. In particular, we write $u(m, \sigma, \theta, \tilde{\pi})$ as the utility of a user who has type σ and chooses the pure action $\theta \in \Theta$ when there are m existing answers and subsequent users adopt $\tilde{\pi}$ as their action rule. We have

$$u(m,\sigma,\theta,\tilde{\pi}) = \begin{cases} -c(m) + \delta g_{\tilde{\pi}}(m+1, \sigma_A) & \text{if } \theta = A \\ \sigma_V + R_V - C_V & \text{if } \theta = V \text{ and } m > 0 \\ 0 & \text{if } \theta = N. \end{cases} \quad (2)$$

Note that the reward for answering is discounted with δ since the reward will be received starting from the next time slot.

Based on the definition of action rule, we can write the utility of a user who adopts an action rule π as

$$\mathcal{U}(m,\sigma,\pi,\tilde{\pi}) = \sum_{\theta\in\Theta} \pi_\theta(m,\sigma)\cdot u(m,\sigma,\theta,\tilde{\pi}).$$

3.4 Solution Concept

The proposed game can be formally defined as a tuple $\mathcal{G} = (\mathcal{N}, \Pi, \mathcal{U})$, where $\mathcal{N}$ is the set of users, Π represents the set of action rules and $\mathcal{U}$ is the utility function. Since users act sequentially and $\mathcal{N}$ is countably infinite, the proposed game $\mathcal{G}$ is a sequential game with infinite horizon. To study the proposed game $\mathcal{G}$, we adopt the solution concept of symmetric Nash equilibrium, which can be formally defined as follows.

Definition 1. *An action rule $\hat{\pi}$ is a symmetric Nash equilibrium of the proposed game $\mathcal{G}$ if and only if*

$$\hat{\pi} \in \arg\max_{\pi\in\Pi} \mathcal{U}(m,\sigma,\pi,\hat{\pi}), \quad \forall m \geq 0, \sigma \in \Omega. \quad (3)$$

To show SNE is a valid solution concept for $\mathcal{G}$, we prove in next section that there exists a unique SNE that has a threshold structure. Such an SNE can be computed efficiently using dynamic programming and is easy for users to follow. We also demonstrate that under mild conditions any Nash equilibrium for $\mathcal{G}$ is equivalent to the unique SNE, which shows the generality of SNE as a solution concept for $\mathcal{G}$. Moreover, based on the definition, an SNE in $\mathcal{G}$ must contains equilibrium actions in every state m, and therefore is subgame perfect [18], [19].

4 Equilibrium Analysis

In this section, we conduct equilibrium analysis to understand how users participate sequentially under the presence of answering-voting externality. Particularly, the answering-voting externality is expressed through the reward function for answering g_π, which is the key to analyze the proposed game. We will first explore several properties of the reward function g_π. These properties enable us to establish the existence and uniqueness as well as the threshold structure of the SNE. We will then develop a dynamic programming algorithm to obtain the SNE efficiently and further discuss properties of the SNE.

We first show that for any action rule π, the reward function g_π can be upper bounded by a decreasing function of m, as illustrated below.

Proposition 1. *$\forall \pi \in \Pi$, we have*

$$g_\pi(m,q) \leq \max\left\{\frac{(R_u + R_d)q - R_d}{(1-\delta)m}, 0\right\} \quad \forall m \geq 1, q \in [0,1]. \quad (4)$$

Proof. We prove Proposition 1 by invoking another equivalent expression of $g_\pi(m,q)$ that follows directly from its definition as

$$g_\pi(m,q) = \mathbb{E}\left\{\sum_{t=0}^{\infty}\delta^t \frac{P_\pi^V(Y_t)}{Y_t}[(R_u+R_d)q - R_d]\,\middle|\, m,\pi\right\}, \quad (5)$$

where the expectation is taken over the randomness of user types and action rules. The time slot is indexed by t and $t = 0$ stands for the current time slot. We denote by $\{Y_t\}_{t=0}^{\infty}$ the discrete random process of the state. Conditioned on the current state m, we have $Y_0 = m$. By relaxing (5), we have

$$g_\pi(m,q) \leq \mathbb{E}\left\{\sum_{t=0}^{\infty}\delta^t \frac{1}{Y_t}[(R_u+R_d)q - R_d]\,\middle|\, m,\pi\right\}. \quad (6)$$

If $[(R_u + R_d)q - R_d] \leq 0$, since $Y_t \geq 0$ and $\delta \geq 0$, we have $g_\pi(m,q) \leq 0$.

On the other hand, if $[(R_u + R_d)q - R_d] > 0$, the expression inside the expectation in (6) decreases with respect to Y_t. Then, given the current state m, $\{Y_t = m\}_{t=0}^{\infty}$ is the one that achieves the highest value among all realizations of $\{Y_t\}_{t=0}^{\infty}$. Therefore, we have

$$g_\pi(m,q) \leq \left\{\sum_{t=0}^{\infty}\delta^t \frac{1}{m}[(R_u+R_d)q - R_d]\right\} = \frac{(R_u+R_d)q - R_d}{(1-\delta)m}. \quad (7)$$

Combining above results we can establish Proposition 1. □

Based on results of Proposition 1, we show in the following that no user will have the incentive to answer the question if the number of existing answers is large enough.

Lemma 1. *After reaching a certain state, action A will be strictly dominated by action N, i.e., $\exists \tilde{m} \geq 0$ such that $\forall m \geq \tilde{m}$, $u(m,\sigma,A,\tilde{\pi}) < u(m,\sigma,N,\tilde{\pi})$.*

Proof. Let us consider a user's utility of choosing action A. For any action rule $\tilde{\pi}$, we have

$$u(m,\sigma,A,\tilde{\pi}) \leq -c(m) + \delta \max\left\{\frac{(R_u+R_d)\sigma_A - R_d}{(1-\delta)(m+1)}, 0\right\} \tag{8}$$

$$\leq -c(m) + \frac{\delta R_u}{(1-\delta)(m+1)}. \tag{9}$$

The inequality in (8) follows from Proposition 1. Note the right hand side expression in (9) is strictly decreasing in m and $\lim_{m\to\infty}\{-c(m) + \frac{\delta R_u}{(1-\delta)(m+1)}\} \leq -c(0) < 0$. Therefore, there exists $\tilde{m} \geq 0$ such that $\forall m \geq \tilde{m}$, we have $u(m,\sigma,A,\tilde{\pi}) < 0 = u(m,\sigma,N,\tilde{\pi})$, which implies that action A is strictly dominated by action N and thus users will have no incentive to choose action A. □

Lemma 1 shows that the state in the proposed game will stop growing after a certain value. Therefore, the last state is an absorbing state, which represents the largest possible number of answers a question can have. Due to the existence of such an absorbing state, we can then establish the existence of SNE as demonstrated in the following theorem.

Theorem 1. *There always exists a symmetric Nash equilibrium for the proposed game $\mathcal{G}$.*

Proof. We explicitly construct an SNE action rule $\hat{\pi}$ to show the existence result. From Lemma 1, we know that there exists $\tilde{m} \geq 0$ such that $\forall m \geq \tilde{m}$, we have $u(m,\sigma,A,\tilde{\pi}) < 0 = u(m,\sigma,N,\tilde{\pi})$.

For $m \geq \tilde{m}$, we choose $\hat{\pi}$ such that $\pi_V(m,\sigma) = \mathbf{1}(\sigma_V + R_V - C_V \geq 0)$, $\pi_N(m,\sigma) = 1 - \pi_V(m,\sigma)$ and $\pi_A(m,\sigma) = 0$. Since action A is strictly dominated by action N and the utility of choosing action V is $\sigma_V + R_V - C_V$, this particular choice of $\hat{\pi}$ is the best response for users at state $m \geq \tilde{m}$ independent of other users' action rule.

