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Cooperation Enforcement in Autonomous MANETs under Noise and Imperfect Observation

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***Abstract*— In autonomous mobile ad hoc networks (MANET) where each user is its own authority, the issue of cooperation enforcement must be solved first to enable network functioning, such as packet forwarding, which becomes very difficult under noise and imperfect monitoring. In this paper, we focus on cooperation enforcement in autonomous mobile ad hoc networks under noise and imperfect observation and study the basic packet-forwarding function using the repeated game models with imperfect information. A belief-based packet forwarding framework is proposed to obtain cooperation-enforcement strategies solely based on each node's own past actions and its private imperfect observation of other nodes' information. The simulation results illustrate that the proposed belief-based packet forwarding approach can enforce the cooperation with only a small performance degradation compared with the unconditionally cooperative outcomes when noise and imperfect observation exist.**

I. INTRODUCTION

Mobile ad hoc networks (MANET) have drawn extensive attention in recent years due to the increasing demands of its potential applications [1], [2]. In traditional crisis or military situations, the nodes in a MANET usually belong to the same authority and work in a fully cooperative way of unconditionally forwarding packets for each other to achieve their common goals. Recently, the MANETs are also envisioned to be deployed for civilian applications [3]–[11], where nodes typically do not belong to a single authority and may not pursue a common goal. Consequently, fully cooperative behaviors cannot be directly assumed as the nodes are selfish to maximize their own interests. We refer to such networks as autonomous (or self-organized) MANETs.

However, before ad hoc networks can be successfully deployed in an autonomous way, the issue of cooperation enforcement must be resolved first. One way to enforce cooperation among selfish nodes is to use payment-based schemes such as [7], [8], [10], in which a selfish node will forward packets for other nodes only if it can get some payment from those requesters as compensation. For example, a cooperation enforcement approach was proposed in [7] by using a virtual currency called nuglets as payments for packet forwarding, which requires tamper-proof hardware in each node. Another payment-based system, SPRITE [8], releases the requirement of tamper-proof hardware, but requires some online central banking service trusted by all nodes. Another way to enforce cooperation among selfish nodes is to use reputation-based schemes with necessary traffic monitoring mechanisms such as [4]–[6], [9], in which a node determines whether it should forward packets for other nodes or request other nodes to forward packets for it based on their past behaviors. In [4], a reputation-based system was proposed for ad hoc networks to mitigate nodes' misbehaviors, where each node launches a "watchdog" to monitor its neighbors' packet forwarding activities. Following [4], CORE and CONFIDANT systems [5], [6] were proposed to enforce cooperation among selfish nodes which aim at detecting and isolating misbehaving node and thus making it unattractive to deny cooperation. Recently, ARCS was proposed in [9] to further defend against various attacks while providing the incentives for cooperation.

Recently, some efforts have been made towards mathematical analysis of cooperation in autonomous ad hoc networks using game theory, such as [11]–[14]. In [11], Srinivasan et al. provided a mathematical framework for cooperation in ad hoc networks, which focuses on the energy-efficient aspects of cooperation. In [12], Michiardi et al. studied the cooperation among selfish nodes in a cooperative game theoretic framework. In [13], Felegyhazi et al. defined a game model and identified the conditions under which cooperation strategies can form an equilibrium. In [14], Altman et al. studied the packet forwarding problem using a non-cooperative game theoretic framework.

One major drawback of these existing game theoretic analysis on cooperation in autonomous ad hoc networks lies in that all of them have assumed perfect observation, and most of them have not considered the effect of noise on the strategy design. However, in autonomous ad hoc networks, even when a node has decided to forward a packet for another node, this packet may still be dropped due to link breakage or transmission errors. Further, since central monitoring is in general not available in autonomous ad hoc networks, perfect public observation is either impossible or too expensive. Therefore, how to stimulate cooperation and analyze the efficiency of possible strategies in the scenarios with noise and imperfect observation are still open problems for autonomous ad hoc networks.

In this paper we study the cooperation enforcement for autonomous mobile ad hoc networks under noise and imperfect observation and focus on the most basic networking functioning, namely packet forwarding. Considering the nodes need to infer the future actions of other nodes based on their own imperfect observations, in order to optimally quantify the inference process with noise and imperfect observation, a belief-based packet forwarding approach is proposed to stimulate the packet forwarding between nodes and maximize the expected payoff of each selfish node by using repeated game theoretical analysis. Specifically, a formal belief system using Bayes' rule is developed to assign and update beliefs of other nodes' continuation strategies for each node based on its private imperfect information. Further, we not only show that the packet forwarding strategy obtained from the proposed belief-based framework achieves a sequential equilibrium that guarantees the strategy to be cheat-proof but also derive its performance bounds. The simulation results illustrate that the proposed belief-based packet forwarding approach can enforce the cooperation in autonomous ad hoc networks under noise and imperfect observation with only a small performance degradation compared to the unconditionally cooperative outcomes.

The rest of this paper is organized as follows. The system model of self-organized ad hoc networks under noise and imperfect observation is presented in Section II. In Section III, we propose the belief-based packet forwarding framework and carry out the equilibrium and efficiency analysis. In Section IV, the belief-based multi-hop multi-node packet forwarding approach is developed based on two-player strategies. The simulation studies are provided in Section V. Finally, Section VI concludes this paper.

II. System Model

We consider self-organized ad hoc networks where nodes belong to different authorities and have different goals. Assume all nodes are selfish and rational, that is, their objective are to maximize their own payoff, not to cause damage to other nodes. Each node may act as a service provider: packets are scheduled to be generated and delivered to certain destinations; or act as a relay: forward packets for other nodes. The sender will get some payoffs if the packets are successfully delivered to the destination and the forwarding effort of relay nodes will also introduce certain costs.

