Joint Power Control and Blind Beamforming in Wireless Networks

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Abstract— **Traditional power control with beamforming achieves a targeted Signal-to-Interference-Noise-Ratio (SINR) in wireless networks assuming measurement of SINR and direction of arrival (DOA) at the receivers. Blind beamforming is an emerging technique for DOA measurement without consuming extra bandwidth. Here we propose a joint power control and blind beamforming algorithm that reformulates the power control problem and, importantly, does not need additional measurements or sending training sequences. In contrast to traditional power control that achieves a targeted SINR threshold, the scheme achieves a threshold on a quantity available from blind beamforming, which is directly related to bit error rate. We have shown in both analysis and simulation results that the algorithm converges to the desired solution.**

Keywords: Power Control, Blind Beamforming, Multiple Antenna.

I. Introduction

One of the main factors that degrade the network performance in wireless communications is co-channel interference. Power control is one direct approach towards minimizing co-channel interference [1], [2], [3], [4]. The transmitted powers are constantly adjusted. They are increased if the Signal-to-Noise-Ratio's (SINR) at the receivers are low and are decreased if the SINR are high. Such a process improves the quality of weak links and reduces the overall transmitted powers.

As a majority of communication systems often struggle with limited bandwidth constraint, it is desirable for the receiver with multiple antennas to steer to the desired direction without consuming much channel bandwidth. By eliminating training sequence and maximizing channel capacity for true information transmission, blind estimation and beamforming [5], [6], [7], [8], [9] provides a bandwidth efficient solution to channel and direction estimation. Its importance also lies in the practical need for some communication receivers to equalize unknown channels without the assistance and the expense of training sequences.

Current methods of power control assume measurement of channel parameters and SINR at the receivers. Blind beamforming is used to estimate, without use of training sequences, the direction of transmitted signals that suffer from channel distortion and additive noise. In this paper, we present a joint power control and blind beamforming algorithm. Based on a reformulated power control problem, this combined algorithm optimizes the Bit Error Rate (BER) using a quantity directly available from blind beamforming, avoiding additional measurements mentioned above. The method is applied to an uplink scenario. Convergence properties for a low delay-spread channel are discussed. Simulation results illustrate that our algorithm converges to the desired solution and is more robust to channel estimation error compared with power control with minimum variance distortion response (MVDR) beamforming.

The organization of this paper is as follows: In Section II, we give the system model, present the traditional power control problem and a blind beamforming algorithm. In Section III, we give the reformulated power control problem definition. Then an adaptive algorithm is developed and its convergence is analyzed. In Section IV, we have numerical study. In Section V, we give the conclusion.

II. System Model and Blind Beamforming

A. System Model

Consider K distinct cells in wireless networks that co-channel links exist. Each cell consists of one base station and its assigned mobiles. The maximum number of mobile users is D and the number of mobiles can be accurately estimated. Antenna arrays with P elements are used only at base station. We assume coherent detection is possible so that it is sufficient to model this multiuser system by an equivalent base band model. Each link is affected by the multipath slow Rayleigh fading. The maximum multipath number is L. The propagation delay is far less than one symbol period. For uplink case, the i^{th} base station antenna array's output vector is given by:

$$\mathbf{x}_i(t) = \sum_{k=1}^{K}\sum_{d=1}^{D}\sum_{l=1}^{L}\sqrt{G_{ki}^d P_k^d}\,\alpha_{ki}^{dl}\mathbf{a}_{ki}^d(\theta_l)\cdot g_k^d(t)s_k^d(t) + \mathbf{n}_i(t) \tag{1}$$

