

Dynamic Pricing Approach for Spectrum Allocation in Wireless Networks with Selfish Users

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Abstract—Dynamic spectrum allocation becomes a promising approach to increase the spectrum efficiency for wireless networks. In this paper, we consider the spectrum allocation in wireless networks with multiple selfish legacy spectrum holders and unlicensed users as multi-stage dynamic games. A dynamic pricing approach is proposed to optimize overall spectrum efficiency while keeping the participating incentives of the users based on double-auction rules. Moreover, a belief system is developed to assist selfish users to dynamically update their strategies adaptive to the network dynamics and substantially decrease the pricing overhead. The simulation results show that our proposed scheme not only approaches optimal outcomes but also has low overhead.

I. INTRODUCTION

Current static spectrum allocation can be very inefficient considering the bandwidth demands may vary highly along the time dimension or the space dimension. With the development of cognitive radio technologies, dynamic spectrum access becomes a promising approach to increase the efficiency of spectrum usage, which allows unlicensed wireless users to dynamically access the licensed bands from legacy spectrum holders based on leasing agreements.

The FCC began to consider more flexible and comprehensive use of available spectrum in [1], [2]. Then, great attentions have been drawn to explore the open spectrum systems [3], [4] for dynamic spectrum sharing. Traditionally, network-wide spectrum assignment is carried out by a central server, namely, spectrum broker [5], [6]. Recently, distributed spectrum allocation approaches [7], [8] have been well studied to enable efficient spectrum sharing only based on local observations.

Although the existing dynamic spectrum access schemes have achieved some success on enhancing the spectrum efficiency and distributive design, most of them focus on efficient spectrum allocation given fixed topologies and cannot adapt to the dynamics of wireless networks due to node mobility, channel variations or varying wireless traffic. Furthermore, existing cognitive spectrum sharing approaches generally assume that the network users will act cooperatively to maximize the overall system performance. However, with the emerging applications of mobile ad hoc networks envisioned in civilian usage, the users may be selfish and aim to maximize their own interests. Therefore, novel spectrum allocation approaches need to be developed considering the dynamic nature of wireless networks and users' selfish behaviors.

Considering a general network scenario in which multiple primary users (legacy spectrum holders) and secondary users (unlicensed users) coexist, primary users attempt to sell unused

spectrum resources to secondary users for monetary gains while secondary users try to acquire spectrum usage permissions from primary users to achieve certain communication goals, which generally introduces reward payoffs for them. In order to solve the above issues, we consider the spectrum sharing as multistage dynamic games and propose a belief-assisted dynamic pricing approach to optimize the overall spectrum efficiency, meanwhile, keeping the participating incentives of the users based on double-auction rules. The simulation results show that our proposed scheme not only approaches optimal spectrum efficiency but also has low pricing overhead compared to general continuous double auction mechanisms.

The reminder of this paper is organized as follows: The system model of dynamic spectrum allocation is described in Section II. In Section III, we formulate the spectrum allocation as pricing games based on the system model. In Section IV, the belief-based dynamic pricing approach is proposed for the optimal spectrum allocation. The simulation studies are provided in Section V. Finally, Section VI concludes this paper.

II. SYSTEM MODEL

We consider the wireless networks where multiple primary users and secondary users operate simultaneously, which may represent various network scenarios. For instance, the primary users can be the spectrum broker connected to the core network and the secondary users are the base stations equipped with cognitive radio technologies; or the primary users are the access points of a mesh network and the secondary users are the mobile devices. On one hand, considering that the authorized spectrum of primary users may not be fully utilized over time, they prefer to lease the unused channels to the secondary users for monetary gains. On the other hand, since the unlicensed spectrums become more and more crowded, the secondary users may try to lease some unused channels from primary users for more communication gains by providing leasing payments.

In our system model, we assume all users are selfish and rational, that is, their objectives are to maximize their own payoffs, not to cause damage to other users. However, users are allowed to cheat whenever they believe cheating behaviors can help them to increase their payoffs. Generally speaking, in order to acquire the spectrum licenses from regulatory bodies such as FCC, the primary users have certain operating costs. In order to have the reward payoffs, secondary users want to utilize more spectrum resources. The selfishness of both primary and secondary users will prevent them from

revealing their private information such as acquisition costs or reward payoffs, which makes traditional spectrum allocation approaches not applicable.