In this paper we assume that some necessary traffic monitoring mechanisms, such as those described in [4], [8], [9], will be launched by each node to keep tracking of its neighbors' actions. However, it is worth mentioning that we do not assume any public or perfect observation, where a public observation means that when an action happens, a group of nodes in the network will have the same observation, and perfect observation means all actions can be perfectly observed without any mistake. In ad hoc networks, due to its multi-hop nature and the lack of central monitoring mechanism, public observation is usually not possible. Meanwhile, to our best knowledge, these exist no such monitoring mechanisms in ad hoc networks which can achieve perfect observation. Instead, in this paper, we study the cooperation-enforcement strategies based on imperfect private observation. Here, private means that the observation of each node is only known to itself and won't or cannot be revealed to others.

We focus on two scenarios causing imperfect observation in ad hoc networks. One scenario is that the outcome of a forwarding action may be a packet-drop due to link breakage

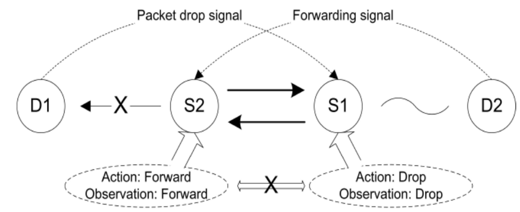

Fig. 1: Packet forwarding in autonomous ad hoc networks under noise and imperfect observation.

or transmission errors. The other scenario is that a node has dropped a packet but is observed as forwarding the packet, which may happen when the watchdog mechanism [4] is used and the node wants to cheat its previous node on the route. Figure 1 illustrates our system model by showing a network snapshot of one-hop packet forwarding between two users at a certain time stage under noisy and imperfect observations. In this figure, there are two source-destination pairs (S_1, D_1) and (S_2, D_2). At this time, node S_1 drops the packet and observes the packet-drop signal of node S_2's action, while node S_2 forwards the packet and observes the forwarding signal of node S_1's action. The action and observation of each node are only known to itself and cannot or will not be revealed to other nodes. Due to transmission error or packet loss, S_2's forwarding action is observed as a packet-drop signal; due to possible cheating behavior between S_1 and D_2, a forfeit forwarding signal may be observed by S_2. Therefore, it is important to design strategies for each node to make the optimal decision solely based on these imperfect private information.

III. Belief-Based Packet Forwarding

Two-player packet forwarding game is studied in this section in attempt to shed light on the solutions to the more complicated multi-player case.

A. Static and Repeated Packet-Forwarding Game Model

We model the process of routing and packet-forwarding between a source node and a relay node as a game. The players of the game are the network nodes. There are two players in this game, denoted by $i \in I = \{1, 2\}$. Each player is able to serve as the relay for the other player and needs the other player to forward packets for him based on current routing selection and topology. Each player chooses his action, i.e., strategy, a_i from the action set $A = \{F, D\}$, where F and D are packet forwarding and dropping actions, respectively. Also, each player observes a private signal ω of the opponent's action from the set $\Omega = \{f, d\}$, where f and d are the observations of packet forwarding and dropping signals, respectively. Since the player's observation cannot be perfect, the forwarding action F of one player may be observed as d by the other player due to link breakage or transmission error. We let such probability be p_f. Also, the noncooperation action D may be observed as the cooperation signal f under certain circumstances. Without loss of generality, let the observation

Player 1 \ Player 2	F	D
F	$g-l, g-l$	$-l, g$
D	$g, -l$	$0, 0$

Fig. 2: Two-player packet forwarding game in strategic form.

error probability be p_e in our system, which is usually caused by malicious cheating behaviors and the packet is actually dropped though forwarding signal f is observed. For each node, the cost of forwarding a group of packets for the other node during one play is ℓ, and the gain it can get for the packets that the other node has forwarded for it is $\tilde{g}$.

We first consider the packet forwarding as a static game [15], which is only played once. Given any action profile $a = (a_1, a_2)$, we refer to $u(a) = (u_1(a), u_2(a))$ as the expected payoff profile. Let a_{-i} and $\text{Prob}(\omega_i|a_{-i})$ be the action of the ith player's opponent and the probability of observation ω_i given a_{-i}, respectively. Then, $u_i(a)$ can be obtained as follows.

$$u_i(a) = \sum_{\omega_i \in \Omega} \tilde{u}_i(a_i, \omega_i, a_{-i}) \cdot \text{Prob}(\omega_i|a_{-i}), \quad (1)$$

where $\tilde{u}_i$ is the ith player's payoff depending on the action profile and his own observation. Then, calculating $u(a)$ for different strategy pairs, we have the strategic form of the static packet forwarding game as a matrix in Figure 2. Note that $g = (1 - p_f) \cdot \tilde{g}$, which can be obtained from (1) considering the possibility of the packet-drop.

To analyze the outcome of a static game, the Nash Equilibrium [15], [16] is a well-known concept, which states that in the equilibrium every player selects a payoff-maximizing strategy given the strategies of other players. Noting that our two-player packet-forwarding game is similar to the setting of the prisoner's dilemma game, the only Nash equilibrium is the action profile $a^* = (D, D)$, and the better cooperation payoff outcome $(g-\ell, g-\ell)$ of the cooperation action profile $\{F, F\}$ cannot be practically realized in the static packet-forwarding game due to the greediness of the players. However, generally speaking, the above packet forwarding game will be played many times in real ad hoc networks. It is natural to extend the above static game model to a multistage game model [15]. Considering that the past packet-forwarding behaviors do not influence the feasible actions or payoff function in the current period, the multistage packet forwarding game can be further analyzed using the repeated game model [15], [16]. Basically, in the repeated games, the players face the same static game in every period, and the player's overall payoff is a weighted average of the payoffs in each stage over time.