where G_{ki}^d is path loss, α_{ki}^{dl} is fading loss, P_k^d is transmitted power, $\mathbf{a}_{ki}^d(\theta_l)$ is the i^{th} base station array response vector to the signal from the d^{th} mobile in the k^{th} cell at direction θ_l, $g_k^d(t)$ is shaping function, $s_k^d(t)$ is message symbol and $\mathbf{n}_i(t)$ is thermal noise vector. We assume slow fading and the channels are stable within a frame of hundreds of symbols. Define the impulse response from the d^{th} mobile in the k^{th} cell to the p^{th} element of the i^{th} base station as: $h_{ki}^{dp} = \sum_{l=1}^{L}\alpha_{ki}^{dl}a_{ki}^{dp}(\theta_l)r_{ki}^{dpl}$, where r_{ki}^{dpl} includes the effect of the transmitter, receiver filter, and shaping function $g_k^d(t)$. The vector form is $\mathbf{h}_{ki}^d = [h_{ki}^{1d}, \ldots, h_{ki}^{Pd}]^T$. We assume the transmitted signals from different sources are uncorrelated and zero mean, and the additive noise is spatially and temporally white. Let $\mathbf{w}_i^d$ be the beamforming weight vector for the d^{th} mobile in the i^{th} cell. Without loss of generality, let $\|(\mathbf{w}_i^d)^H\mathbf{h}_{ii}^d\|^2 = 1$. The d^{th} user's SINR at the its associated i^{th} base station's beamformer output is:

$$\Gamma_i^d = \frac{P_i^d G_{ii}^d}{\sum\sum_{(k,j)\neq(i,d)} P_k^j G_{ki}^j \|(\mathbf{w}_i^d)^H\mathbf{h}_{ki}^j\|^2 + (\mathbf{w}_i^d)^H N_i \mathbf{w}_i^d} \tag{2}$$

where N_i is the thermal noise power.

B. Traditional Power Control with MVDR Beamforming

If the channel responses are estimated, the beamforming vector can be calculated by MVDR method, which minimizes the total interferences at the output of beamformer, while the gain for the desired user d is kept as a constant. The MVDR problem can be defined as:

$$\min_{\mathbf{w}_i^d} \|(\mathbf{w}_i^d)^H\mathbf{x}_i\|^2 , \tag{3}$$

$$\text{subject to } \|(\mathbf{w}_i^d)^H\mathbf{h}_{ii}^d\|^2 = 1, \ i = 1, ..., M$$

Define correlation matrix as $\Phi_i = E[\mathbf{x}_i\mathbf{x}_i^H]$. The optimal weight vector is given by:

$$\hat{\mathbf{w}}_i^d = \frac{\Phi_i^{-1}\mathbf{h}_{ii}^d}{(\mathbf{h}_{ii}^d)^H\Phi_i^{-1}\mathbf{h}_{ii}^d}. \tag{4}$$

In traditional power control, each link's transmitted power is selected so that its SINR is equal to or larger than a fixed and predefined targeted SINR threshold required to maintain the link quality, while minimizing the overall transmitted power of all the links. The power control problem to achieve the desired threshold γ_i^d for the d^{th} user in the i^{th} cell is defined as:

$$\min \sum_{i=1}^{K}\sum_{d=1}^{D} P_i^d , \tag{5}$$

$$\text{subject to } (\mathbf{I} - \mathbf{BF})\mathbf{P} \geq \mathbf{u}$$

where $\mathbf{u} = [u_1^1, \dots u_1^D, \dots, u_K^1, \dots, u_K^D]^T$, $k = (i-1)\cdot D + d, 1 \leq d \leq D$, $u_k^d = \gamma_i^d(\mathbf{w}_i^d)^H N_i \mathbf{w}_i^d / G_{ii}^d$, $\mathbf{P} = [P_1^1, \dots P_1^D, \dots P_K^1, \dots, P_K^D]^T$, $j = (i'-1)\cdot D + d', 1 \leq d' \leq D$, $B = diag\{\gamma_1^1, \dots \gamma_1^D, \dots \gamma_K^D, \dots, \gamma_K^D\}$ and

$$[\mathbf{F}]_{kj} = \begin{cases} 0 & \text{if } j = k, \\ \frac{G_{i'i}^{d'}\|(\mathbf{w}_i^d)^H\mathbf{h}_{i'i}^{d'}\|^2}{G_{ii}^d} & \text{if } j \neq k \end{cases}. \tag{6}$$