Specifically, we consider the collection of the available spectrums from all primary users as a spectrum pool, which totally consists of N non-overlapping channels. Assume there are J primary users and K secondary users, indicated by the set $\mathbf{P} = \{p_1, p_2, \dots, p_J\}$ and $\mathbf{S} = \{s_1, s_2, \dots, s_K\}$, respectively. We represent the channels authorized to primary user p_i using a vector $\mathbf{A}_i = \{a_i^j\}_{j \in \{1, 2, \dots, n_i\}}$, where a_i^j represents the channel index in the spectrum pool and n_i is the total number of channels which belong to user p_i . Define \mathbf{A} as the set of all the channels in the spectrum pool. Moreover, denote the acquisition costs of user p_i 's channels as the vector $\mathbf{C}_i = \{c_i^j\}_{j \in \{1, 2, \dots, n_i\}}$, where the j th element represents the acquisition cost of the j th channel in \mathbf{A}_i . For simplicity, we write c_i^j as c_j^i . As for secondary user s_i , we define her/his payoff vector as $\mathbf{V}_i = \{v_i^j\}_{j \in \{1, 2, \dots, N\}}$, where the j th element is the reward payoff if this user successfully leases the j th channel in the spectrum pool.

III. PRICING GAME MODEL

In this paper, we model the dynamic spectrum allocation problem as a pricing game to study the interactions among the players, i.e., the primary and secondary users. Based on the discussion in the previous section, we are able to have the payoff functions of the players in our dynamic game. Specifically, if primary user p_i reaches agreements of leasing all or part of her/his channels to secondary users, the payoff function of this primary user can be written as follows.

$$U_{p_i}(\phi_{\mathbf{A}_i}, \alpha_i^{\mathbf{A}_i}) = \sum_{j=1}^{n_i} (\phi_{a_i^j} - c_j^i) \alpha_i^{a_i^j}, \quad (1)$$

where $\phi_{\mathbf{A}_i} = \{\phi_{a_i^j}\}_{j \in \{1, 2, \dots, n_i\}}$ and $\phi_{a_i^j}$ is the payment that user p_i obtains from the secondary user by leasing the channel a_i^j in the spectrum pool. Note that $\alpha_i^{\mathbf{A}_i} = \{\alpha_i^{a_i^j}\}_{j \in \{1, 2, \dots, n_i\}}$ and $\alpha_i^{a_i^j} \in \{0, 1\}$ which indicates if the j th channel of user p_i has been allocated to a secondary user or not. For simplicity, we denote $\alpha_i^{a_i^j}$ as α_j^i . Similarly, the payoff function of secondary user s_i can be modeled as follows.

$$U_{s_i}(\phi_{\mathbf{A}}, \beta_i^{\mathbf{A}}) = \sum_{j=1}^N (v_i^j - \phi_j) \beta_j^i, \quad (2)$$

where $\phi_{\mathbf{A}} = \{\phi_j\}_{j \in \{1, 2, \dots, N\}}$, $\beta_i^{\mathbf{A}} = \{\beta_j^i\}_{j \in \{1, 2, \dots, N\}}$. Note that $\beta_j^i \in \{0, 1\}$ illustrates if secondary user s_i successfully leases the j th channel in the spectrum pool or not. Hence, the strategies of the primary users and secondary users are actually defined by $\alpha_i^{\mathbf{A}_i}$ and $\beta_i^{\mathbf{A}}$, respectively.

From the above discussion, we can see that the players may have conflict interests with each other. Specifically, the primary users want to earn as much payments as possible by leasing the unused channels and the secondary users aim to accomplish their communication goals by providing the

least possible payments for leasing the channels. Moreover, the spectrum allocation involves multiple channels over time. Therefore, the spectrum users involved in the spectrum allocation process construct a multistage non-cooperative pricing game [9], [10]. Also, the selfish users will not reveal their private information to others unless some mechanisms have been applied to guarantee that it is not harmful to disclose the private information. Generally, such non-cooperative game with incomplete information is difficult to study as the players do not know the perfect strategy profile of others. However, based on our game setting, the well-developed auction theory [11] can be applied to formulate and analyze our pricing game.