Let ω_i^t be the privately observed signal of the ith player in period t. Suppose that the game begins in period 0 with the null history h^0. In this game, a private history for player i at period t, denoted by h_i^t, is a sequence of player i's past actions and signals, i.e., $h_i^t = \{a_i^\tau, \omega_i^\tau\}_{\tau=1}^{t-1}$. Let $H_i^t = (A \times \Omega)^t$ be the set of all possible period-t histories for the ith player. Denote the infinite packet-forwarding repeated game with imperfect private histories by $G(p, \delta)$, where $\delta \in (0, 1)$ is the discount factor and $p = (p_f, p_e)$. Assume that $p_f < 1/2$ and $p_e < 1/2$. Then, the overall discounted payoff for player $i \in I$ is defined as follows [15].

$$U_i(\delta) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i^t(a_1^t(h_1^t), a_2^t(h_2^t)). \quad (2)$$

Folk Theorems for infinite repeated games [15] assert that there exists $\hat{\delta} < 1$ such that any feasible and individually rational payoff can be enforced by an equilibrium for all $\delta \in (\hat{\delta}, 1)$ based on the public information shared by players. However, one crucial assumption for the Folk Theorems is that players share common information about each other's actions. In contrast, the nature of our repeated packet forwarding game for autonomous ad hoc networks determines that the nodes' behaviorial strategies can only rely on the private information histories including their own past actions and imperfectly observed signals. Such a minor game-setting change from the public observation to the private observation due to noise and imperfect observation will make a substantial difference in analyzing the efficiency of the packet-forwarding game. In the situation of imperfect private observation, no recursive structure [17] exists for the forwarding strategies since the player decides their actions according to various private histories. Each node must conduct statistical inference to detect potential deviations and estimate what others are going to do next, which can become extremely complex due to the imperfect observation [18], [19].

B. Belief-Based Packet Forwarding Approach

In order to have an efficient and robust forwarding strategy based on each node's own observation and actions, we propose a belief-based packet forwarding approach enlightened by [19].

First, we define two strategies, i.e., σ_F and σ_D. Let σ_F be the trigger cooperation strategy, which means that the player forwards packets at current stage, and at the next stage the player will continue to forward packets only if it observes the other player's forwarding signal f. Let σ_D be the defection strategy, which means that the player always drops packets regardless of its observation history. Such strategies are also called continuation strategies [19]. Since both of the two strategies also determine the player's following actions at every private history, the strategy path and expected future payoffs caused by any pair of the two strategies are fully specified. Let $V_{\alpha,\beta}(p, \delta), \alpha, \beta \in \{F, D\}$ denote the repeated game payoff of σ_α against σ_β, which can be illustrated by the following Bellman equations [20] for different pairs of continuation strategies.

$$V_{FF} = (1-\delta)(g-\ell) + \delta((1-p_f)^2 V_{FF} + p_f(1-p_f)V_{FD} + p_f(1-p_f)V_{DF} + p_f^2 \cdot V_{DD}), \quad (3)$$

$$V_{FD} = -(1-\delta)\ell + \delta((1-p_f)(1-p_e)V_{DD} + p_f(1-p_e)V_{DD} + p_e(1-p_f)V_{FD} + p_f p_e V_{FD}), \quad (4)$$

$$V_{DF} = (1-\delta)g + \delta((1-p_f)(1-p_e)V_{DD} + p_e(1-p_f)V_{DF} + p_f(1-p_e)V_{DD} + p_e p_f V_{DF}), \quad (5)$$

$$V_{DD} = (1-\delta)\cdot 0 + \delta((1-p_e)^2 V_{DD} + p_e(1-p_e)V_{DD} + p_e(1-p_e)V_{DD} + p_e^2 \cdot V_{DD}). \quad (6)$$

Note that the first term in the above equations represents the normalized payoffs of current period, while the second term illustrates the expected continuation payoffs considering four possible outcomes due to the imperfect observation. By solving the above equations, $V_{\alpha,\beta}(p,\delta)$ can be easily obtained. Then, we have $V_{DD} > V_{FD}$, for any δ, p. Furthermore, if $\delta > \delta_0$, then $V_{FF} > V_{DF}$, where δ_0 can be calculated as

$$\delta_0 = \frac{\ell}{(1-p_f-p_e)g - [p_f(1-p_f)-p_e]\ell}. \quad (7)$$

Note that the first term in the above equations represents the normalized payoffs of current period. Suppose that player i believes that his opponent is playing either σ_F or σ_D, and is playing σ_F with probability μ. Then the difference between his payoff of playing σ_F and the payoff of playing σ_D is given by

$$\triangle V(\mu;\delta,p) = \mu\cdot(V_{FF}-V_{DF}) - (1-\mu)\cdot(V_{DD}-V_{FD}). \quad (8)$$

Hence $\triangle V(\mu)$ is increasing and linear in μ and there is a unique value $\pi(p,\delta)$ to make it zero, which can be obtained as follows.

$$\pi(\delta,p) = \frac{-V_{FD}(\delta,p)}{V_{FF}(\delta,p) - V_{DF}(\delta,p) - V_{FD}(\delta,p)}, \quad (9)$$

where $\pi(p,\delta)$ is defined so that there is no difference for player i to play σ_F or σ_D when player j plays σ_F with probability $\pi(\delta,p)$ and σ_D with probability $1-\pi(\delta,p)$. For simplicity, $\pi(\delta,p)$ may be denoted as π under the circumstances with no confusion. In general, if node i holds the belief that the other node will help him to forward the packets with a probability smaller than $1/2$, node i is inclined not to forward packets for the other node. Considering such situation, we let δ be such that $\pi(\delta,p) > 1/2$.