If the spectral radius of $\mathbf{BF}$, $\rho(\mathbf{BF})$, i.e., the maximum absolute eigenvalue of $\mathbf{BF}$, is inside the unit circle, the system has feasible solutions. By Perron-Frobenius theorem, the optimum power vector for this problem is $\hat{\mathbf{P}} = (\mathbf{I} - \mathbf{BF})^{-1}\mathbf{u}$. The optimal solution of the power vector is achieved when the equations of SINR constraint are held, i.e., $\Gamma_i^d = \gamma_i^d, \forall i, d$. It has been shown that this is a NP hard problem. Many adaptive algorithms [1], [2], [3], [4] have been developed to decrease the system complexity. If the spectral radius $\rho(\mathbf{BF})$ is less than 1, the optimal power vector exists and the following decentralized iteration converges to the optimal power vector. The algorithm can be briefly expressed by:

Beamforming: MVDR
Power Update: $\mathbf{P}^{n+1} = \mathbf{BFP}^n + \mathbf{u}$

At each iteration, transmitters update their powers based on the interference power measured at the receivers and the link gain transmitted by the feedback channel.

C. Blind Beamforming

The traditional power control needs additional measurement of DOA. Here we use Iterative Least Squares Projection (ILSP) algorithm [5] for blind beamforming. Moreover the channel compensation and symbol detection can be implemented at the same time without extra modules.

Consider maximum D co-channel mobile transmitters inside the i^{th} cell. The d^{th} mobile generates binary data $s_i^d(n)$ with power P_i^d transmitted over a low delay spread multipath Rayleigh fading channel $\mathbf{h}_{ii}^d$. The sampled antenna output at the base station is given by:

$$\mathbf{x}_i(n) = \sum_{d=1}^{D}\mathbf{h}_{ii}^d\sqrt{P_i^d G_{ii}^d}s_i^d(n) + \mathbf{v}_i(n). \tag{7}$$

Here $\mathbf{v}_i(n)$ includes the i^{th} base station antenna thermal noise and all the co-channel interferences from the other cells.

$$\mathbf{v}_i(n) = \mathbf{n}_i(n) + \sum_{k=1,k\neq i}^{K}\sum_{d=1}^{D}\mathbf{h}_{ki}^d\sqrt{P_k^d G_{ki}^d}s_k^d(n) \tag{8}$$

where $\mathbf{n}_i(n)$ is the $P \times 1$ sampled thermal noise vector.

ILSP algorithm works with a shifting window on data blocks of size N. Assume that the channel is constant over the available N symbol periods, we obtain the following formulation of the l^{th} block data

$$\mathbf{X}_i(l) = \mathbf{A}_i\mathbf{S}_i(l) + \mathbf{V}_i(l) \tag{9}$$

where $\mathbf{X}_i = [\mathbf{x}_i(lN+1)\ \mathbf{x}_i(lN+2)\dots\mathbf{x}_i((l+1)N)]$, $\mathbf{V}_i = [\mathbf{v}_i(lN+1)\ \mathbf{v}_i(lN+2)\dots\mathbf{v}_i((l+1)N)]$ and $\mathbf{S}_i = [\mathbf{s}_i(lN+1)\ \mathbf{s}_i(lN+2)\dots\mathbf{s}_i((l+1)N)]$, $\mathbf{s}_i(n) = [s_i^1(n)\dots s_i^D(n)]^T$ and $\mathbf{A}_i = [\sqrt{P_i^1 G_{ii}^1}\mathbf{h}_{ii}^1\ \dots\ \sqrt{P_i^D G_{ii}^D}\mathbf{h}_{ii}^D]$.

ILSP algorithm uses the finite alphabet property of the input to implement a least squares algorithm that has good convergence properties for channel with low delay spread. The algorithm is carried out in two steps. The first step is a least square minimization problem defined as:

$$\min_{\mathbf{A}_i,\mathbf{S}_i} f(\mathbf{A}_i,\mathbf{S}_i;\mathbf{X}_i) = \|\mathbf{X}_i(l) - \mathbf{A}_i\mathbf{S}_i(l)\|^2 \tag{10}$$

