Relay Selection in Multi-Node Cooperative Communications: When to Cooperate and Whom to Cooperate with?

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***Abstract*— In this paper, we propose a new cooperative communication protocol, which achieves high bandwidth efficiency while guaranteeing full diversity order. The proposed scheme considers *relay selection* via the available partial channel state information (CSI) at the source and the relays. More precisely, the source determines when it needs to cooperate with one relay only among arbitrary N relays and which relay to cooperate with in case of cooperation, i.e., *"When to cooperate?"* and *"Whom to cooperate with?"*. In case of cooperation, the source employs the optimal relay, which has the maximum instantaneous scaled harmonic mean function of its source-relay and relay-destination channels' gains. For the symmetric scenario, we prove that full diversity is guaranteed and that a significant increase of the bandwidth efficiency is achieved. Furthermore, we show the tradeoff between the achievable bandwidth efficiency and the corresponding error rate. Finally, the obtained analytical results are verified through computer simulations.**

I. Introduction

Recently, cooperative communications for wireless networks have gained much interest due to its ability to mitigate fading in wireless networks through achieving *spatial diversity*, while resolving the difficulties of installing multiple antennas on small communication terminals. In cooperative communication, relays are assigned to help a source in forwarding its information to its destination, hence forming a virtual antenna array. Various cooperative diversity protocols were proposed and analyzed in [1]-[8].

In this paper, we propose a cooperative protocol based on the *relay selection* technique using the availability of the partial channel state information (CSI) at the source and the relays. The main objective of this scheme is to achieve higher bandwidth efficiency than that of the conventional cooperative schemes, while guaranteeing full diversity order. More precisely, we consider a multi-relay scenario where arbitrary N relays are available and we discuss two main questions: *"When to cooperate?"* and *"Whom to cooperate with?"*. The rationale behind our proposed protocol is that there is no need for any relay to forward the source's information if the direct link between the source and the destination is of high quality. In addition, the source picks one relay only, which is referred to as the *optimal* relay, to cooperate with in case it needs help. This optimal relay is the one which has the maximum instantaneous value of a metric, which is a scaled version of the harmonic mean function of its source-relay and relay-destination channels' gains, among the N relays. For the symmetric scenario, we provide an approximate expression of the bandwidth efficiency and obtain an upper bound on the Symbol Error Rate (SER) performance. Finally, we provide a few tradeoff curves between the bandwidth efficiency and the SER.

The rest of this paper is organized as follows. In Section II, we present the conventional single-relay decode-and-forward cooperative scenario, which leads to the motivation behind choosing an appropriate metric to indicate the relay's ability to help. Furthermore, we propose the multi-node relay-selection decode-and-forward cooperative scenario. In Section III, we derive an approximate expression of the bandwidth efficiency and an upper bound on the SER performance for the symmetric scenario. Section IV presents the bandwidth efficiency-SER tradeoff curves for different number of relays and some simulation results which verify the analytical results. Finally, Section V concludes the paper.

II. Motivation and Proposed Relay-Selection Protocol

In this section, we present the system model of the conventional single-relay decode-and-forward cooperative scenario along with the SER results obtained in [4]. In addition, we introduce the proposed multi-node relay-selection decode-and-forward cooperative scenario.

A. Motivation for Relay-Selection Criterion

The conventional single-relay decode-and-forward cooperative scheme has been presented and analyzed in [4]. The single-relay communication system consists of a source s, its destination d, and a relay r. The transmission protocol requires two consecutive phases as follows. In the first phase, the source broadcasts its information to the relay and the destination. The received symbols at the destination and the relay can be modeled as $y_{s,d} = \sqrt{P_1}\, h_{s,d}\, x + \eta_{s,d}$ and $y_{s,r} = \sqrt{P_1}\, h_{s,r}\, x + \eta_{s,r}$, respectively, where P_1 is the source transmitted power, x is the transmitted information symbol, and $\eta_{s,d}$ and $\eta_{s,r}$ are additive noises. In addition, $h_{s,d}$ and $h_{s,r}$ are the source-destination and source-relay channel coefficients, respectively. If the relay decodes the received symbol correctly, it forwards the decoded symbol to the destination in the second phase, otherwise it remains idle. The received symbol at the destination is modeled as $y_{r,d} = \sqrt{\tilde{P}_2}\, h_{r,d}\, x + \eta_{r,d}$, where $\tilde{P}_2 = P_2$ if the