For $m < \tilde{m}$, we construct $\hat{\pi}$ using backward induction. Recall from (2) that a user's utility at state m depends on other users' action rule only for states starting from $m+1$. In other words, modifying other users' action rule for states $m' \leq m$ will not affect a user's best response at state m. Based on this observation, we iteratively set $\hat{\pi}$ from $m = \tilde{m}-1$ to 0 to be the best response of users as follows:

$$\hat{\pi}(m,\sigma) \in \arg\max_{\mathbf{p}\in P_\Theta} \mathbf{p} \cdot [\, u(m,\sigma,A,\hat{\pi}) \quad u(m,\sigma,V,\hat{\pi}) \quad u(m,\sigma,N,\hat{\pi}) \,]^T, \tag{10}$$

where $\mathbf{p} = [\, p_A \quad p_V \quad p_N \,]$ is a probability distribution over Θ and P_Θ represents the set of all probability distributions over Θ.

With the constructed action rule $\hat{\pi}$, we have $\hat{\pi} \in \arg\max_{\pi\in\Pi}\mathcal{U}(m,\sigma,\pi,\hat{\pi})$, $\forall m \geq \tilde{m}, \sigma \in \Omega$ since $\pi_V(m,\sigma) = \mathbf{1}(\sigma_V + R_V - C_V \geq 0)$, $\pi_N(m,\sigma) = 1 - \pi_V(m,\sigma)$ and $\pi_A(m,\sigma) = 0$ is the best response for users in state $m \geq \tilde{m}$ regardless of others' action rule. Moreover, from (10) we have $\hat{\pi} \in \arg\max_{\pi\in\Pi}\mathcal{U}(m,\sigma,\pi,\hat{\pi})$, $\forall 0 \leq m < \tilde{m}, \sigma \in \Omega$. Therefore, the constructed action rule $\hat{\pi}$ is an SNE, which proves Theorem 1. □

Once the existence of SNE has been established, we can obtain a tighter bound on $g_{\hat{\pi}}$ and the absorbing state for SNE action rules, as demonstrated below.

Corollary 1. *If $\hat{\pi}$ is an SNE, then*

$$g_\pi(m,q) \leq \max\left\{\frac{P_V[(R_u+R_d)q - R_d]}{(1-\delta)m}, 0\right\} \quad \forall m \geq 1, q \in [0,1], \tag{11}$$

where $P_V = \mathbb{E}_\sigma[\mathbf{1}(\sigma_V + R_V - C_V \geq 0)]$.

Proof. Corollary 1 can be proved in a very similar way as Proposition 1. The only modification we need is to use a tighter bound for $P_{\hat{\pi}}^V$, i.e., $P_{\hat{\pi}}^V \leq P_V$, since in SNE users will choose action V only if the utility for voting is greater than 0. □

Corollary 2. *If $\hat{\pi}$ is an SNE, then $\hat{\pi}_A(m,\sigma) = 0, \forall m \geq \overline{m}, \sigma \in \Omega$, where $\overline{m} = \lceil m^* \rceil$ such that $c(m^*) = \frac{\delta P_V R_u}{(1-\delta)(m^*+1)}$.*

Proof. Corollary 2 can be proved following the same steps as in Lemma 1 and use the tighter bound of $g_{\hat{\pi}}$ given by Corollary 1. □

Next, we show that given an arbitrary action rule π (not necessarily an SNE), a higher quality answer will almost always receive a larger reward than a lower quality answer does. Our results are summarized in the following proposition.

Proposition 2. *Given an action rule π and $m \geq 1$, $g_\pi(m,q)$ is a continuous function of q. Moreover, if $\exists q \in [0,1]$ such that $g_\pi(m,q) \neq 0$, then $\forall 0 \leq q_1 < q_2 \leq 1$, we have $g_\pi(m,q_1) < g_\pi(m,q_1)$.*

Proof. Let us consider the time series expression of $g_\pi(m,q)$ in (5). Since the expectation is irrelevant to q, we have

$$g_\pi(m,q) = \mathbb{E}\left\{\sum_{t=0}^{\infty} \delta^t \frac{P_\pi^V(Y_t)}{Y_t}\middle| m,\pi\right\}[(R_u+R_d)q - R_d], \tag{12}$$

which is linear in q and thus a continuous function of q. Moreover, we have

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \delta^t \frac{P_\pi^V(Y_t)}{Y_t}\middle| m,\pi\right\} \geq 0. \tag{13}$$

If the equality holds, then $g_\pi(m,q) = 0, \forall q \in [0,1]$. On the other hand, since $R_u > 0$ and $R_d > 0$, it follows that $g_\pi(m,q)$ is strictly increasing in q. □

Proposition 2 shows that the reward function $g_\pi(m,q)$ is strictly increasing in answer quality q, except for the extreme case where no users will vote at all. This implies that users with higher abilities will have an advantage for answering the question. Such a property can be employed to greatly simplify the SNE, which we show in the following theorem.

Theorem 2. *There exists a pure strategy SNE that has a threshold structure in each state, i.e.,* $\forall m \geq 0,\ \sigma_V \in [V_{min}, V_{max}]$, $\exists \hat{a}(m,\sigma_V) \in [0,1]$ *and* $\hat{\sigma}_V = C_V - R_V$ *such that*

$$\begin{cases} [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [1,\ 0,\ 0] \\ \hfill \text{if } \sigma_A > \hat{a}(m,\sigma_V) \\ [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [0,\ 1,\ 0] \\ \hfill \text{if } \sigma_A \leq \hat{a}(m,\sigma_V) \text{ and } \sigma_V \geq \hat{\sigma}_V \text{ and } m \geq 1 \\ [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [0,\ 0,\ 1] \\ \hfill \text{otherwise.} \end{cases} \tag{14}$$

The above action rule is the unique SNE in the sense that other possible SNEs can only differ from it for $\sigma_A = \hat{a}(m,\sigma_V)$ *or* $\sigma_V = \hat{\sigma}_V$.

Proof. Define $U(m,\sigma_V)$ as the maximum utility that a user with voting preference σ_V can receive at state m other than choosing action A, i.e., $U(m,\sigma_V) \triangleq \max\{\sigma_V + R_V - C_V, 0\} \cdot \mathbf{1}(m \geq 1)$. Note that $U(0,\sigma_V) = 0$ since action V is not an option when $m = 0$.

Let us consider an arbitrary SNE $\hat{\pi}$. We first show that there exists a threshold $\hat{a}(m,\sigma_V)$ such that users will choose action A in $\hat{\pi}$ only if their abilities are larger than the threshold. We know

$$\begin{aligned} u(m,\sigma,A,\hat{\pi})|_{\sigma_A=0} &= -c(m) + \delta g_{\hat{\pi}}(m+1,0) \\ &\leq -c(m) < 0 \leq U(m,\sigma_V). \end{aligned} \tag{15}$$

Recall $u(m,\sigma,A,\hat{\pi})|_{\sigma_A=1} = -c(m) + \delta g_{\hat{\pi}}(m+1,1)$ and $g_{\hat{\pi}}(m,\sigma_A)$ is a continuous function of σ_A. If

$$u(m,\sigma,A,\hat{\pi})|_{\sigma_A=1} \geq U(m,\sigma_V), \tag{16}$$

then there exists a solution $\sigma_A^* \in [0,1]$ to

$$-c(m) + \delta g_{\hat{\pi}}(m+1,\sigma_A^*) = U(m,\sigma_V). \tag{17}$$

We set $\hat{a}(m,\sigma_V) = \sigma_A^*$. On the other hand, if (16) does not hold, we set $\hat{a}(m,\sigma_V) = 1$ indicating that it is impossible for users to have ability higher than the threshold.