In auction games [11], according to an explicit set of rules, the principles (auctioneers) determine resource allocation and prices on the basis of bids from the agents (bidders). In our spectrum allocation pricing game, the primary users (principles) attempt to sell the unused channels to the secondary users and the secondary users (bidders) compete with each other to buy the permission of using primary users' channels. Moreover, multiple primary and secondary users coexist, which indicates the double auction scenario [11], [12]. It means that not only the secondary users but also the primary users need to compete with each other to make the beneficial transactions possible by eliciting their willingness of the payments in the forms of bids or asks. Generally, the double auction mechanism is highly efficient such as in the New York Stock Exchange (NYSE) or Chicago Merchandize Exchange (CME) and can respond dynamically to changing conditions of auction participants. However, in our spectrum allocation games, either powerful centralized authorities can be pre-assumed or the bandwidth of control channels is very limited. Therefore, we aim to develop an efficient pricing approach for spectrum allocation, which adapts to spectrum dynamics by simple message exchanges.

IV. DYNAMIC PRICING FOR EFFICIENT SPECTRUM ALLOCATION

A. Static Pricing Game and Competitive Equilibrium

Assume that the available channels from the primary users are leased for usage of certain time period T . Also, we assume that the cost of the primary users and reward payoffs of the secondary users remain unchanged over this period. Before this spectrum sharing period, we define a trading period τ , within which the users exchange their information of bids and asks to achieve agreements of spectrum usage. The time period $T + \tau$ is considered as one stage in our pricing game. We first study the interactions of the players in static pricing games. Note that the users' goals are to maximize their own payoff functions. As for the primary users, the optimization problem can be written as follows.

$$O(p_i) = \max_{\phi_{\mathbf{A}_i}, \alpha_i^{\mathbf{A}_i}} U_{p_i}(\phi_{\mathbf{A}_i}, \alpha_i^{\mathbf{A}_i}), \quad \forall i \in \{1, 2, \dots, J\} \quad (3)$$

$$\text{s.t. } U_{\hat{s}_{a_i^j}}(\{\phi_{-a_i^j}, \phi_{a_i^j}\}, \beta_i^{\mathbf{A}}) \geq U_{\hat{s}_{a_i^j}}(\{\phi_{-a_i^j}, \tilde{\phi}_{a_i^j}\}, \beta_i^{\mathbf{A}}), \\ \hat{s}_{a_i^j} \neq 0, \alpha_i^j \in \mathbf{A}_i. \quad (4)$$

where $\tilde{\phi}_{\alpha_i^j}$ is any feasible payment and $\phi_{-\alpha_i^j}$ is the payment vector excluding the element of the payment for the channel α_i^j . Note that $\hat{s}_{\alpha_i^j}$ is defined as follows.

$$\hat{s}_{\alpha_i^j} = \begin{cases} s_k & \text{if } \beta_k^{\alpha_i^j} = 1, \\ 0 & \text{if } \beta_k^{\alpha_i^j} = 0, \forall k \in \{1, 2, \dots, K\}. \end{cases} \quad (5)$$

Thus, (4) is the incentive compatible constraint [11]. It means that the secondary users have incentives to provide the optimal payment because they cannot have extra gains by cheating on the primary users. Similarly, the optimization problem can be written for the secondary users as follows.

$$O(s_i) = \max_{\phi_{\mathbf{A}}, \beta_i^{\mathbf{A}}} U_{s_i}(\phi_{\mathbf{A}}, \beta_i^{\mathbf{A}}), \quad \forall i \in \{1, 2, \dots, K\} \quad (6)$$

$$\text{s.t. } U_{\hat{p}_j}(\{\phi_{-j}, \phi_j\}, \beta_i^{\mathbf{A}}) \geq U_{\hat{p}_j}(\{\phi_{-j}, \tilde{\phi}_j\}, \beta_i^{\mathbf{A}}), \\ \hat{p}_j \neq 0, \beta_i^j = 1. \quad (7)$$

where \hat{p}_j is defined as

$$\hat{p}_j = \begin{cases} p_k & \text{if } \beta_i^j = 1, j \in \mathbf{A}_k, \alpha_k^j = 1 \\ 0 & \text{otherwise, } \forall k \in \{1, 2, \dots, J\}. \end{cases} \quad (8)$$

Similarly, (7) is the incentive compatible constraint for the primary users, which guarantees that the primary user will give the usage permission of their channels to the secondary users so that they can receive the optimal payments.