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relay decodes the symbol correctly, otherwise $\tilde{P}_2 = 0$, $\eta_{r,d}$ is an additive noise, and $h_{r,d}$ is the relay-destination channel coefficient. Power is distributed between the source and the relay subject to the power constraint $P_1 + P_2 = P$.

The channel coefficients $h_{s,d}$, $h_{s,r}$, and $h_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta^2_{s,d}$, $\delta^2_{s,r}$, and $\delta^2_{r,d}$, respectively. The noise terms $\eta_{s,d}$, $\eta_{s,r}$, and $\eta_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variance N_0. It has been shown in [4] that the SER for M-PSK signalling can be upper bounded as

$$Pr(e) \leq \frac{N_0^2}{b^2\ \delta^2_{s,d}\ P\ P_1}\left(\frac{A^2}{r\ \delta^2_{s,r}} + \frac{B}{(1-r)\ \delta^2_{r,d}}\right), \quad (1)$$

where $r = \frac{P_1}{P}$ is referred to as *power ratio*, $b = sin^2(\pi/M)$, $A = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \sin^2\theta\ \ d\theta = \frac{M-1}{2M} + \frac{\sin(\frac{2\pi}{M})}{4\pi}$, and $B = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \sin^4\theta\ d\theta = \frac{3(M-1)}{8M} + \frac{\sin(\frac{2\pi}{M})}{4\pi} - \frac{\sin(\frac{4\pi}{M})}{32\pi}$. Moreover, it was shown in [4] that the SER upper bound in (1) is tight at high enough signal-to-noise ratio (SNR).

The SER upper bound in (1) can be decomposed into two terms. The first term $\frac{N_0^2}{b^2\ \delta^2_{s,d}\ P\ P_1}$ is independent of the relay, while the second term $m = \frac{A^2}{r\ \delta^2_{s,r}} + \frac{B}{(1-r)\ \delta^2_{r,d}}$ depends on the relay channels, the modulation scheme, and the power ratio. Therefore the second term gives a measure, in some sense, about how much help a relay can provide to the source. A modified metric can be obtained by inverting m and formulating the result in a standard harmonic mean function [10] as

$$m' = \frac{2\ q_1\ q_2}{m} = \frac{2\ q_1\ q_2\ \delta^2_{s,r}\ \delta^2_{r,d}}{q_1\ \delta^2_{r,d} + q_2\ \delta^2_{s,r}} = \mu_H(q_1\ \delta^2_{r,d}, q_2\ \delta^2_{s,r})\ , \quad (2)$$

where $q_1 = \frac{A^2}{r}$ and $q_2 = \frac{B}{(1-r)}$. In order to minimize the SER in (1), (2) should be maximized.

We note that the expression in (2) is a function of the average channels gain $\delta^2_{s,r}$ and $\delta^2_{r,d}$. The average channels' gains cannot be utilized to determine whether the source needs the relay's help or not, as it may result in having a direct transmission between the source and the destination during the whole transmission, which results in a diversity order equal to 1. Therefore instead of having an average metric for the relay (2), we obtain an *instantaneous* metric. This instantaneous metric is the same as the average one, but with the average channels' gains being replaced by the instantaneous channels' gains and it is given by

$$\beta_m = \mu_H(q_1\ \beta_{r,d}, q_2\ \beta_{s,r}) = \frac{2\ q_1\ \ q_2\ \beta_{s,r}\ \beta_{r,d}}{q_1\ \beta_{r,d} + q_2\ \beta_{s,r}}\ , \quad (3)$$

where $\beta_{s,r} = |h_{s,r}|^2$ and $\beta_{r,d} = |h_{r,d}|^2$. Thus, the instantaneous value of (3) can be used to give an instantaneous indication about the relay's ability to cooperate with the source.