Here matrix $\mathbf{A}_i$ and $\mathbf{S}_i$ is unstructured and continuous. In the second step, each element of the solution $\mathbf{S}_i$ is projected to its closest discrete values $\hat{\mathbf{S}}_i$. Then a better estimate of $\hat{\mathbf{A}}_i$ is obtained by minimizing $f(\mathbf{A}_i,\hat{\mathbf{S}}_i;\mathbf{X}_i)$ with respect to $\mathbf{A}_i$, keeping $\hat{\mathbf{S}}_i$ fixed. We continue this process until the estimate of $\hat{\mathbf{A}}_i$ and $\hat{\mathbf{S}}_i$ is convergent. ILSP algorithm is given in Table I:

TABLE I

ILSP Algorithm

1.	Given $\hat{A}_{i,0}$, iteration index $m = 0$;
2.	$m = m + 1$ a. $\bar{S}_{i,m} = A^{+}_{i,m-1} X_i$, where $A^{+}_{i,m-1} = (\hat{A}^{*}_{i,m-1}\hat{A}_{i,m-1})^{-1}\hat{A}^{*}_{i,m-1}$ b. projection onto finite alphabet $\hat{S}_{i,m} = proj[\bar{S}_{i,m}]$ c. $\hat{A}_{i,m} = X_i \hat{S}^{+}_{i,m}$, where $\hat{S}^{+}_{i,m} = \hat{S}^{*}_{i,m}(\hat{S}_{i,m}\hat{S}^{*}_{i,m})^{-1}$
3.	Repeat until $(\hat{A}_{i,m}, \hat{S}_{i,m}) \approx (\hat{A}_{i,m-1}, \hat{S}_{i,m-1})$.

III. Joint Power Control and Blind Beamforming

A. Reformulation of Power Control

In this paper, we use BPSK modulation for simplicity of simulation and analysis. The other PAM or MQAM modulation methods can be easily extended. It has been shown in [5], the error probability of ILSP algorithm is given by:

$$P_r(s_i^d) = Q\left(\frac{1}{\sqrt{Var[\hat{s}_i^d(n)]}}\right) \tag{11}$$

where each element $\hat{s}_i^d(n)$ has $E[\hat{s}_i^d(n)] = s_i^d(n)$ and $Var[\hat{s}_i^d(n)] = \sigma_i^2(\mathbf{A}_i^H\mathbf{A}_i)^{-1}_{dd}$. Here $\sigma_i^2 = E[\mathbf{v}_i(n)^H\mathbf{v}_i(n)]$. Because there are a large number of co-channel interference sources, by the law of large numbers, we can assume $\mathbf{v}_i(n)$ is additive white Gaussian noise.

Now we consider the joint power control and blind beamforming problem. The key is the quantity $Var[\hat{s}_i^d(n)]$ which is directly related to error performance. We can use it as a constraint to the power control problem. We want to minimize the overall transmitted power while each user's BER is less than some desired value. The reformulated power control problem with ILSP is given by:

$$\min \sum_{i=1}^{K}\sum_{d=1}^{D} P_i^d \tag{12}$$

$$\text{subject to } Var(\hat{s}_i^d(n)) \le var_0,\ \forall i, d$$

where var_0 is a threshold to achieve the desired BER.

B. Adaptive Algorithm

In order to control the powers in the co-channel scenario, we develop the following algorithm. The algorithm is initialized by some feasible power allocation vector $\mathbf{P}(0)$ and some approximate channel estimation $\hat{\mathbf{A}}_{i,0}$. First ILSP blind beamforming algorithm is applied to estimate the channel response and the transmitted signal. Then we can get newer estimated $\hat{\mathbf{A}}_i$. The estimated interference plus noise power $\hat{\sigma}_i^2$ can be calculated from the estimated transmitted signal and channel responses. Then var_d is calculated for the d^{th} user. If this value is too large, it means that the BER for the d^{th} user is too large and consequently the d^{th} user's power needs to be increased. If var_d is too small, then it is unnecessary to have so much power for the d^{th} user. The algorithm is stopped by comparing the power vector of two consecutive iterations. The iteration number is m'. The adaptive algorithm for joint power control and blind beamforming is given in Table II