It is worth mentioning that equation (8) is applicable to any period. Thus, if a node's belief of an opponent's continuation strategy being σ_F is μ, in order to maximize its expected continuation payoff, the node prefers σ_F to σ_D if $\mu > \pi$ and prefers σ_D to σ_F if $\mu < \pi$. Starting with any initial belief μ, the ith player's new belief when he takes action a_i and receives signal ω_i can be defined using Bayes' rule [15] as follows.

$$\mu(h_i^{t-1},(F,f)) = \frac{\mu(h_i^{t-1})(1-p_f)^2}{\mu(h_i^{t-1})(1-p_f) + p_e\cdot(1-\mu(h_i^{t-1}))}, \quad (10)$$

TABLE I: Belief-based Two-player Packet Forwarding Algorithm

1. Initialize using system parameter configuration (δ, p_e, p_f)**:** Node i initializes his belief μ_i^1 of the other node as $\pi(\delta,p)$ and chooses the forwarding action in period 1.
2. Belief update based on the private history: Update each node's belief μ_i^{t-1} into μ_i^t using (10-13) according to different realizations of private history.
3. Optimal Decision of the player's next move: If the continuation belief $\mu_i^t > \pi$, node i plays σ_F; If the continuation belief $\mu_i^t < \pi$, node i plays σ_D; If the continuation belief $\mu_i^t = \pi$, node i plays either σ_F or σ_D.
4. Iteration: Let $t = t+1$, then go back to Step 2.

$$\mu(h_i^{t-1},(F,d)) = \frac{\mu(h_i^{t-1})(1-p_f)\cdot p_f}{\mu(h_i^{t-1})\cdot p_f + (1-p_e)\cdot(1-\mu(h_i^{t-1}))}, \quad (11)$$

$$\mu(h_i^{t-1},(D,f)) = \frac{\mu(h_i^{t-1})(1-p_f)\cdot p_e}{\mu(h_i^{t-1})\cdot(1-p_f) + p_e\cdot(1-\mu(h_i^{t-1}))}, \quad (12)$$

$$\mu(h_i^{t-1},(D,d)) = \frac{\mu(h_i^{t-1})p_f\cdot p_e}{\mu(h_i^{t-1})\cdot p_f + (1-p_e)\cdot(1-\mu(h_i^{t-1}))}. \quad (13)$$

Based on the above discussion, we propose a two-player belief-based packet forwarding algorithm in Table I.

C. Efficiency Analysis

In this part, we show that the behaviorial strategy obtained from the proposed algorithm with well-defined belief systems is a sequential equilibrium and further analyze its performance bounds.

First, we briefly introduce the equilibrium concepts of the repeated games with imperfect information. As for the infinitely repeated game with perfect information, the Nash Equilibrium concept is a useful concept for analyzing the game outcomes. But, since the threats in Nash equilibria may not be credible and become empty threats, the subgame perfect equilibrium [16] is defined to eliminate those equilibria in which the players' threats are incredible. However, the above equilibrium criteria for the infinitely repeated game require that perfect information can be obtained for each player. In our packet forwarding game, each node is only able to have his own strategy history and form the beliefs of other nodes' future actions through imperfect observation. Sequential Equilibrium [16] is a well-defined counterpart of subgame perfect equilibrium under such circumstance, which guarantees that any deviations will be unprofitable.

In our packet-forwarding game with private history and observation, the proposed strategy with belief-system can be denoted as (σ^*, μ), where $\mu = \{\mu_i\}_{i\in I}$ and $\sigma^* = \{\sigma_i^*\}_{i\in I}$. By studying (10), we find that there exists a point ϕ such that $\mu(h_i^{t-1},(F,f)) < \mu(h_i^{t-1})$ as $\mu(h_i^{t-1}) > \phi$ while $\mu(h_i^{t-1},(F,f)) > \mu(h_i^{t-1})$ as $\mu(h_i^{t-1}) < \phi$. Here, ϕ can be calculated as $\phi = [(1-p_f)^2 - p_e]/(1-p_f-p_e)$. It is easy to show that $\mu(h_i^{t-1},(a_i,\omega_i)) < \mu(h_i^{t-1})$ when $(F,d), (D,f)$ and (D,d) are reached. Since we initialize the belief with π we have $\mu_i^t \le \phi$ after any belief-updating operation if $\pi < \phi$. Considering the belief updating in the scenario that $\pi \ge \phi$ becomes trivial, we assume $\pi < \phi$ thus $\mu_i^t \in [0,\phi]$ and

$\phi \geq 1/2$. Then, let the proposed packet-forwarding strategy profile σ^* be defined as: $\sigma_i^*(\mu_i) = \sigma_F$ if $\mu_i > \pi$ and $\sigma_i^*(\mu_i) = \sigma_D$ if $\mu_i < \pi$; if $\mu_i = \pi$, the node forwards packets with probability π and drops them with probability $1 - \pi$. Similar to [19], we have the following two theorems.

Theorem 1: The proposed strategy profile σ^* with the belief system μ from Table I is a sequential equilibrium for $\pi \in (1/2, \phi)$.

Theorem 2: Given g and ℓ, there exist $\tilde{\delta} \in (0,1)$ and $\tilde{p}$ for any small positive τ such that the average payoff of the proposed strategy σ^* in the packet-forwarding repeated game $G(p,\delta)$ is greater than $g - \ell - \tau$ if $\delta > \tilde{\delta}$ and $p_e, p_f < \tilde{p}$.

Theorem 1 shows that the strategy profile σ^* and the belief system μ obtained from the proposed algorithm is a sequential equilibrium, which not only responds optimally at every history but also has consistency on zero-probability histories. Thus, the cooperation can be enforced using our proposed algorithm since the deviation will not increase the players' payoffs. Then, Theorem 2 addresses the efficiency of the equilibrium and shows that when the p_e and p_f are small enough, our proposed strategy approaches the cooperative payoff $g - \ell$. However, in real ad hoc networks, considering the mobility of the node, channel fading and the cheating behaviors of the nodes, it may be not practical to assume very small p_e and p_f values. A more useful and important measurement is the performance bounds of the proposed strategy given some fixed p_e and p_f values. We further develop the following theorem studying the lower bound and upper bound of our strategy to provide a performance guideline. In order to model the prevalent data application in current ad hoc networks, we assume the game discount factor is very close to 1.