B. Proposed Relay-Selection Protocol

The conventional multi-node cooperative communication system consists of a source s, its destination d, and N relays, as shown in Fig. 1. In the conventional multi-node decode-and-forward cooperative communications [6], each relay receives the transmitted symbols from the source and the previous relays, applies maximal-ratio combining (MRC) [9] on the received signals, and re-transmits the decoded symbols if they have been correctly decoded. This protocol achieves full diversity order, however, it requires $N+1$ phases to complete the transmission and consequently the bandwidth efficiency is $R = \frac{1}{N+1}$ symbol per channel use (SPCU). Therefore, the objective of our proposed relay-selection scenario is to increase the bandwidth efficiency, while guaranteeing full diversity order. The basic idea of the proposed multi-node relay-selection cooperative scenario depends on selecting one relay among the N relays to cooperate with the source, if it needs cooperation.

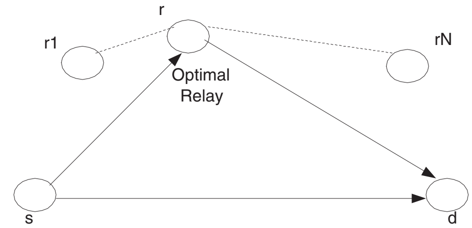

Fig. 1. Multi-node relay-selection cooperative communication system.

There are two main questions to be answered. The first question is how the optimal relay is selected, in case of cooperation and its answer follows from the motivation described earlier. The scaled harmonic mean function of the source-relay and relay-destination channels' gains is an appropriate measure on how much help a relay can offer. Thus the optimal relay is the one, which has the maximum scaled harmonic mean function of its source-relay and relay-destination channels' gains among all the N relays. With the optimal relay being selected, the system consists of the source s, the destination d, and the optimal relay r, as shown in Fig. 1. The second question is how the source determines whether to cooperate with this optimal relay or not, and its answer is explained in the sequel while explaining the transmission protocol.

Let the metric for each relay be defined as the scaled harmonic mean function of its source-relay and relay-destination channels' gains as $\beta_i = \mu_H(q_1\ \ \beta_{r_i,d}, q_2\ \ \beta_{s,r_i})\ \ ,for\ \ i = 1, 2, \cdots, N$. Consequently, the optimal relay has a metric which is equal to $\beta_{max} = \max\{\ \beta_1, \beta_2, \ldots, \beta_N\ \ \}$. The transmission protocol of the proposed scheme can be described as follows. In the first phase, the source computes the ratio $\beta_{s,d}/\beta_{max}$ and compares it to a constant, referred to as *cooperation threshold* α. If $\frac{\beta_{s,d}}{\beta_{max}} \geq \alpha$, then the source decides to use direct transmission only. This mode is referred to as the *direct-transmission* mode. Let $\phi = \{\ \beta_{s,d} \geq \alpha\ \beta_{max}\ \}$ be the event of direct transmission. The received symbol at the destination can then be modeled as

$$y^{\phi}_{s,d} = \sqrt{P}\ h_{s,d}\ x + \eta_{s,d}, \quad (4)$$

where P is the total transmitted power.