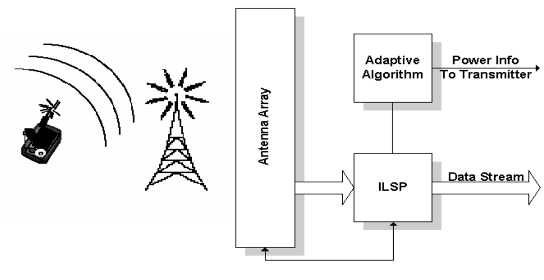

Fig. 1. Joint Power Control and Blind Estimation System

TABLE II

Joint Power Control and Blind Estimation Algorithm

1.	Given P(0), var_0, $m' = 0$ and $\hat{A}_i = \hat{A}_{i,0}$.
2.	Received data block at base station i, i. ILSP Blind Estimation to get $\hat{A}_i$ and $\hat{\sigma}_i^2$ ii. For each mobile d inside i^{th} cell, $var_d = \hat{\sigma}_i^2(\hat{A}_i^H\hat{A}_i)^{-1}_{dd}$ $P_i^d(m'+1) = \frac{var_d}{var_0}P_i^d(m')$ iii. $\hat{A}_{i,0} = \hat{A}_i$
3.	$m' = m' + 1$. Go to step 2; Repeat until $P_i^d(m') \approx P_i^d(m'-1)$, $\forall i, d$.

With the adaptive algorithm, we can construct a joint power control and blind beamforming system as shown in Fig. 1. The adaptive algorithm module gets estimation of users' channel and DOA from ILSP module. The targeted transmitted powers are updated. Then the power control information is sent back to mobiles.

C. Analysis and Convergence of the Algorithm

In this subsection, we analyze and prove the convergence of our proposed algorithm. Consider the transmission from the d^{th} mobile to its associated i^{th} base station. $\mathbf{h}_{ii}^d$ and G_{ii}^d give the channel and link gain. $\mathbf{A}_i$ is the channel response matrix. We want to find the expression of var_d in the power updated equation in step 2 of the adaptive algorithm. Then we will analyze the condition that our algorithm is converged. We have

$$[\mathbf{A}_i^H\mathbf{A}_i]_{jk} = \sqrt{P_i^j P_i^k G_{ii}^j G_{ii}^k}(\mathbf{h}_{ii}^j)^H\mathbf{h}_{ii}^k \tag{13}$$

It can be shown by deduction that

$$det(\mathbf{A}_i^H\mathbf{A}_i) = P_{ii}^1 G_{ii}^1 \dots P_{ii}^D G_{ii}^D f_1(\mathbf{h}_{ii}) \tag{14}$$

where $f_1(\mathbf{h}_{ii})$ is a real function of channel responses $\mathbf{h}_{ii}^d, \forall d$. Then it follows that

$$(\mathbf{A}_i^H \mathbf{A}_i)_{dd}^{-1} = \frac{\Pi_{j=1,j\neq d}^{D} P_{ii}^j G_{ii}^j f_2(\mathbf{h}_{ii})}{\Pi_{j=1}^{j=D} P_{ii}^j G_{ii}^j f_1(\mathbf{h}_{ii})} = \frac{f_3(\mathbf{h}_{ii})}{P_{ii}^d G_{ii}^d} \quad (15)$$

where $f_2(\mathbf{h}_{ii})$ and $f_3(\mathbf{h}_{ii})$ are real functions of channel responses $\mathbf{h}_{ii}^d, \forall d$. We assume that co-channel interference plus thermal noise in (8) are additive white Guaussian noise with variance

$$\sigma_i^2 = \sum_{k\neq i}^{K} \sum_{d=1}^{D} \|\mathbf{h}_{ki}^d\|^2 G_{ki}^d P_{ki}^d + N_i \quad (16)$$

Now we can calculate $Var[\hat{s}_i^d(n)]$ that directly connect to BER as:

$$Var[\hat{s}_i^d(n)] = \frac{\sigma_i^2}{P_{ii}^d G_{ii}^d} f_3(\mathbf{h}_{ii}) \quad (17)$$

The key result is that $Var(\hat{s}_k(n))$ is independent of the transmitted powers of other mobiles in the same cell. Use this value and put into the step 2 of our proposed algorithm. The power update equation can be expressed as:

$$P_i^d(n+1) = \frac{\sum_{j\neq i}^{K} \sum_{d=1}^{D} \|\mathbf{h}_{ji}^d\|^2 G_{ji}^d P_{ji}^d + N_i}{G_{ii}^d var_0} f_3(\mathbf{h}_{ii}) \quad (18)$$