Theorem 3: Given the fixed (p_e, p_f) and discount factor of the repeated game δ_G close to 1, the payoff of the proposed algorithm in Table I is upper bounded by

$$\bar{U} = (1 - \kappa) \cdot (g - \ell), \tag{14}$$

where

$$\kappa = \frac{p_f \cdot [g(1 - p_f) + \ell]}{(1 - p_f - p_e)(g - \ell)}. \tag{15}$$

The lower bound of the performance will approach the upper bound when the discount factor of the repeated game δ_G approaches 1 and the packet forwarding game is divided into N sub-games as follows: the first sub-game is played in period $1, N+1, 2N+1, ...$ and the second sub-game is played in period $2, N+2, 2N+2, ...$, and so on. The optimal N is

$$N = \lfloor \log \underline{\delta} / \log \delta_G \rfloor, \tag{16}$$

where $\underline{\delta} = \ell / \{[(1 - p_f)^2 - p_e] \cdot g + \ell \cdot p_e\}$. The proposed strategy is played in each sub-game with equivalent discount factor δ_G^N.

Proof: By substituting $V_{\alpha,\beta}$ obtained from (3)-(6) into (9), we have

$$\pi(\delta, p) = \frac{\ell}{g - \ell} \cdot \frac{1 - \delta(1 - p_f)^2}{\delta(1 - p_f - p_e)}. \tag{17}$$

Then, since the node i is indifferent of forwarding or dropping packets if his belief of the other node is equal to π the expected payoff of the node i at the sequential equilibrium σ^*, μ can be written as

$$V(\pi, \delta, p) = \pi(\delta, p) \cdot V_{DF}(\delta, p) + (1 - \pi(\delta, p)) \cdot V_{DD}(\delta, p). \tag{18}$$

It is easy to show that $V(\pi(\delta, p), \delta, p)$ is a decreasing function in δ when $\delta \in (0,1)$. Then, the upper bound of the expected payoff can be obtained by letting δ be the smallest feasible value. As we have shown in (7), δ needs to be greater than δ_0 for having non-trivial solutions. Considering $\pi(\delta, p) \leq \phi$, we obtain another constraint on δ, which can be written as follows.

$$\delta \geq \underline{\delta} = \frac{\ell}{[(1 - p_f)^2 - p_e] \cdot g + \ell \cdot p_e}. \tag{19}$$

Since $\underline{\delta} > \delta_0$, we obtain the upper bound of the payoff of the proposed algorithm as (14) by substituting $\underline{\delta}$ into (18).

However, the discount factor of our game is usually close to 1. Generally, $\underline{\delta}$ is a relatively smaller value in the range of $(0,1)$. In order to emulate the optimal discount factor $\underline{\delta}$, we introduce the following game partition method. We partition the original repeated game $G(p, \delta_G)$ into N distinct sub-games as the theorem illustrates. Each sub-game can be regarded as a repeated game with the discount factor δ_G^N. The optimal sub-game number N, which minimizes the gap between δ_G^N and $\underline{\delta}$, can be calculated as $N = \lfloor \log \underline{\delta} / \log \delta_G \rfloor$.

As there is always difference between δ_G^N and $\underline{\delta}$, it is more important to study the maximal gap, which results in the lower bound of the payoff using our game partition method. Similar to [21], we can show that by using the optimal N, $\delta_G^N \in [\underline{\delta}, \bar{\delta}]$, where $\bar{\delta} = \underline{\delta}/\delta_G$. Substituting $\bar{\delta}$ into (18), we have the lower bound of the payoff of our proposed algorithm with the proposed game partition method. When δ_G approaches 1, $\bar{\delta}$ approaches $\underline{\delta}$ and the payoff of our algorithm can achieve the payoff upper-bound. ■

In the above theorem, the idea of dividing the original game into some sub-games is useful to maintain the efficiency when δ approaches one for our game setting. A larger δ indicates that future payoffs are more important for the total payoff, which results in more number of sub-games. Since there are multiple sub-games using the belief-based forwarding strategy, even if the outcomes of some sub-games become the non-cooperation case due to the observation errors, cooperation plays can still continue in other sub-games to increase the total payoff.

IV. Belief-Based Multi-Node Multi-Hop Packet Forwarding

In the previous sections, we mainly focus on the two-player case, while in an ad hoc network there usually exist many nodes. In this section, we model the interactions among selfish nodes in an autonomous ad hoc network as a multi-player packet forwarding game, and investigate the optimal cooperation strategies based on the two-player belief-based approach.

A. Multi-Node Multi-Hop Game Model

In this section, we consider autonomous ad hoc networks where nodes can move freely inside a certain area. For each node, packets are scheduled to be generated and sent to certain destinations. Different from the twoplayer packet forwarding game, the multi-player packet forwarding game studies multi-hop packet forwarding which involves the interactions and beliefs of all the nodes on the route. Before studying the belief-based strategy in this scenario, we first model the multi-player packet forwarding game as follows:

- There are M players in the game, which represent M nodes in the network. Denote the player set as $I_M = \{1, 2, ..., M\}$.
- For each player $i \in I_M$, he has groups of packets to be delivered to certain destinations at different time. The payoff of successfully delivering a group of packets is denoted by $\tilde{g}$.
- For each player $i \in I_M$, forwarding a packet for another player will incur some cost ℓ.
- Due to the multi-hop nature of ad hoc networks, the destination player j may not lie in the sending player i's direct transmission range. Player i needs to not only find the possible routes leading to the destination (i.e., route discovery), but also choose an optimal route from multiple routing candidates to help forwarding the packets (i.e., route selection).
- Each player only knows his own past action and imperfect observation of other players' action. Note that the information history consisting of the above two parts is private to each player.

Similar to [11], we assume the network operates in discrete time. In each time slot, one node is randomly selected from the M nodes as the sender. The probability that the sender finds r possible routes is given by $q_r(r)$ and the probability that each route needs 1h hops is given by $q_\hbar(\hbar)$ (assume at lease one hop is required in each time slot). Note that the $\hbar$ relays on each route are selected from the rest of nodes with equal probability and $\hbar \leq \lfloor \tilde{g}/\ell \rfloor$. Assume each routing session lasts for one slot and the routes remain unchanged within each time slot. In our study, we consider that delicate traffic monitoring mechanisms such as receipt-submission approaches [8] are in place, hence the sender is able to have the observation of each node on the forwarding route.