Let us consider a user with type $\sigma = (\sigma_A, \sigma_V)$. When $\sigma_A > \hat{a}(m,\sigma_V)$, it implies that (16) holds and thus we have

$$\begin{aligned} u(m,\sigma,A,\hat{\pi}) &= -c(m) + \delta g_{\hat{\pi}}(m+1,\sigma_A) \\ &> -c(m) + \delta g_{\hat{\pi}}(m+1,\sigma_A^*) = U(m,\sigma_V), \end{aligned} \tag{18}$$

which shows it is optimal to choose action A with probability 1, i,e, $\hat{\pi}_A(m,\sigma) = 1$. Similarly, when $\sigma_A < \hat{a}(m,\sigma_V)$, we have $u(m,\sigma,A,\hat{\pi}) < U(m,\sigma_V)$, which shows action A is strictly dominated and thus $\hat{\pi}_A(m,\sigma) = 0$. When $\sigma_A = \hat{a}(m,\sigma_V)$, there exists at least one action from $\{V, N\}$ that has the same utility as choosing action A. Therefore $\hat{\pi}_A(m,\sigma) = 0$ is optimal.

Next, for cases where action A is dominated, i.e., $\sigma_A \leq \hat{a}(m,\sigma_V)$, users will only consider action V and action N. Recall that the utility of choosing action V is $\sigma_V + R_V - C_V$. Therefore, it is the best response for users to choose $\hat{\pi}_V(m,\sigma) = \mathbf{1}(\sigma_V \geq C_V - R_V) \cdot \mathbf{1}(m \geq 1)$ and $\hat{\pi}_N(m,\sigma) = 1 - \hat{\pi}_V(m,\sigma)$.

Therefore, the action rule given in (14) is an SNE. Moreover, such an action rule is essentially a pure strategy action rule in that users will choose one action with probability 1 in all situations.

To prove Theorem 2, we are left to show that the action rule given in (14) is also a unique SNE. According to Proposition 2, we know $g_{\hat{\pi}}(m,\sigma_A)$ is strictly increasing in σ_A when (16) holds due to $g_{\hat{\pi}}(m+1,1) > 0$. Therefore, the solution to (17) and thus the threshold $\hat{a}(m,\sigma_V)$ is unique. Since $\hat{\pi}$ is an arbitrary SNE in our proof, the uniqueness of $\hat{a}(m,\sigma_V)$ implies that, in any SNE, users will follow the same action rule in (14) except for those with $\sigma_A = \hat{a}(m,\sigma_V)$ or $\sigma_V = C_V - R_V$. □

From Theorem 2, the SNE of the proposed game not only exists, but also is unique and in the form of pure strategy. Moreover, such a unique pure strategy SNE has a threshold structure at every state: users will choose answering only if their ability σ_A is greater than a threshold function $\hat{a}(m,\sigma_V)$; otherwise users will choose either to vote or not to participate based on a constant threshold $\hat{\sigma}_V$ on their voting preferences. Such a threshold structure greatly simplifies the action space of users. As a result, the SNE can be expressed equivalently using a threshold function $\hat{a}$ and a constant $\hat{\sigma}_V$. We show in the following that this equivalent form of SNE can be efficiently obtained through a dynamic programming algorithm.

Algorithm 1. A Dynamic Programming Algorithm to Find the Unique SNE

$\hat{\sigma}_V \leftarrow C_V - R_V$
$\hat{a}(m,\sigma_V) \leftarrow 1$ **for** $m \geq \bar{m}, \sigma_V \in [V_{min}, V_{max}]$,
$g_{\hat{\pi}}(\overline{m}, q) \leftarrow \frac{P_V[(R_u+R_d)q-R_d]}{(1-\delta)\bar{(m)}}$
for $m = \overline{m} - 1 : 0$ **do**
  $U(m,\sigma_V) \leftarrow \max\{0, \sigma_V + R_V - C_V\} \cdot \mathbf{1}(m \geq 1)$
  if $\delta g_{\hat{\pi}}(m+1,1) - c(m) \leq U(m,\sigma_V)$ **then**
    $\hat{a}(m,\sigma_V) \leftarrow 1$
  else
    $\hat{a}(m,\sigma_V) \leftarrow a$ where $\delta g_{\hat{\pi}}(m+1,a) - c(m) = U(m,\sigma_V)$
  end if
  if $m \geq 1$ **then**
    $P_{\hat{\pi}}^A(m) \leftarrow \int \mathbf{1}(\sigma_A \leq \hat{a}(m,\sigma_V)) dF(\sigma)$
    $P_{\hat{\pi}}^V(m) \leftarrow \int [\mathbf{1}(\sigma_A \leq \hat{a}(m,\sigma_V)) \cdot \mathbf{1}(\sigma_V \geq \hat{\sigma}_V)] dF(\sigma)$
    $g_{\hat{\pi}}(m,q) \leftarrow \frac{\left\{\frac{P_{\hat{\pi}}^V(m)}{m}[(R_u+R_d)q-R_d] + \delta P_{\hat{\pi}}^A(m) g_{\hat{\pi}}(m+1,q)\right\}}{1-\delta(1-P_{\hat{\pi}}^A(m))}$
  end if
end for
Output $(\hat{a}, \hat{\sigma}_V)$

Corollary 3. *The thresholds that define the SNE in* $\mathcal{G}$, *i.e.,* $(\hat{a}, \hat{\sigma}_V)$, *can be obtained using Algorithm 1.*

Proof. From Corollary 2, we know that for $m \geq \overline{m}$, no users will choose action A in SNE. Therefore, we can set $\hat{a}(m, \sigma_V) = 1$ for $m \geq \overline{m}$ and $\sigma_V \in [V_{min}, V_{max}]$. Moreover, as $P^A_{\hat{\pi}}(\overline{m}) = 0$, we can derive from (1) the expression of $g_{\hat{\pi}}(\bar{m}, q)$ as shown in Algorithm 1. Then, based on $g_{\hat{\pi}}(\overline{m}, q)$, we can iteratively calculate the threshold from $m = \overline{m} - 1$ to 0, following the steps outlined in the proof of Theorem 2. □

We can obtain a stronger sense of uniqueness for equilibrium in $\mathcal{G}$ when the distribution of user types is atomless.

Corollary 4. *If the distribution of user types, $F(\sigma_A, \sigma_V)$, is atomless, then Nash equilibria of $\mathcal{G}$ differ from each other only for 0 mass users.*

Proof. We prove Corollary 4 using mathematical inductions. We first show that there exists $\tilde{m} > 0$ such that, $\forall m \geq \tilde{m}$, all Nash equilibria of $\mathcal{G}$ will be the same except for 0 mass users. Let $\bar{g}(m, q)$ represent the largest possible reward a user can get by answering the question regardless subsequent users' actions. We have

$$\bar{g}(m,q) \leq \mathbb{E}\left\{ \sum_{t=0}^{\infty} \delta^t \frac{1}{Y_t}[(R_u + R_d)q - R_d] \middle| m, \pi \right\} \leq \max\left\{ \frac{(R_u + R_d)q - R_d}{(1-\delta)m}, 0 \right\}.$$

Denote by $\bar{u}(m, \sigma, A)$ the maximum utility a user can receive by choosing action A. We have $\bar{u}(m, \sigma, A) \leq -c(m) + \delta \max\left\{ \frac{(R_u+R_d)q-R_d}{(1-\delta)m}, 0 \right\}$. Similar as in the proof of Lemma 1, there exists $\tilde{m} > 0$ such that $\forall m \geq \tilde{m}$, $\bar{u}(m, \sigma, A) < 0$, which implies action A is strictly dominated by action N. Therefore, in any Nash equilibrium of $\mathcal{G}$, users will choose action V if $\sigma_V > C_V - R_V$ and action N if $\sigma_V < C_V - R_V$ for $\forall m \geq \tilde{m}$. Nash equilibria can only differ from each other for users with $\sigma_V = C_V - R_V$, which accounts for 0 mass due the atomless assumption of $F(\sigma_A, \sigma_V)$.