On the other hand, if $\frac{\beta_{s,d}}{\beta_{max}} < \alpha$, then the source employs the optimal relay r to transmit its information as in the conventional single-relay decode-and-forward cooperative protocol [4]. This mode is denoted by *relay-cooperation* mode and can be described as follows. In the first phase, the source broadcasts its symbol to both the optimal relay and the destination. The received symbols at the destination and the optimal relay can be modeled as

$$y_{s,d}^{\phi^c} = \sqrt{P_1}\ h_{s,d}\ x + \eta_{s,d}, \qquad y_{s,r}^{\phi^c} = \sqrt{P_1}\ h_{s,r}\ x + \eta_{s,r}, \quad (5)$$

respectively, where ϕ^c denotes the complement of the event ϕ. The optimal relay decodes the received symbol and retransmits the decoded symbol if correctly decoded in the second phase, otherwise it remains idle. The received symbol at the destination is modeled as

$$y_{r,d}^{\phi^c} = \sqrt{\tilde{P}_2}\ h_{r,d}\ x + \eta_{r,d}, \quad (6)$$

where $\tilde{P}_2 = P_2$ if the relay decodes the symbol correctly, otherwise $\tilde{P}_2 = 0$. Power is distributed between the source and the optimal relay subject to the power constraint $P_1 + P_2 = P$. The terms presented in (4)-(6) are defined as in Section II-A. In addition, the channel coefficients and noise terms are modeled as in Section II-A.

The optimal relay decides whether to forward the received information or not according to the quality of the received signal. For mathematical tractability, we assume that the relay can decide whether the information is decoded correctly or not. Practically, this can be done at the relay by applying a simple SNR threshold on the received data. Although, it can lead to some error propagation, but for practical ranges of operating SNR, the event of error propagation can be assumed to be negligible.

We assume that the channels are reciprocal as in the time division duplex (TDD) mode, hence each relay knows its source-relay and relay-destination channels' gains and calculates their harmonic mean function. Then, each relay sends this metric to the source through a feedback channel. Furthermore, we assume that the source knows its source-destination channel's gain. Thus, the source uses its source-destination channel's gain and the maximum metric of the relays, to determine whether to cooperate with one relay only or not. Finally, the source sends a control signal to the destination and the relays to indicate its decision and the optimal relay it is going to cooperate with, in case of cooperation. This procedure is repeated every time the channels' gains vary. We assume that the channels' gains vary slowly so that the overhead resulting from sending the relays' metrics is negligible. We should note here that the source and the relays are not required to know the phase information of their channels.

III. Performance Analysis

In this section, we calculate an approximate expression of the bandwidth efficiency and an upper bound on the SER performance for the multi-node relay-selection decode-and-forward cooperative scenario.

A. Bandwidth Efficiency Analysis

We derive the achievable bandwidth efficiency of the multi-node relay-selection decode-and-forward cooperative scheme as follows. The cumulative distribution function (CDF) of β_i for $i = 1, 2, \cdots, N$, denoted by $P_{\beta_i}(.)$, can be written as given in [10] as $P_{\beta_i}(\beta_i) = 1 - \frac{\beta_i}{t_{1,i}} \exp(-\frac{t_{2,i}}{2}\ \beta_i)\ K_1(\frac{\beta_i}{t_{1,i}})$ where $t_{1,i} = \sqrt{q_1\ q_2\ \delta_{s,r_i}^2\ \delta_{r_i,d}^2}$, $t_{2,i} = \frac{1}{q_2\ \delta_{s,r_i}^2} + \frac{1}{q_1\ \delta_{r_i,d}^2}$, and $K_1(x)$ is the first-order modified Bessel functions of the second kind, defined in [[11], (9.6.22)]. The CDF of β_{max} is calculated as $P_{\beta_{max}}(\beta) = \prod_{i=1}^{N} P_{\beta_i}(\beta)$, and the probability density function (PDF) of β_{max} is written as

$$p_{\beta_{max}}(\beta) \approx \sum_{j=1}^{N} p_{\beta_j}(\beta) \Bigg(\prod_{i=1, i\neq j}^{N} \Big(1 - \exp(-\frac{t_{2,i}}{2}\ \beta) \Big) \Bigg), \quad (7)$$

where we approximated $K_1(.)$ as given in [[11], (9.6.9)] by $K_1(x) \approx \frac{1}{x}$.