B. Belief-Based Strategy Design

In this part, we develop efficient belief-based strategies for multi-hop packet forwarding games based on the proposed two-player approach. Since a successful packet transmission through a multi-hop route depends on the actions of all the nodes on the route, the belief-based forwarding system needs to consider the observation error caused by each node, which makes a direct design of the belief system for the multi-player case very difficult. However, the proposed two-player algorithm can be applied to solve the multi-player packet forwarding problem by considering the multi-node multi-hop game as many two-player games between the source and each relay node.

Let R_i^t denote the set of players on the forwarding route of player i in tth period. Let $\mu_{i,j}$ denote the sender i's belief value of the node j on the route. The proposed forwarding strategy for the multi-player case is illustrated as follows.

Belief-based Multi-hop Packet Forwarding (BMPF) Strategy: In the multi-node multi-hop packet forwarding game, given the discount factor δ_G and $p = (p_e, p_f)$, the sender and relay nodes act as follows during different phases of routing process.

- Game partition and belief initialization: Partition the original game into N sub-games according to (16). Then, each node initializes its belief of other nodes as $\pi(\delta_G^N, p)$ and forwards packets with probability $\pi(\delta_G^N, p)$.
- Route participation: The selected relay node on each route participates in the routing if and only if its beliefs of the sender and other forwarding nodes are greater than π.
- Route selection: The sender selects the route with the largest $\mu_i = \Pi_{j \in R_i} \mu_{ij}$ with $\mu_{ij} > \pi$ from the route candidates.
- Packet forwarding: The sender updates its belief of each relay node's continuation strategy using (10)-(13) and decides the following actions based on its belief.

In the above strategy, the belief value of each node plays an important role. The nodes who intentionally drop packets will be gradually isolated by other nodes since the nodes who have low belief value of the misbehaved nodes will not cooperate with them or participate in the possible routes involving these nodes. As each node is selfish and trying to maximize its own payoff, all nodes are inclined to follow the above strategy and obtain the optimal payoff. In order to formally show the cooperation enforcement, we have the following theorem. Note that the proposed strategy simplifies the complicated multi-player packet-forwarding game by considering multiple two-player packet-forwarding games between the sender and relay nodes. But, the equivalent two-player gain g here is different from that in Table I, which needs to further cope with the error propagation and routing diversity depending on the routing statistics such as $q_r(r)$ and $q_\hbar(\hbar)$.

Theorem 4: The forwarding strategy and belief system specified by the BMPF Strategy lead to a sequential equilibrium for the multi-player packet forwarding game.

Proof: A sequential equilibrium for the game with imperfect information is not only sequential rational but also consistent. First, we prove the sequential rationality of the proposed strategy using the one-step deviation property [16], which indicates that (σ, μ) is sequentially rational if and only if no player i has a history at which a change in $\sigma_i(h_i)$ increases his expected payoff.

In route participation stage, we assume each forwarding node $j \in R_i$ has built up a belief value of the sender i as μ_{ji} and the belief values of any other relay node $k \in R_i$. One-step deviation property is considered for the following three subcases for any forwarding node j: First, if $\mu_{ji} > \pi$ and $\mu_{jk} > \pi, k \neq j$, a one-step deviation is not to participate

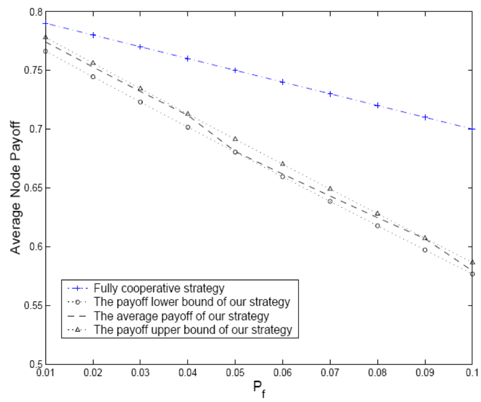

Fig. 3: The average payoffs of the cooperative strategy and proposed strategy.

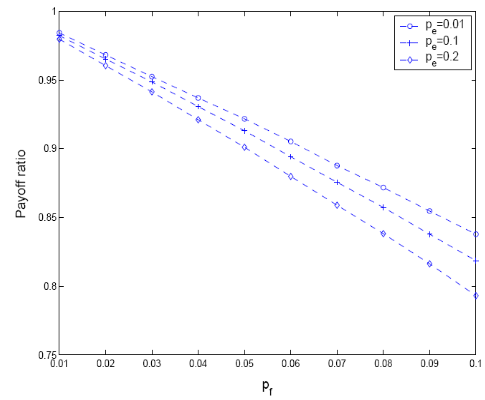

Fig. 4: Payoff ratios of the proposed strategy to the cooperative strategy.

in the routing. In this case, the forwarding node will miss the opportunity of cooperating with the sender, which has been shown to be profitable for the forwarding node in (8). Second, if $\mu_{ji} < \pi$ and $\mu_{jk} > \pi, k \neq j$, a one-step deviation is to participate in the routing. Since the relay node j will drop the packet from the sender i, the equivalent cooperation gain g in Table I will decrease due to packet-drop of the participated nodes, which also decreases the future gain of node j. Although node j does not afford the cost to forward packets for node i, its future gain will be damaged due to a smaller g. Thus, one-step deviation is not profitable in this subcase. Third, if $\mu_{ji} < \pi$ and there exists node k such that $\mu_{jk} < \pi$ the noncooperation forwarding behavior may happen since node j's belief of node k is lower than the threshold π. Such possible noncooperation outcome may decrease the expected equivalent gain g, which results in future payoff loss as (14) shows. Therefore, in all of the above three subcases of the route participation stage, one-step deviation from the BMPF Strategy cannot increase the payoffs of the nodes.