Next, it suffices to show that $\forall m_0$, if the statement holds for $\forall m \geq m_0 + 1$, then it is true for $m = m_0$. Since in any equilibrium, users will choose the same action for $m \geq m_0 + 1$, the expression of $g_\pi(m, q)$ in (1) holds for $m \geq m_0 + 1$. Then, according to the proof of Theorem 2, users will choose the same action in state m_0 except for users with $\sigma_A = \hat{a}(m, \sigma_V)$ or $\sigma_V = C_V - R_V$. These users account for 0 mass due to the atomless assumption of $F(\sigma_A, \sigma_V)$, which concludes the proof. □

From Corollary 4, we know that all Nash equilibria of $\mathcal{G}$ are essentially equivalent if the user type distribution is atomless. Therefore, in this case, the SNE we explicitly constructed in (14) is the unique Nash equilibrium of $\mathcal{G}$.

The essence of the SNE lies in the threshold function $\hat{a}(m, \sigma_V)$, which determines the portion of users who will answer the question at each stage. How will this threshold vary for different m and σ_V? In particular, how do the voting preferences of users impact their decisions on whether or not to answer the question? Is it to a user's advantage to provide an early answer? And as answers accumulate, will it become more competitive for users to answer the question? In the following, we will show properties of the threshold function that help to answer these questions. Our results are summarized in the following two propositions.

Proposition 3. *In SNE, $\forall m \geq 0$, the threshold of user ability for answering, i.e., $\hat{a}(m, \sigma_V)$, is increasing in σ_V. Moreover, the threshold is lower bounded as*

$$\hat{a}(m, \sigma_V) \geq \frac{R_d}{R_u + R_d}, \quad \forall m \geq 0, \sigma_V \in [V_{min}, V_{max}]. \tag{19}$$

Proof. For any $m \geq 0$, it suffices to prove the results for the case where $g_{\hat{\pi}}(m+1, q)$ is strictly increasing in q. Otherwise, we have $g_{\hat{\pi}}(m+1, q) = 0$, $\forall q \in [0,1]$, which implies $\hat{a}(m, \sigma_V) = 1$ and Proposition 3 holds.

Let us consider two voting preferences σ_{V1} and σ_{V2} such that $1 \geq \sigma_{V1} \geq \sigma_{V2} \geq 0$. If $-c(m) + \delta g_{\hat{\pi}}(m+1, 1) \leq \max\{0, \sigma_{V1} + R_V - C_V\}$, then according to Algorithm 1, we have $\hat{a}(m, \sigma_{V1}) = 1 \geq \hat{a}(m, \sigma_{V2})$. Otherwise, we have

$$\begin{aligned} &-c(m) + \delta g_{\hat{\pi}}(m+1, \hat{a}(m, \sigma_{V1})) \\ &\quad = \max\{0, \sigma_{V1} + R_V - C_V\} \\ &\quad \geq \max\{0, \sigma_{V2} + R_V - C_V\} \\ &\quad = -c(m) + \delta g_{\hat{\pi}}(m+1, \hat{a}(m, \sigma_{V2})). \end{aligned}$$

Since $g_{\hat{\pi}}$ is strictly increasing in answer quality, we can conclude that $\hat{a}(m, \sigma_{V1}) \geq \hat{a}(m, \sigma_{V2})$. Therefore, $\hat{a}(m, \sigma_V)$ is increasing in σ_V.

To show the lower bound, note from the expression of $g_{\hat{\pi}}$ in (5) that

$$g_{\hat{\pi}}\left(m, \frac{R_d}{R_u + R_d}\right) = 0 \leq g_{\hat{\pi}}(m, \hat{a}(m, \sigma_V)), \quad \forall m \geq 0, \sigma_V \in [V_{min}, V_{max}], \tag{20}$$

which implies that $\hat{a}(m, \sigma_V) \geq \frac{R_d}{R_u+R_d}$ due to the monotonicity of $g_{\hat{\pi}}$. □

Proposition 4. *In the SNE $\hat{\pi}$, $\forall q \in [0,1]$, $g_{\hat{\pi}}(m, q)$ is decreasing in m. In addition, the threshold of user ability for answering, i.e., $\hat{a}(m, \sigma_V)$, is increasing in m for any given $\sigma_V \in [V_{min}, V_{max}]$.*

Proof. We first show that $g_{\hat{\pi}}(m, q)$ is a decreasing function of m using mathematical induction. From Corollary 2, we know that users will not choose action A at the absorbing state $\overline{m}$ in SNE. Therefore, we have $P^A_{\hat{\pi}}(\overline{m}) = 0$ and

$$g_{\hat{\pi}}(\overline{m}, q) = \frac{P^V_{\hat{\pi}}(\overline{m})[(R_u + R_d)q - R_d]}{(1-\delta)\overline{m}}. \tag{21}$$

Then, $\forall m$ such that $1 \geq m \geq \overline{m} - 1$, we show in the following that if

$$g_{\hat{\pi}}(m+1, q) \leq \frac{P^V_{\hat{\pi}}(m+1)[(R_u + R_d)q - R_d]}{(1-\delta)(m+1)}, \tag{22}$$

we can derive $g_{\hat{\pi}}(m, q) \geq g_{\hat{\pi}}(m+1, q)$ and, as a result,

$$g_{\hat{\pi}}(m, q) \leq \frac{P^V_{\hat{\pi}}(m)[(R_u + R_d)q - R_d]}{(1-\delta)m}. \tag{23}$$

Assume the above conclusion does not hold, i.e., $g_{\hat{\pi}}(m,q) < g_{\hat{\pi}}(m+1,q)$. Then, according to the monotonicity of $g_{\hat{\pi}}$ with respect to answer quality q, we have $\hat{a}(m,\sigma_V) \geq \hat{a}(m+1,\sigma_V)$, which implies $P^V_{\hat{\pi}}(m) \geq P^V_{\hat{\pi}}(m+1)$. Moreover, from the optimality form expression of $g_{\hat{\pi}}$ in (1), we have

$$\begin{aligned} & g_{\hat{\pi}}(m,q) - g_{\hat{\pi}}(m+1,q) \\ & = \frac{\frac{P^V_{\hat{\pi}}(m)}{m}[(R_u+R_d)q - R_d] - (1-\delta)g_{\hat{\pi}}(m+1,q)}{1-\delta(1-P^A_{\hat{\pi}}(m))} \\ & \geq \frac{\left\{\frac{P^V_{\hat{\pi}}(m)}{m} - \frac{P^V_{\hat{\pi}}(m+1)}{m+1}\right\}[(R_u+R_d)q - R_d]}{1-\delta(1-P^A_{\hat{\pi}}(m))} \geq 0, \end{aligned} \tag{24}$$

which contradicts the assumption. Therefore, $g_{\hat{\pi}}(m,q) \geq g_{\hat{\pi}}(m+1,q)$ must hold. Moreover, from (24), we can also show that

$$g_{\hat{\pi}}(m+1,q) \leq \frac{P^V_{\hat{\pi}}(m)[(R_u+R_d)q - R_d]}{(1-\delta)m}. \tag{25}$$

Substituting the above inequality into (1), we can then derive (23).

Therefore, we can conclude that $g_{\hat{\pi}}(m,q)$ is an increasing function of m for any given $q \in [0,1]$, which proves the first part of Theorem 2. The second part of Theorem 2 can then be verified easily using this result and the monotonicity property of $g_{\hat{\pi}}$ with respect to answer quality q. □

The above proposition shows that there exists an advantage for answering the question earlier: the answers that are posted earlier will receive more rewards than those posted later. Moreover, since it is more profitable to answer the question when there are fewer answers, more users will choose answering at the earlier state of the game. As answers accumulate, it becomes more and more competitive to answer the question; users are gradually driven away from answering the question, which is left to a selective group of high ability users, until the question reaches the absorbing state where no more answers will be posted.