The expression in (7) is complex and will lead to more complex and intractable expressions. For simplicity, we consider the symmetric scenario where all the relays have the same source-relay and relay-destination channel variances, i.e., $\delta_{s,r_i}^2 = \delta_{s,r}^2$ and $\delta_{r_i,d}^2 = \delta_{r,d}^2$ for $i = 1, 2, \ldots, N$. Consequently, let $t_1 = \sqrt{q_1\ q_2\ \delta_{s,r}^2\ \delta_{r,d}^2}$ and $t_2 = \frac{1}{q_2\ \delta_{s,r}^2} + \frac{1}{q_1\ \delta_{r,d}^2}$.

The probability of the direct-transmission mode can be calculated as

$$\begin{aligned} Pr(\phi) = Pr(\beta_{s,d} \geq \alpha\ \beta_{max}) = \int_0^\infty P_{\beta_{max}}(\frac{\beta_{s,d}}{\alpha}) \\ \cdot p_{\beta_{s,d}}(\beta_{s,d})\ d\beta_{s,d} \approx \sum_{n=0}^{N} \tbinom{N}{n} \frac{2\alpha\ (-1)^n}{2\alpha + t_2\ \delta_{s,d}^2\ n}\ . \end{aligned} \quad (8)$$

In addition, the probability of the relay-cooperation mode is $Pr(\phi^c) = 1 - Pr(\phi)$. Since the bandwidth efficiency of the direct-transmission mode is 1 SPCU, and that of the relay-cooperation mode is 1/2 SPCU, thus the average bandwidth efficiency is calculated as

$$\begin{aligned} R &= Pr(\phi) + \frac{1}{2}\ Pr(\phi^c) \\ &\approx \frac{1}{2} + \sum_{n=0}^{N} \tbinom{N}{n} \frac{(-1)^n \alpha}{2\alpha + \left(\frac{1-r}{B\ \delta_{s,r}^2} + \frac{r}{A^2\ \delta_{r,d}^2}\right) \delta_{s,d}^2\ n}\ . \end{aligned} \quad (9)$$

B. SER Analysis

In the sequel, we consider the M-PSK modulation scheme. The probability of symbol error, or SER, is defined as

$$Pr(e) = Pr(e/\phi) \cdot Pr(\phi) + Pr(e/\phi^c) \cdot Pr(\phi^c)\ , \quad (10)$$

where $Pr(e/\phi) \cdot Pr(\phi)$ represents the SER of the direct-transmission mode and $Pr(e/\phi^c) \cdot Pr(\phi^c)$ represents the relay-cooperation mode SER. The SER of the direct-transmission mode can be calculated as follows. First, the instantaneous direct-transmission SNR is $\gamma^\phi = \frac{P\ \beta_{s,d}}{N_0}$. In addition, the conditional direct-transmission SER can be written as given in [12] as

$$Pr(e/\phi, \beta_{s,d}) = \Psi(\gamma^{\phi}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp(-\frac{b\,\gamma^{\phi}}{\sin^2\theta})\,d\theta\ , \quad (11)$$

where $b = sin^2(\pi/M)$. It can be shown that

$$Pr(e/\phi)\ Pr(\phi) \approx \sum_{n=0}^{N} \binom{N}{n} (-1)^n \times F_1\big(1 + \frac{t_2\ \delta_{s,d}^2\ n}{2\alpha} + \frac{b\ P}{N_0\ sin^2\theta}\ \delta_{s,d}^2\big)\ , \quad (12)$$

where $F_1\big(x(\theta)\big) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{x(\theta)}\ d\theta$.