$k = (i-1)\cdot D+d, 1 \leq d \leq D$, $j = (i'-1)\cdot D+d', 1 \leq d' \leq D$, In matrix form, we define a matrix $\mathbf{Q}$ as

$$[\mathbf{Q}]_{kj} = \begin{cases} G_{i'i}^{d'} f_4(\mathbf{h})/G_{ii}^d & if\ i' \neq i \\ 0 & Otherwise \end{cases} \quad (19)$$

where $f_4(\mathbf{h})$ is a real function. The matrix expression of (18) for the whole network can be written as:

$$\mathbf{P}(n+1) = \frac{1}{var_0} \mathbf{Q}\mathbf{P}(n) + \mathbf{u} \quad (20)$$

where

$$u_j = \frac{f_3(\mathbf{h}_{ii}) N_i}{G_{ii}^d var_0} \quad (21)$$

Following the same proof in [3], an optimum power vector exists and the power update converges if spectrum radius $\rho(\mathbf{Q}) < var_0$. Under this condition, the system is balanced in var_0. From the simulation results, we can see that our algorithm converges fast to the desired var_0 related to specific BER. When var_0 is too small and less than $\rho(\mathbf{Q})$, the system is not feasible and there is no power allocation solution.

IV. Simulation Results

A network with 50 base stations is simulated as shown in Fig. 2. Each hexagonal cell's radius is 1000m. Two adjacent cells do not share the same channel. In each cell, one base station is placed at the center. Two mobiles are placed randomly with uniform distribution. Each mobile transmits BPSK data over four multi-path fading channels. Each base station employs four elements antenna array. The transmit frame has 1000 data symbols. Power updates are jointly performed using the proposed algorithm.

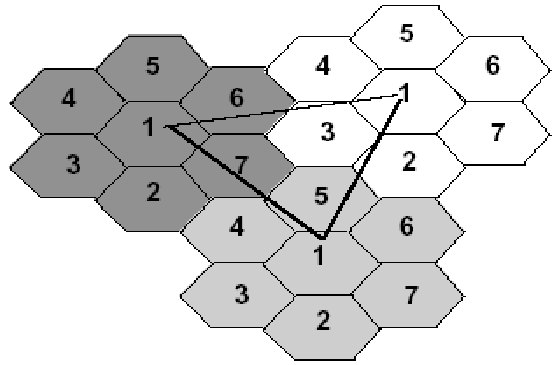

Fig. 2. Simulation Setup

In Fig. 3, we show the analytic and numerical performance of ILSP, compared with MVDR with perfect channel estimation. The numerical result with co-channel interference matches analytic results well especially at high SINR range, which proves our assumption that the sum of co-channel interference can be treated as additive white Guaussian noise. MVDR with perfection channel estimation has about 2 dB performance gain over ILSP. However it needs additional training sequence to estimate the channel and direction of arrival.

In reality, perfect channel estimation is hard to get. In Fig. 4, we show the effect of channel estimation error on the traditional power control with MVDR beamforming. Here we fix desired user's SINR. When the channel estimation is poor, the estimation error will greatly degrade the performance of MVDR method. We can see that when the channel estimation error is greater than 0.75%, ILSP algorithm outperforms the traditional MVDR. Consequently, our proposed algorithm is more practical in the real wireless communication networks with inaccurate channel and DOA estimation.

In Fig. 5, we show the numerical result of BER and overall power vs. var_0. When var_0 is decreasing, BER decreases and overall power increases slightly. We can define a threshold of var_0 for the desired BER. Within a reasonable BER range such as BER $= 10^{-3}$ to BER $= 10^{-5}$, the overall power and BER are converged. After var_0 decreases to a specific point, both BER and overall power increase quickly. This is because the co-channel interferences are too large and $\rho(\mathbf{Q}) > 1$, consequently there is no feasible power control solution, i.e. no matter how large the transmitted powers are, the receivers can not get enough SINR to ensure the desired BER. This proves that our algorithm behaviors exactly the same as traditional power control algorithm except that our algorithm directly ensures BER instead of each user's SINR.