5 Extensions to Endogenous Effort

In the previous section, we have studied the sequential user behavior in social computing systems under the homogenous effort model, which assumes that the quality of answer equals the user's ability and all users incur the same cost for creating an answer. Such a model corresponds to cases where the domain knowledge and the expertise of users are essential in answering the question, such as focused Q&A sites like Stack Overflow. A more general setting is that, in addition to strategic decisions on whether to answer the question or not, users can also decide endogenously how much effort to put to produce their answers. In this section, we study the proposed game under such an endogenous effort model and show that our previous results can be extended naturally to incorporate this more general setting.

We now refer actions in the action set Θ as main actions. In addition to main actions, users will choose another action $e \in [0,1]$, which represents the amount of effort in producing answers. Similar as in the homogeneous effort case, we consider mixed strategies for main actions and denote by π the main action rule. We further write $\Pi_E = \Pi \times [0,1]$ as the set of action rules. Let $u_E(m,\sigma,\theta,e,\tilde{\pi})$ represent the utility of a user with type σ who arrives at state m and choose action $\theta \in \Theta$ and $e \in [0,1]$ will receive provided that other users adopt main action rule $\tilde{\pi}$. We have

$$u_E(m,\sigma,\theta,e,\tilde{\pi}) = \begin{cases} -c(m,e) + \delta g_{\tilde{\pi}}(m+1,\phi(\sigma_A,e)) & \text{if } \theta = A \\ \sigma_V + R_V - C_V & \text{if } \theta = V \text{ and } m > 0 \\ 0 & \text{if } \theta = N. \end{cases} \tag{26}$$

Then, the utility of choosing action rule π can be written as $\mathcal{U}_E(m,\sigma,\pi,e,\tilde{\pi}) = \sum_{\theta\in\Theta}\pi_\theta(m,\sigma)\cdot u_E(m,\sigma,\theta,e,\tilde{\pi})$. Therefore, the proposed game with endogenous effort can be formally defined as a tuple as $\mathcal{G}_E = (\mathcal{N},\Pi_E,\mathcal{U}_E)$.

From (26), we can see that the effort of a user impacts his utility of choosing action A and thus his optimal action rule. On the other hand, however, the choice of effort only has local impact in the sense that given the state m and other users' main action rule $\tilde{\pi}$, a user's utility will not depend on other users' efforts. Moreover, we would like to note that properties of the reward function for answering in Proposition 1 and Proposition 2 are derived with respect to the answer quality q, and thus hold for the endogenous effort case with $q = \phi(\sigma_A, e)$.

For the endogenous effort case, the SNE can be formally defined as follows.

Definition 2. *An action rule pair $(\hat{\pi},\hat{e})$ is a symmetric Nash equilibrium of $\mathcal{G}_E$ if and only if*

$$(\hat{\pi},\hat{e}) \in \arg\max_{(\pi,e)\in\Pi_E} \mathcal{U}_E(m,\sigma,\pi,e,\hat{\pi}) \quad \forall m \geq 0, \sigma \in \Omega. \tag{27}$$

As before, we are interested in whether there exists an SNE for the proposed game with endogenous effort and if so, what is the structure of the SNE. We answer these questions in the following theorem.

Theorem 3. *There exists an SNE in $\mathcal{G}_E$ such that users choose their main actions according to the following threshold structure:*

$$\begin{cases} [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [1,\ 0,\ 0] \\ \qquad\qquad \text{if } \sigma_A > \hat{a}(m,\sigma_V) \\ [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [0,\ 1,\ 0] \\ \qquad\qquad \text{if } \sigma_A \leq \hat{a}(m,\sigma_V) \text{ and } \sigma_V \geq \hat{\sigma}_V \text{ and } m \geq 1 \\ [\hat{\pi}_A(m,\sigma),\ \hat{\pi}_V(m,\sigma),\ \hat{\pi}_N(m,\sigma)] = [0,\ 0,\ 1] \\ \qquad\qquad \text{otherwise.} \end{cases} \tag{28}$$

Moreover, conditioned on choosing action A, each user chooses an effort $\hat{e}(m,\sigma_A)$ based on the state m and his ability σ_A.

Proof. To prove Theorem 3, we first show that there exists an absorbing state in any SNE of $\mathcal{G}_E$. From Proposition 1 and the monotonicity of $c(m,e)$ in e, we have $u_E(m,\sigma,A,e,\hat{\pi}) \leq -c(m,0) + \frac{\delta R_u}{(1-\delta)(m+1)}$. Therefore, there exists

$\tilde{m} \geq 0$ such that $\forall m \geq \tilde{m}$, the utility of choosing action A is strictly less than 0, which implies that action A is strictly dominated by action N.

Next, we construct an SNE, $(\hat{\pi}, \hat{e})$, and show that it satisfies conditions in Theorem 3. For $m \geq \tilde{m}$, since the probability of choosing action A is 0 for all user types, the main action rule in (28) with $\hat{a}(m, \sigma_V) = 1$ and $\hat{\sigma}_V = C_V - R_V$ is the best response for all users regardless of other users' main action rule. The choice of effort is irrelevant in this case.

For $m < \tilde{m}$, the $(\hat{\pi}, \hat{e})$ can be constructed by iteratively picking the best response backward from $m = \tilde{m} - 1$ to 0. At each state m, let

$$\hat{e}(m, \sigma_A) \in \arg\max_{e \in [0,1]} \{-c(m, e) + \delta g_{\hat{\pi}}(m+1, \phi(\sigma_A, e))\}. \tag{29}$$

The best response is to choose action A with probability 1 and exert effort $\hat{e}(m, \sigma_A)$ if

$$\begin{aligned} &-c(m, \hat{e}(m, \sigma_A)) + \delta g_{\hat{\pi}}(m+1, \phi(\sigma_A, \hat{e}(m, \sigma_A))) \\ &\quad > \max\{0, \sigma_V + R_V - C_V\}. \end{aligned} \tag{30}$$

Otherwise it is optimal to choose action V with probability 1 if $m \geq 1$ and $\sigma_V + R_V - C_V > 0$, and to choose action N in other cases.

To show that $\hat{\pi}$ satisfies (28) for state $m < \tilde{m}$. The key is to show the utility of answering with optimal effort is increasing in user's ability. Consider $0 \leq \sigma_{A1} \leq \sigma_{A2} \leq 1$. We have

$$\begin{aligned} &-c(m, \hat{e}(m, \sigma_{A1})) + \delta g_{\hat{\pi}}(m+1, \phi(\sigma_{A1}, \hat{e}(m, \sigma_{A1}))) \\ &\quad \leq -c(m, \hat{e}(m, \sigma_{A1})) + \delta g_{\hat{\pi}}(m+1, \phi(\sigma_{A2}, \hat{e}(m, \sigma_{A1}))) \end{aligned} \tag{31}$$

$$\leq -c(m, \hat{e}(m, \sigma_{A2})) + \delta g_{\hat{\pi}}(m+1, \phi(\sigma_{A2}, \hat{e}(m, \sigma_{A2}))). \tag{32}$$

The inequality in (31) follows from the fact that $g_{\hat{\pi}}$ is increasing in answer quality q and $q = \phi(\sigma_A, e)$ is an increasing function of σ_A. The inequality in (32) is based on the definition of $\hat{e}$ in (29). Therefore, a user with higher ability can obtain a higher utility by choosing action A than one with lower ability. From (30), such a monotonicity property leads to the threshold structure for answering where the threshold $\hat{a}(m, \sigma_V)$ is the solution $a \in [0, 1]$ to the following equation:

$$\begin{aligned} &-c(m, \hat{e}(m, a)) + \delta g_{\hat{\pi}}(m+1, \phi(a, \hat{e}(m, a))) \\ &\quad = \max\{0, \sigma_V + R_V - C_V\}. \end{aligned} \tag{33}$$

When the above equation does not have a solution in [0, 1], the threshold $\hat{a}(m, \sigma_V)$ can be set as 0 if the left hand side is greater or 1 if otherwise. □

From Theorem 3, we see that there exists an SNE for the proposed game with endogenous effort that has a very similar structure as the SNE for homogenous effort model. The difference here is that the calculation of the threshold function for answering now takes into account different possible efforts. In other words, to decide whether or not to answer the question, a user must first find his optimal effort and then evaluate his utility for answering using this optimal effort. Moreover, we note that the SNE characterized in Theorem 3 may not be a unique one as there may be multiple optimal efforts and the quality function ϕ may not be strictly increasing.