For the relay-cooperation mode, maximal-ratio combining (MRC) [9] is applied at the destination. The output of the MRC [9] is written as $y^{\phi^c} = \frac{\sqrt{P_1}\ h^*_{s,d}}{N_0}\ y^{\phi^c}_{s,d} + \frac{\sqrt{\tilde{P_2}}\ h^*_{r,d}}{N_0}\ y^{\phi^c}_{r,d}$. Consequently, the instantaneous SNR of the MRC output is calculated as $\gamma^{\phi^c} = \frac{P_1\beta_{s,d} + \tilde{P}_2\beta_{r,d}}{N_0}$. The conditional SER of the relay-cooperation mode is given in [4] as

$$Pr(e/\phi^c, \beta_{s,d}, \beta_{s,r}, \beta_{r,d}) = \Psi(\gamma^{\phi^c})|_{\tilde{P}_2=0}\ \Psi(\frac{P_1\beta_{s,r}}{N_0}) + \Psi(\gamma^{\phi^c})|_{\tilde{P}_2=P_2}\ \left(1 - \Psi(\frac{P_1\beta_{s,r}}{N_0})\right)\ . \quad (13)$$

Let $Pr(X/\phi^c, \beta_{s,d}, \beta_{s,r}, \beta_{r,d}) = \Psi(\gamma^{\phi^c})\Psi(\frac{P_1\beta_{s,r}}{N_0})$ and $Pr(Y/\phi^c, \beta_{s,d}, \beta_{s,r}, \beta_{r,d}) = \Psi(\gamma^{\phi^c})$. It can be shown that

$$Pr(X/\phi^c)Pr(\phi^c) = \int_{\underline{\tilde{\beta}}} \frac{1}{\pi^2}\int_{\theta_1=0}^{\frac{(M-1)\pi}{M}} \int_{\theta_2=0}^{\frac{(M-1)\pi}{M}} \frac{1 - \exp\Big(-\big(P_1C(\theta_1) + \frac{1}{\delta^2_{s,d}}\big)\alpha\beta_{max}\Big)}{1 + P_1\ C(\theta_1)\delta^2_{s,d}}\, p_{\underline{\tilde{\beta}}}(\underline{\tilde{\beta}}) \cdot \exp\Big(-\big(P_2C(\theta_1)\beta_{r,d} + P_1C(\theta_2)\beta_{s,r}\big)\Big)d\theta_2\ d\theta_1\ d\underline{\tilde{\beta}}\, , \quad (14)$$

where $\underline{\tilde{\beta}} = [\beta_{s,r},\ \beta_{r,d}]$ and $C(\theta) = \frac{b}{N_0\sin^2\theta}$.

It is difficult to get a closed-form expression for (14), thus, we obtain an upper bound via a worst-case scenario. More precisely, we replace $\beta_{s,r}$ and $\beta_{r,d}$ in (14) by their worst-case values in terms of β_{max}. Then, we average (14) over β_{max} only. Since $\beta_{max} = \mu_H(q_1\ \beta_{r,d}, q_2\ \beta_{s,r})$, we can write $\frac{1}{\beta_{max}} = \frac{1}{2\ q_2\ \beta_{s,r}} + \frac{1}{2\ q_1\ \beta_{r,d}}$. Thus, $\beta_{max} \leq 2\ q_2\ \beta_{s,r}$ and $\beta_{max} \leq 2\ q_1\ \beta_{r,d}$. Therefore, we replace $\beta_{s,r}$ and $\beta_{r,d}$ by their worst values in terms of β_{max} as $\beta_{s,r} \longrightarrow \frac{\beta_{max}}{2\ q_2}$ and $\beta_{r,d} \longrightarrow \frac{\beta_{max}}{2\ q_1}$. Therefore, (14) can be upper bounded as

$$Pr(X/\phi^c)Pr(\phi^c) \leq \frac{1}{\pi^2}\int_{\theta_1=0}^{\frac{(M-1)\pi}{M}} \frac{d\theta_1}{1 + P_1\ C(\theta_1)\delta^2_{s,d}} \int_{\theta_2=0}^{\frac{(M-1)\pi}{M}} \Bigg(M_{\beta_{max}}\Big(\frac{\tilde{P}_2\ C(\theta_1)}{2\ q_1} + \frac{P_1\ C(\theta_2)}{2\ q_2}\Big) - M_{\beta_{max}}\Big(\big(P_1C(\theta_1) + \frac{1}{\delta^2_{s,d}}\big)\alpha + \frac{\tilde{P}_2C(\theta_1)}{2\ q_1} + \frac{P_1C(\theta_2)}{2\ q_2}\Big)\Bigg)d\theta_2, \quad (15)$$