In route selection stage, two subcases need to be considered for one-step deviation test. First, if the largest μ_i with $\mu_{ij} < \pi, \exists j$ is selected as the forwarding route, the noncooperation interaction between the sender i and relay j exists, which decreases the expected equivalent gain g and then lower the future payoffs. Second, if not the route with largest μ_i is selected, the expected gain g can still be increased by another route with larger successful forwarding probability. Thus, one-step deviation is not profitable in the route selection stage.

Further, Theorem 1 can be directly applied here to prove the sequential rationality for every packet-forwarding stage. To sum up, the BMPF Strategy is sequential rational for the multi-node multi-hop packet-forwarding game. Besides, Following the definition of the consistency for sequential equilibria [16], it is straightforward to prove it for our BMPF Strategy. Therefore, the proposed multi-player packet-forwarding strategy is a sequential equilibrium. ■

Since the above theorem has proved that the BMPF Strategy is a sequential equilibrium, the cooperation among the nodes can be enforced and no selfish node will deviate from the equilibrium. As all nodes follow the proposed strategy, the expected gain g in Table I can be written as follows.

$$g = \tilde{g} \cdot E_{r,\hbar}[1 - [1 - (\pi(1 - p_e))^{\hbar}]^r] - E(\hbar) \cdot \pi\ell, \qquad (20)$$

where $E(\hbar)$ is the expected number of hops and $E_{r,\hbar}$ represents the expectation with respect to the random variables r and $\hbar$. The first term on the right hand side (RHS) of (20) is the expected gain of the sender considering multiple hops and possible routes; the second term on the RHS is the expected forwarding cost of sender i for returning the forwarding favor of the other relay nodes on its route. Note that π in (20) is also affected by g as shown in (17), which makes the computation of g more complicated. However, as we show in Theorem 3, the optimal π approaches ϕ when δ approaches $\underline{\delta}$. Considering the situations when δ_G approaches 1, π can be very close to ϕ as $\underline{\delta}$ is approached. Then, we can approximate g by substituting π with ϕ in (20), which is only determined by p_f and p_e.

V. SIMULATION

In this section, we investigate the cooperation enforcement results of our proposed belief-based packet forwarding approach by simulation.

We first focus our simulation studies on one-hop packet forwarding scenarios in ad hoc networks, where the two-player belief-based packet forwarding approach can be directly applied to. Let $M = 100$, $g = 1$ and $\ell = 0.2$ in our simulation. In each time slot, any one of the nodes is picked with equal probability as the relay node for the sender. For comparison, we define the cooperative strategy, in which we assume every node will unconditionally forward packets with no regard to other nodes' past behaviors. Such cooperative strategy is not implementable in autonomous ad hoc networks. But it can serve as a loose performance upper bound of the proposed strategy to measure the performance loss due to noise and imperfect observation.

Figure 3 shows the average payoff and performance bounds of the proposed belief-based strategy for different p_f by comparing them with the cooperative payoff. Note that $p_e = 0.01$ and $\delta_G = 0.99$. It can be seen from Figure 3 that our proposed approach can enforce cooperation with only small

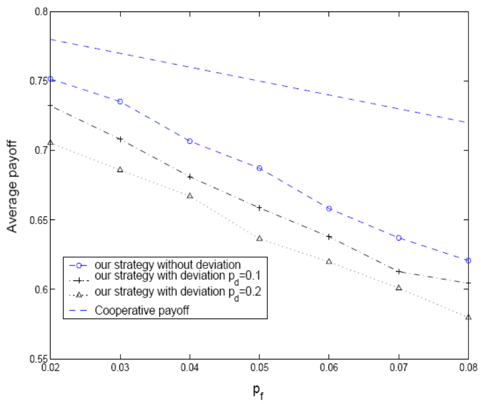

Fig. 5: Payoff comparison of the proposed strategy and deviating strategies.

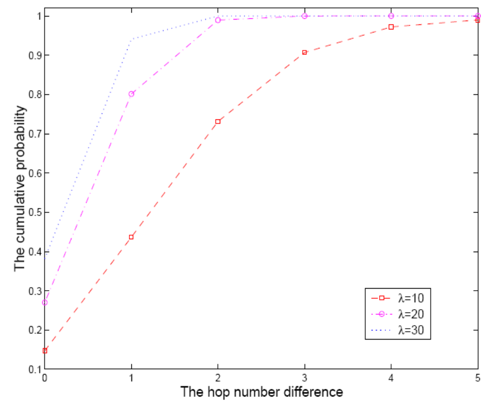

Fig. 6: The cumulative probability mass function of the hop-number difference between the $\tilde{\hbar}(n_i, n_j)$ and $h_{\min}(n_i, n_j)$.

performance loss compared to the unconditionally cooperative payoff. Further, this figure shows that the average payoff of our proposed strategy satisfies the theoretical payoff bounds developed in Theorem 3. The fluctuation of the payoff curve of our strategy is because only integer number of sub-games can be partitioned into from the original game. Figure 4 shows the ratio of the payoffs of our strategy to those of the cooperative strategy for different p_e and p_f . Here we let $\delta_G = 0.999$ to approach the payoff upper bound. It can be seen from Figure 4 that even if p_f is as large as 0.1 due to link breakage or transmission error, our cooperation enforcement strategy can still achieve as high as 80% of the cooperative payoff.

In order to show that the proposed strategy is cheat-proof among selfish users, we define the deviation strategies for comparison. The deviation strategies differ from the proposed strategy only when the continuation strategy σ_F and observation F are reached. The deviation strategies will play σ_D with deviating probability p_d instead of playing σ_F as the proposed belief-based strategy specifies. Figure 5 compares the nodes' average payoffs of the proposed strategy, cooperative strategy and deviation strategies with different deviating probabilities. Note that $\delta_G = 0.999$ and $pe = 0.1$. This figure shows that the proposed strategy has much better payoffs than the deviating strategies.