TABLE 1
(a) Reputation Updating Rule; (b) Statistics of the Dataset

(a)		(b)	
Action	Reputation change	Item	Count
Answer is upvoted	+10	Questions	430 K
Answer is downvoted	−2 (−1 to voter)	Answers	731 K
Answer is accepted	+15 (+2 to accepter)	Votes	1.32 M

6 Empirical Evaluations

In this section, we use real-world data from a popular Q&A site Stack Overflow to valid our model. In particular, we investigate how qualitative observations obtained from the data compare with predictions of our model. We will first introduce the dataset and then present our evaluation results.

6.1 Dataset Description

Stack Overflow is one of the most popular Q&A site, where questions are strictly restricted to be factual and programming-related. Questions in Stack Overflow are generally hard and thus usually require strong domain knowledge and expertise to answer, which makes it a good fit for our homogenous effort model. Besides question asking and answering, voting is another popular type of user activities on Stack Overflow, which is designed to provide additional information regarding the quality of answers as well as long-lasting incentives for users to answer questions. The model of Stack Overflow has been proved successful and adopted by over 100 other focused Q&A websites under the StackExchange [20].

Different types of user activities in Stack Overflow are connected through an incentive mechanism that is built with reputation points. We list in Table 1a how reputation points are gained and lost by actions related to our discussions. Note that, to prevent abuse, downvotes are discouraged in a sense that the voter will lose one reputation point by casting a downvote. Moreover, in Stack Overflow, the user who asks the question can select an answer as the selected answer, which brings slightly more reputation points to the contributor than a regular upvote does. In addition to the listed actions, reputation of a user can change in many other ways such as offering or wining a bounty associated with a question. Overall, a user's reputation summarizes his activities on Stack Overflow since registration and roughly measures the amount of expertise he has as well as the level of respect he received from his peers.

The user activity data on Stack Overflow is publicly available through the Stack Exchange Data Explorer [21]. We collect questions that are posted in the first Quarter of 2013, i.e. from January 1st, 2013 to March 31st, 2013. We

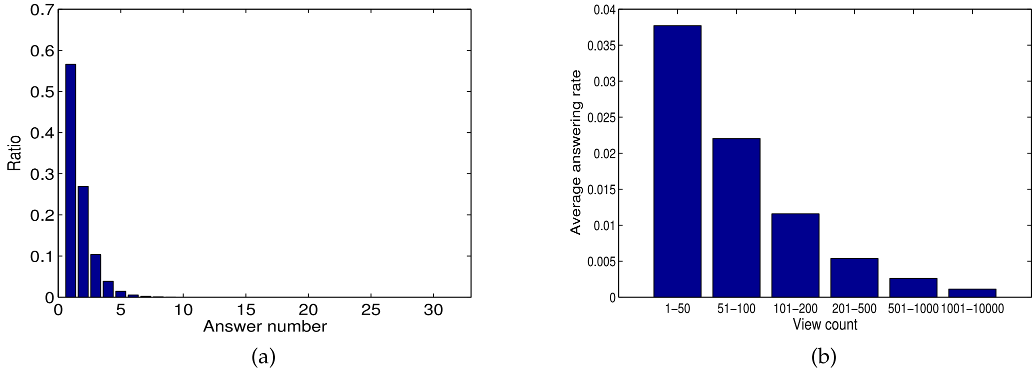


Fig. 2. (a) The distribution of answer count; (b) The average answering rate by different view count intervals.

include all the answers and votes that are related to these questions (as of March 2014) into our dataset. Note that we only impose time restrictions on questions and but not on the related answers and votes. The user reputation score is also based on values at the data collection time, i.e., March 2014. We consider questions that receive at least one answer and further exclude questions that are closed for various reasons such as being marked as subjective or duplicate. In addition, to fit the data into our model, we regard the action of accepting an answer simply as a regular upvote. That is, we treat the user who asks the question the same as other users with respect to voting. The statistics of our dataset are shown in Table 1b.

6.2 Observations and Validations

6.2.1 The Saturation Phenomenon

In our analysis, the existence of SNE is based on an observation that the number of answers to a question stops increasing after a certain value, which makes our game equivalent to a finite sequential game. To verify such an observation, we first show in Fig. 2a the distribution of answer count for questions in our dataset. The maximum answer count is 33 and we can see that the distribution is concentrated around the lower end. We further investigate how the answering rate varies with the view count of a question. The answering rate is defined as the number of answers to a question divided by the number of users who view this question. Our results are shown in Fig. 2b. We found that the answering rate drops quickly as the view count increases. This illustrates that as users keep arriving to the question and as answers accumulate, it is getting harder for the question to obtain new answers. Therefore, there exists a saturation phenomenon in terms of answers to a question, which justifies our observation.

6.2.2 The Advantage of Higher Ability

A key prediction derived from our model is that the reward function for answering is monotonically increasing in answer quality, as stated in Proposition 2. In homogenous effort settings, this means a user with higher ability can receive a higher reward by answering the question than a user with lower ability does. Such a prediction serves as the foundation of our equilibrium analysis and leads directly to the threshold structure of the equilibrium. To justify such a prediction, we investigate how the average score of answers varies with the contributors' abilities. We define the answer score as the number of positive votes an answer has minus the number of negative votes, which is a good indication of the reward a user can obtain from his answer. Since user ability is not directly observable from the data, we use reputation as a rough approximation of a user's ability. In particular, we quantize the reputation using a set of logarithmic boundary values as $\{0, 100, 1{,}000, 5{,}000, 20{,}000, 1e7\}$. Roughly speaking, a user with a higher reputation level is more likely to have a higher ability in answering the question. We show in Fig. 3a our results for answers with different time ranks, which is defined as the order by which they are posted. For example, an answer with time rank 1 means

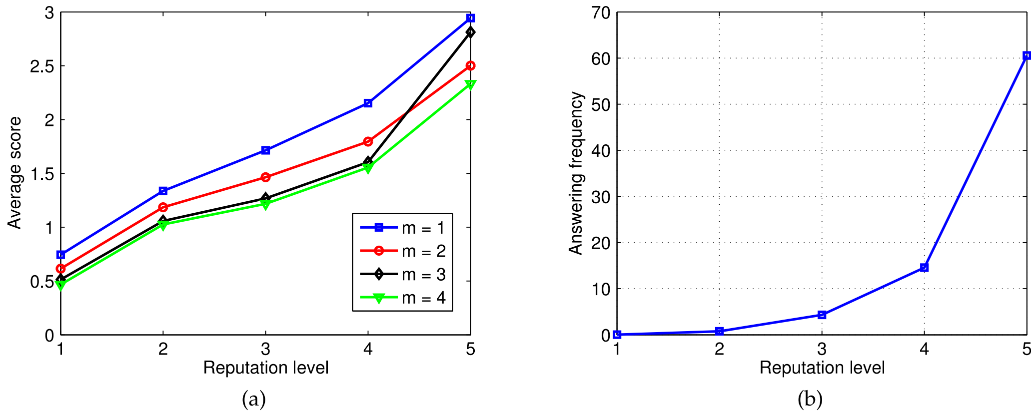


Fig. 3. (a) The average score of answers versus the reputation level of users; (b) The relative frequency of answering versus reputation level.