N	r	α	R	CG
1	0.5744	1.3	0.8018	0.3676
2	0.5528	0.84	0.7880	0.2135
3	0.5409	0.68	0.7824	0.1421
4	0.5334	0.6	0.7781	0.1033

TABLE I

Optimum values of power ratio and cooperation threshold for unity channel variances.

where $M_{\beta_{max}}(.)$ is the moment generation function (MGF) of β_{max} and can be written as $M_{\beta_{max}}(\gamma) \approx N\sum_{n=0}^{N-1}\binom{N-1}{n}\ (-1)^n\ M_{\beta_m}(\gamma + \frac{n\ t_2}{2})$. The $M_{\beta_m}(.)$ is the MGF of the harmonic mean function of two independent exponential random variables obtained in [10]. The same technique can be implemented to get an upper bound for $Pr(Y/\phi^c)Pr(\phi^c)$.

Using this worst-case approximation it can be shown that at high SNR $\gamma = \frac{P}{N_0}$, the total SER of the multi-node relay-selection decode-and-forward symmetric cooperative scenario is upper bounded as

$$Pr(e) \quad \leq \quad (CG\cdot\gamma)^{-(N+1)}\ , \quad (16)$$

where CG denotes the coding gain and is equal to

$$CG = \Bigg(\frac{N!\ \big(\frac{1-r}{B\ \delta^2_{s,r}} + \frac{r}{A^2\ \delta^2_{r,d}}\big)^{N-1}}{b^{N+1}\ \delta^2_{s,d}}\Bigg)^{-\frac{1}{(N+1)}} \cdot \Bigg(\frac{\big(\frac{1-r}{B\ \delta^2_{s,r}} + \frac{r}{A^2\ \delta^2_{r,d}}\big)\ I(2\ N+2)}{(2\ \alpha)^N} + \Big(\frac{A^2\delta^2_{r,d}}{r} + \frac{B\delta^2_{s,r}}{1-r}\Big) \frac{\big(A^{2N}\ I(2\ N+2) + B^N\ A\ I(2\ N)\big)}{r^{N+1}\ (1-r)^N}\Bigg)^{-\frac{1}{(N+1)}}\ , \quad (17)$$

where $I(p) = \frac{1}{\pi}\int_{\theta=0}^{\frac{(M-1)\pi}{M}} \sin^p\theta\ \ d\theta$. (16) shows that full diversity order of $N+1$ is guaranteed as long as $\alpha > 0$. In (16), we have applied the approximation $M_{\beta_m}(\gamma) \approx \frac{q_1\ \delta^2_{r,d} + q_2\ \delta^2_{s,r}}{2\ \gamma}$, derived in [5], which is valid at high enough SNR.

IV. Bandwidth Efficiency-SER Tradeoff and Simulation Results

In this section, we present the bandwidth efficiency-SER tradeoff curves, which are used to obtain the optimum cooperation threshold. Furthermore, we show some simulation results.

A. Bandwidth Efficiency-SER Tradeoff

First we obtain the optimum power ratio, which minimizes the SER upper bound in (16) through exhaustive numerical search. Table I presents the obtained optimum power ratios for different number of relays. Fig. 2 depicts the bandwidth efficiency-SER tradeoff curves for different number of relays at SNR equal to 20 dB. This tradeoff is the achievable bandwidth efficiency and SER for different values of cooperation threshold. At certain SER value, the maximum achievable bandwidth efficiency, while guaranteeing full diversity order, can be obtained.