In Fig. 6, we shown the distribution of number of itera-

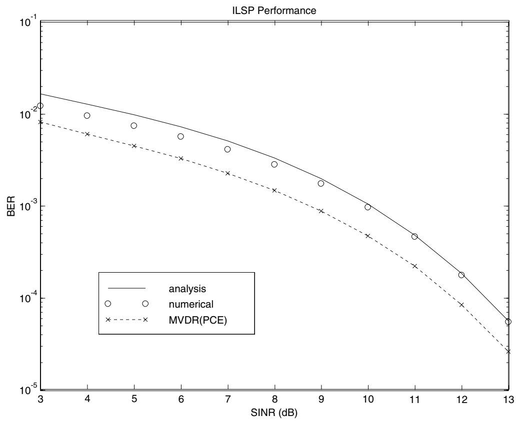

Fig. 3. ILSP Performance

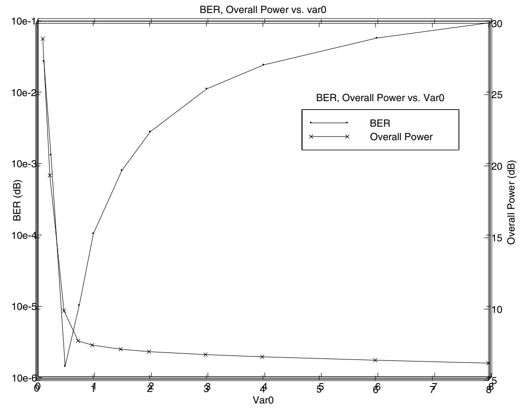

Fig. 5. BER, Overall Power vs. var_0

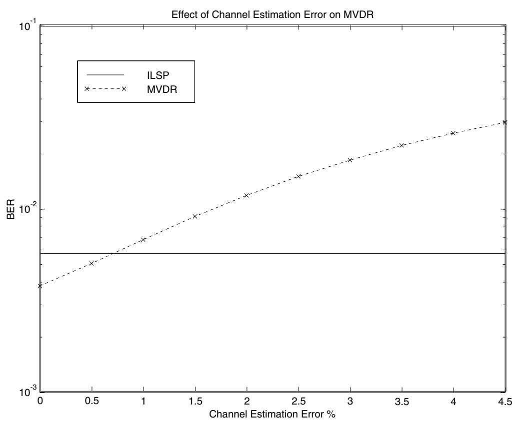

Fig. 4. Effect of Channel Estimation Error

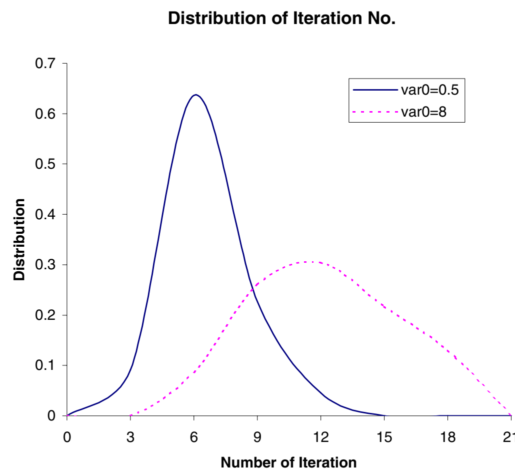

Fig. 6. Convergence of the Algorithm

tion to converge for our proposed algorithm with the different desired var_0. We can see that our algorithm converges within a small number of iterations. When var_0 is small, i.e. the desired BER is small, the algorithm converges more faster. When var_0 is within the range that the system is feasible, our algorithm is converged in a small number of iterations.

V. Conclusion

We have proposed a novel combined power control and blind beamforming algorithm that reformulates the power control problem in terms of a quantity directly related to the error performance of the estimation. First, this approach uses a more appropriate optimization of symbol error rate instead of a theoretically defined SINR, and this can be extended to symbol estimation algorithm other than ILSP. Secondly, the algorithm does not require additional measurements of interference or SINR. Theoretical result for the convergence of the algorithm is obtained and is supported by simulation results. Performance results show that our algorithm performances well in the real wireless communication networks.

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