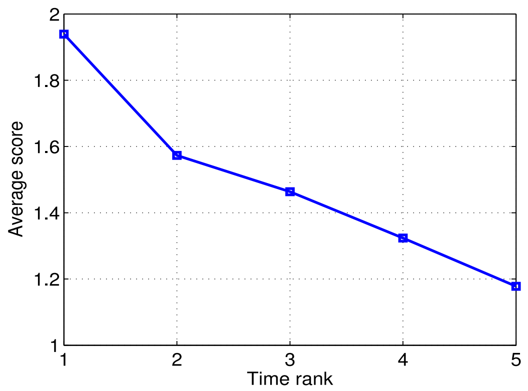


Fig. 4. The average score versus the time rank of answers.

it is the first answer to the question. The time rank of an answer also represents the earliest state m at which the answer can receive votes. From Fig. 3a, we can see that for any state m, users with higher abilities can receive more rewards by answering the question. Therefore, observations obtained from the data agree with the qualitative predictions of our model.

We further investigate how the frequency of answering varies with user abilities. We group answers by the contributors' reputation levels. We then calculate the number of answers in each group and normalize it using the population size of each group to show the frequency of answering. Our results are shown in Fig. 3b. We can see that the frequency of answering increases drastically as user ability increases, which shows an evidence of threshold structures in users' decision makings. With threshold structures, users with higher abilities are more likely to answer the questions. Since different types of questions may have different thresholds, the average frequency of answering therefore is monotonically increasing in user abilities.

6.2.3 The Advantage of Answering Earlier

Another important prediction derived from our model is that the reward for an answer decreases with respect to its time rank, as stated in Proposition 4. That is, there is an advantage for answering earlier. To compare such a prediction with observations made from real-world data, we can first observe from Fig. 3a that, for most reputation levels, the score of answer decreases in m. We further show in Fig. 4 the curve of average score of answers versus the time rank. We can see that answers that are posted earlier receive higher scores on average, which is consistent with our qualitative predictions.

7 Numerical Simulations

In this section, we investigate through numerical simulations how our model can help to provide insights on the design of incentive mechanisms for a wide range of social computing systems.

7.1 Simulation Settings

Recall that a mechanism in our model is defined by a set of three parameters $\{R_V, R_u, R_d\}$, which specify how the system should reward voting and answering respectively. The system designer adjusts these parameters to steer user behavior on the site. Depending on the characteristics of applications, system designers may be interested in optimizing different metrics. It would be very difficult to study a complete set of utility functions that include all scenarios. In this paper, we consider a general function that covers many typical use case scenarios in social computing. Denote by q_k and t_k the quality and arrival time of the kth answer. Let K represent the number of received answers. The system designer's utility function can be written as

$$U^s(K, q_1, t_1, \ldots, q_K, t_K) = K^{-\alpha} \sum_{k=1}^{K} \beta^{t_k} q_k, \tag{34}$$

where $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. We show below three typical use case scenarios that can be captured by the above objective function with different choices of α and β.

1) Use Case I: $\alpha = 0$ and $\beta = 1$, where the objective function becomes the sum of qualities. In this case, the diversity of answers is valuable. The system designer prefers a large number of reasonable answers over a few near-perfect ones. Moreover, answers have long-lasting values that will not decay over time.
2) Use Case II: $\alpha = 0$ and $\beta < 1$. In this case, the diversity of answers is valuable but the question is time sensitive. The system designer prefers answers to arrive sooner rather than later.
3) Use Case III: $\alpha = 1$ and $\beta = 1$, where the objective function becomes the average quality of answers. In this case, individual answer quality rather than diversity is valuable to the system designer. Moreover, answers have long-lasting values in this case.

We assume user types are drawn identically and independently according to the probability density function (PDF) $f(\sigma_A, \sigma_V) = \frac{\lambda e^{-\lambda \sigma_A}}{2(1-e^{-1})}$ over $[0,1] \times [-1,1]$. That is, we assume σ_A and σ_V are independently distributed; σ_V follows a uniform distribution and σ_A follows a truncated exponential distribution with parameter λ. Note that the larger λ is, the more rare high ability users are. Unless otherwise stated, we set by default $\lambda = 1$. We assume $C_V = 0.2$ and set the discounting factor $\delta = 0.9$.

For the homogenous effort model, we choose $c(m) = 1 + 0.1m$. For the endogenous effort model, we assume $c(m, e) = 0.1m + 5e^2$. We adopt $\phi(\sigma_A, e) = (\frac{\gamma + \sigma_A}{\gamma + 1})e$ as the quality function, where $\gamma \geq 0$ is a parameter that controls how much the answer quality depends on a user's ability. The larger γ is, the less dependent the answer quality is on a user's ability (and thus more dependent on the amount of effort).

7.2 Simulation Results for Homogenous Effort

In the first simulation, we investigate the impact of R_V on the system designer's utility. Our results for all three use cases are shown in Fig. 5 where we set $R_u = 2$ and $R_d = 1$. In all cases, when R_V is small, the system designer's utility increases quickly as R_V increases. This is because a higher reward for voting stimulates more users to vote rather than to leave without participation, which creates a stronger incentive for answering. Nevertheless, as the value of R_V

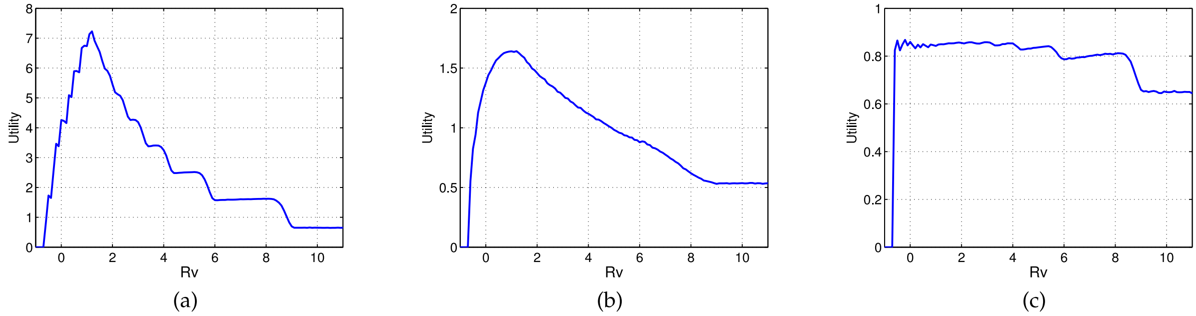


Fig. 5. The system designer's utility versus R_V: (a) Use Case I: $\alpha = 0$ and $\beta = 1$; (b) Use Case II: $\alpha = 0$ and $\beta = 0.9$; (c) Use Case III: $\alpha = 1$ and $\beta = 1$.

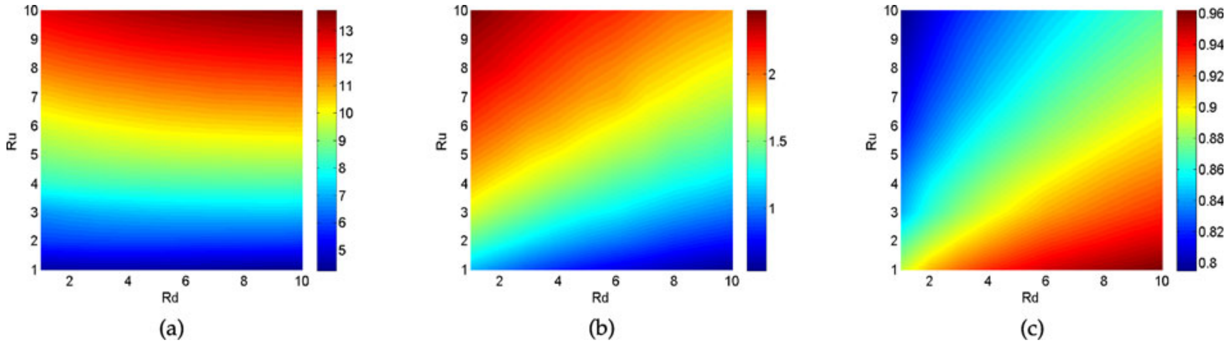


Fig. 6. The system designer's utility versus R_u and R_d: (a) Use Case I: $\alpha = 0$ and $\beta = 1$; (b) Use Case II: $\alpha = 0$ and $\beta = 0.9$; (c) Use Case III: $\alpha = 1$ and $\beta = 1$.

keeps increasing, it starts driving users away from answering since voting becomes more profitable. When diversity is valuable for the system designer such as in Use Case I and II, the system designer's utility will decrease after R_V passes an optimal value. It can be further observed that the optimal value is around 1.2 which is just enough to make voting preferable over no participation for all users. For Use Case III, since the average quality of answers is less sensitive to R_V when R_V is large, the system designer's utility fluctuates within a small range. If R_V is large enough, no users will have the incentive to answer the question when voting is an option.