Fig. 2 depicts that the bandwidth efficiency-SER tradeoff curve achieved by four relays is the best among the plotted curves in the low SER region. In general, increasing

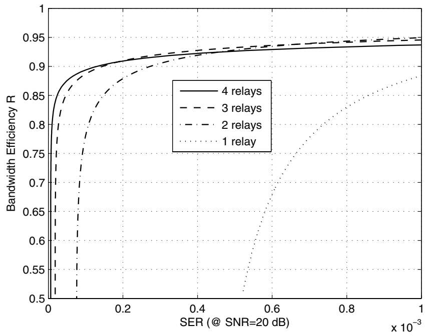

Fig. 2. Bandwidth efficiency versus SER at SNR=20 dB using unity channel variances.

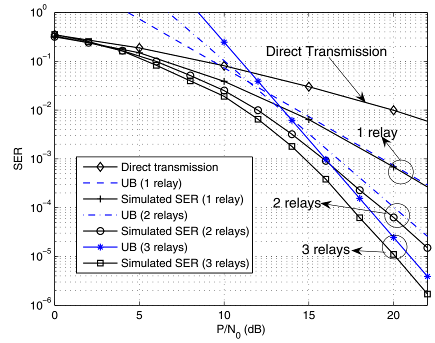

Fig. 3. SER simulated with optimum power ratio and SER upper bound curves with QPSK modulation and unity channel variances.

the number of relays does not necessarily lead to a better bandwidth efficiency-SER tradeoff for the following reason. As the bandwidth efficiency corresponds to the number of channel uses required to transmit a fixed-modulation symbol. Therefore, the more relays into the system, the higher the number of available paths between the source and the destination, the higher the probability that one of them is more effective than the direct source-destination path, the higher the probability of the relay-cooperation mode, and finally the higher the probability that the bandwidth efficiency is closer to 1/2 SPCU. Therefore, increasing the number of relays reduces both the SER and the bandwidth efficiency. Thus in general, increasing the number of relays does not necessarily improve the bandwidth efficiency-SER tradeoff curve.

As an *example* of choosing the optimum cooperation threshold, we choose an optimization metric that is the product of the coding gain and bandwidth efficiency. This optimization metric can be written as $\max_{\alpha}(CG \cdot R)$, where R and CG are obtained from (9) and (17), respectively. The optimum values of α are presented in Table I. Finally, Table I shows that the bandwidth efficiency is boosted up from 0.2 to 0.77 SPCU for $N = 4$ relays, while guaranteeing full diversity order (16).

B. Simulation Results

In the sequel, we assume that the noise variance is $N_0 = 1$. For fair comparison, the SER curves are plotted as a function of P/N_0. Finally, QPSK signalling is used in all the simulations. Fig. 3 depicts the SER performance employing one, two, and three relays for unity channel variances. We plot the simulated SER curves using the optimum power ratios and the optimum cooperation thresholds obtained in Table I. Moreover, we plot the SER upper bounds (16), which achieve full diversity order. It is obvious that the simulated SER curves are bounded by these upper bounds, hence they achieve full diversity order as well. The direct-transmission SER curve is plotted as well to show the significant improvement resulting from employing the relays in a cooperative scenario.

V. Conclusion

In this paper, we have proposed a new multi-node relay-selection decode-and-forward cooperative scenario, which utilizes the partial CSI available at the source and the relays. The main objective of this work is to achieve high bandwidth efficiency and to guarantee full diversity order. We have proven that full diversity order is guaranteed as long as there is a positive probability of having cooperation. We have shown that the bandwidth efficiency is boosted up from 0.2 to 0.77 SPCU for $N = 4$ relays and unity channel variances case. As for the optimum cooperation threshold, we have shown the bandwidth efficiency-SER tradeoff curves, which determine the optimal cooperation threshold. Finally, we have presented some simulation results to verify the obtained analytical results.

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