Then, we study the performance of the proposed multi-hop multi-node packet forwarding approach. Before evaluating the performance of our proposed strategy, we first need to obtain the routing statistics such as $q_r(r)$ and $q_{\hbar}(\hbar)$. An autonomous ad hoc network is simulated with $\mathcal{M}$ nodes randomly deployed inside a rectangular region of $10\gamma \times 10\gamma$ according to the 2-dimension uniform distribution. The maximal transmission range $\gamma = 100$m for each node, and each node moves according to the random waypoint model [22]. Dynamic Source Routing (DSR) [22] is used as the underlying routing to discover possible routes. Let $\lambda = \mathcal{M}\pi/100$ denote the normalized node density, i.e., the average number of neighbors for each node in the network. Note that each source-destination pair is formed by randomly picking two nodes in the network. Moreover, multiple routes with different number of hops may exist for each source-destination pair. Since the routes with the minimum number of hops achieve the lowest costs, without loss of generality, we only consider the minimum-hop routes as the routing candidates.

In order to study the routing statistics, we first conduct simulations to study the hop number on the minimum-hop route for source-destination pairs. Let $h_{\min}(n_i, n_j) = \lceil \text{dist}(n_i, n_j)/\gamma \rceil$ denote the ideal minimum number of hops needed to traverse from node i to node j, where $dist(n_i, n_j)$ denotes the physical distance between node i and j, and let $\tilde{\hbar}(n_i, n_j)$ denote the number of hops on the actual minimum-hop route between the two nodes. Note that we simulate 10^6 samples of topologies to study the dynamics of the routing in ad hoc networks. Firstly, Figure 6 shows the approximated cumulative probability mass function (CMF) of the difference between the $\tilde{\hbar}(n_i, n_j)$ and $h_{\min}(n_i, n_j)$ for different node densities. Based on these results, the average number of hops associated to the minimum-hop route from node i to j can be approximated using the $dist(n_i, n_j)$, γ, and the corresponding CMF of hop difference, which also gives the statistics of $q_{\hbar}(\hbar)$. Besides, it can be seen from Figure 6 that lower node density results in having a larger number of hops for the minimum-hop routes, since the neighbor nodes are limited for packet forwarding in such situations. Secondly, we study the path diversity of the ad hoc networks by finding the maximum number of minimum-hop routes for the source-destination pair. Note that there may exist the scenarios where the node may be on multiple minimum-hop forwarding routes for the same source-destination pair. For simplicity, we assume during the route discovery phase, the destination randomly picks one of such routes as the routing candidate and feedbacks the routing information of all node-disjoint minimum-hop routes to the source. Figure 7 shows the CMF of the number of the least-hop routes for different hop number when the node density is 30. This figure actually shows the $q_r(r)$ statistics when the ideal minimum hop number is given. Based on the routing statistics given in Figure 6 and 7, we are able to obtain the expected equivalent two-player payoff table for multi-node and multi-hop packet forwarding scenarios using (20).

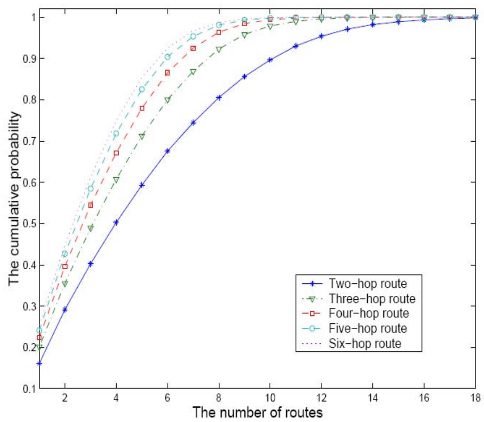

Fig. 7: The cumulative probability mass function of the number the minimum-hop route when the node density is 30.

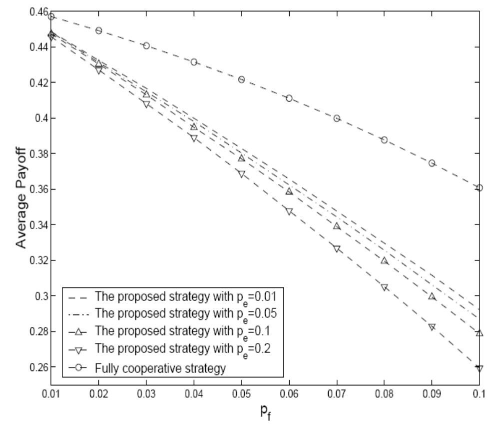

Fig. 8: Average payoffs of the proposed strategy in multi-node multi-hop scenarios.

We compare the payoff of our approach with that of the cooperative one in Figure 8. Note that multi-hop forwarding will incur more costs to the nodes since one successful packet delivery involves the packet forwarding efforts of many relay nodes. Also, the noise and imperfect observation will have more impact on the performance as each node's incorrect observation will affect the payoffs of all other nodes on the selected route. We can see from Figure 8 that our proposed strategy maintains high payoffs even when the environment is noisy and the observation error is large. For instance, when $p_e = 0.2$ and $p_f = 0.1$, our proposed strategy still achieves over 70% payoffs of the unconditionally cooperative payoff.

VI. Conclusion

In this paper, we study the cooperation enforcement in autonomous ad hoc networks under noise and imperfect observation. By modeling the packet forwarding, as a repeated game with imperfect information, we develop the belief-based packet forwarding framework to enforce cooperation in the scenarios with noise and imperfect observation. We show that the behaviorial strategy with well-defined belief system from our proposed approach not only achieves the sequential equilibrium, but also maintains high payoffs for both two-player and multi-player cases. Notice that only each node's action history and imperfect private observation are required for the proposed strategy. The simulation results illustrate that the proposed belief-based packet forwarding approach achieves stable and near-optimal equilibria in ad hoc networks under noise and imperfect observation.

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