From the above simulation, we can abstract an important principle towards the design of incentive mechanisms: voting should be encouraged but not too much! In practice, the reward for voting should be designed large enough to make voting preferable over no participation for a large fraction of users but relatively small compared to the reward for answering. Moreover, when the system designer is uncertain about the optimal value, it would be safer to overestimate than to underestimate, especially for cases where a few near-perfect answers are desired.

Next, we study how the system designer's utility depends on R_u and R_d. Recall that a user will receive R_u points for receiving an upvote and lose R_d points for receiving a downvote. We show our simulation results in Fig. 6 where we set R_V as 1. For Use Case I, the primary factor that influences the system designer's utility is R_u. Since diversity is valuable in this case, a larger R_u will stimulate more users to provide their answers and thus lead to a higher utility for the system designer. The impact of R_d is more visible in Use Case II and Use Case III. We found that, surprisingly, the value of R_d impacts the system designer's utility in two distinct directions for these two cases. In particular, as R_d increases the utility decreases in Use Case II while increases in Use Case III. This can be explained as follows. Recall from Proposition 3 that $\frac{R_d}{R_u+R_d}$ sets a lower bound on user's ability for answering. So roughly speaking, the thresholds of user ability for answering will increase as R_d increases. With higher thresholds, the system designer's utility will be lower in Use Case II, since it takes longer time for answers to accumulate. On the other hand, higher thresholds lead to higher quality, which makes the system designer's utility higher in Use Case III. Moreover, since the diversity of answers is not valuable in Use Case III, the ratio of R_u to R_d is the primary factor that impacts the system designer's utility.

To summarize, we can abstract another principle that could potentially aid the design of incentive mechanisms in practice. When diversity of answers is desired, a high reward should be assigned to users for each upvote they

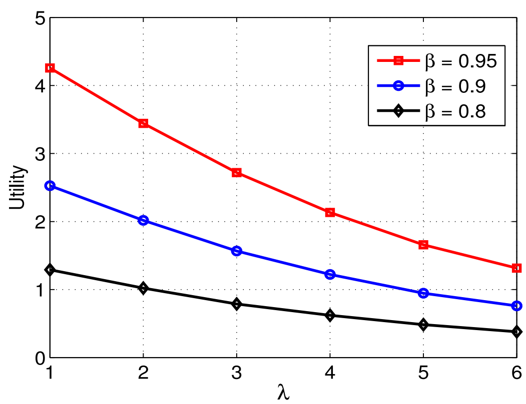
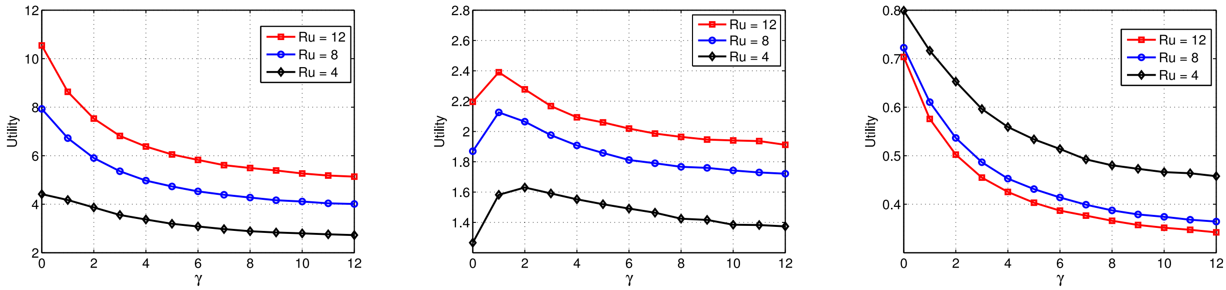


Fig. 7. The system designer's utility versus λ for $\alpha = 0$ and different values of β.



(a) (b) (c)

Fig. 8. The system designer's utility versus γ: (a) Use Case I: $\alpha = 0$ and $\beta = 1$; (b) Use Case II: $\alpha = 0$ and $\beta = 0.9$; (c) Use Case III: $\alpha = 1$ and $\beta = 1$.

receive. Depending on whether the answer quality or the answer timeliness is more valuable, different strategies should be applied to set the punishment for receiving downvotes.

In the third simulation, we study the impact of λ on the system designer's utility. Recall that λ controls the shape of user type distribution; the larger λ is, the more rare high ability users are. The results is shown in Fig. 7. We can see that the system designer's utility decreases as λ increases, which demonstrates the value of high ability users to social computing systems. Therefore, for applications that rely heavily on users' domain knowledge and expertise, it is of key importance to develop and maintain an active community of elite members.

7.3 Simulation Results for Endogenous Effort

Finally, we consider the endogenous effort model in our simulation. In particular, we are interested in how the degree of sensitivity of answer quality with respect to effort influences the system designer's utility. We show curves of utility versus γ for all the three use cases in Fig. 8. We set $R_V = 1$ and $R_d = 2$ in our simulations. We can see that in Use Case I and III, the utility decreases as γ increases while in Use Case II, the utility first increases and then decreases.

Since a larger value of γ implies that the answer quality will be less dependent on user's ability, low ability users will get an advantage for answering with large γs. As a result, the threshold of user ability for answering will decrease as γ increases. On the one hand, lower thresholds lead to low quality on average, which explains why the utility decreases in all the three use cases. On the other hand, lower thresholds implies that answers will arrive earlier, which explains the non-monotonic behavior of the system designer's utility in Use Case II.

8 Conclusions and Future Works

In this paper, we study sequential user behavior in social computing systems from a game-theoretic perspective. Our model explicitly takes into account the answering-voting externality, which can be found in many social computing systems. We begin with a homogenous effort model and prove the existence and uniqueness of a pure strategy SNE To further understand the equilibrium user participation, we show that there exist advantages for users with higher abilities and for answering earlier. As a result, the equilibrium exhibits a threshold structure where the threshold for answering increases as the number of answers increases. Our results derived for the homogenous effort model well captures the essence of the game and can be extended naturally to the more general setting where users endogenously choose their efforts for answering. Our model is verified through evaluations of user behavior data collected from Stack Overflow. In particular, we show that the main qualitative predictions of our model are consistent with observations made from the data. Finally, we study the system designer's problem through numerical simulations and derive several design principles that could potentially guide the design of incentive mechanisms for social computing systems in practice.

The work in this paper can be extended in many directions. For example, it is interesting to model user preferences toward answers. With user preferences, the same answer can be considered to have different qualities and users will vote based on their preferences, which corresponds to the scenario where tasks are highly subjective. It is also worth studying how the mechanism of distributing user attentions among different answers will impact user behaviors. Our current model is equivalent to a mechanism that randomly pops up an answer for users to vote. Other interesting mechanisms may include: (a) ranking answers according to quality; (b) ranking answers based on received votes and (c) ranking answers based on users' preferences. Another interesting extension is to model scenarios where users can explicitly evaluate the qualities of existing answers before choosing their own actions. The user can compare his/her own ability with the qualities of existing answers and form a notion of relative ability, which he/she can then use to choose his/her best action.

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