From (3) and (6), we can see that in order to obtain the optimal allocation and payments, a multi-objective optimization problem needs to be solved, which becomes extremely complicated due to our game setting that only involves incomplete information. Thus, in order to make this problem tangible, we analyze it from the game theory point of view. Considering the double auction scenarios of our pricing game, **Competitive Equilibrium** (CE) [12] is a well-known theoretical prediction of the outcomes. It is the price at which the number of buyers willing to buy is equal to the number of sellers willing to sell. Alternatively, CE can also be interpreted as where the supply and demand match [11]. We describe the supply and demand functions of spectrum resources in Figure 1. Note that CE is also proved to be Pareto optimal in stationary double auction scenarios [13].

B. Belief-Assisted Dynamic Pricing

Considering spectrum dynamics due to mobility, channel variations or wireless traffic variations, the secondary users' reward payoffs and primary users' costs may change over time or spectrum. Thus, c_i^j and v_i^j need to be considered as random variables in dynamic scenarios. Without loss of generality, we assume the homogeneous game settings for the statistics of c_i^j and v_i^j , which satisfy the probability density functions (PDF) $f_c(c)$ and $f_v(v)$, respectively. Therefore, considering dynamic network conditions, we further model the spectrum sharing as a multi-stage dynamic pricing game. Let γ be the discount factor of our multi-stage pricing game. Based on (3) and (6),

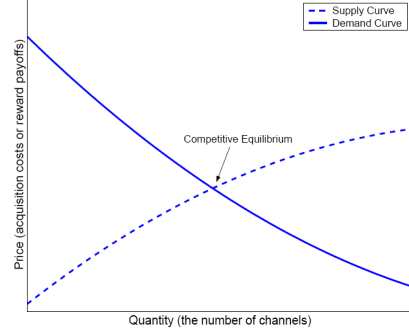


Fig. 1: Illustration of supply and demand functions.

the objective functions for the primary users and secondary users can be rewritten as follows.

$$\tilde{O}(p_i) = \max_{\phi_{\mathbf{A},t}, \alpha_{i,t}^{\mathbf{A}}} E_{c_i^j, v_i^j} \left[\sum_{t=1}^{\infty} \gamma^t \cdot U_{p_i,t}(\phi_{\mathbf{A},t}, \alpha_{i,t}^{\mathbf{A}}) \right], \quad (9)$$

$$\tilde{O}(s_i) = \max_{\phi_{\mathbf{A},t}, \beta_{i,t}^{\mathbf{A}}} E_{c_i^j, v_i^j} \left[\sum_{t=1}^{\infty} \gamma^t \cdot U_{s_i,t}(\phi_{\mathbf{A},t}, \beta_{i,t}^{\mathbf{A}}) \right], \quad (10)$$

where the subscript t indicates the t th stage of the multi-stage game. Generally speaking, there may exist some overall constraints of spectrum sharing such as each secondary user's total budget for leasing spectrum resources or each primary user's total available spectrum supply. Under these constraints, the above problem needs to be further modeled as a dynamic programming process [14], [15] to obtain optimal sequential strategies. However, the major difficulty of dynamic spectrum sharing lies in that how to efficiently and dynamically update the spectrum sharing strategies according to the changing network conditions only based on local information. Therefore, in this paper we don't assume the overall constraints and focus on developing a belief-assisted dynamic pricing approach, which can not only approach CE outcomes but also respond dynamically to networking dynamics while only introducing limited overhead.

Since our pricing game belongs to the non-cooperation games with incomplete information [9], the players need to build up certain beliefs of other players' future possible strategies to assist their decision making. Considering that there are multiple players with private information in the pricing game and what directly affect the outcome of the game are the bid/ask prices, it is more efficient to define one common belief function based on the publicly observed bid/ask prices than generating specific belief of every other player's private information. Hence, enlightened by [12], we consider the primary/secondary users' beliefs as the ratio their bid/ask being accepted at different price levels. At each time during the dynamic spectrum sharing, the ratio of asks from primary users at x that have been accepted can be written as follows.

$$\tilde{r}_p(x) = \frac{\mu_A(x)}{\mu(x)}, \quad (11)$$

where $\mu(x)$ and $\mu_A(x)$ are the number of asks at x and the number of accepted asks at x , respectively. Similarly, at each time during the dynamic spectrum sharing, the ratio of bids from secondary users at y that have been accepted is

$$\tilde{r}_s(y) = \frac{\eta_A(y)}{\eta(y)}, \quad (12)$$

where $\eta(y)$ and $\eta_A(y)$ are the number of bids at y and the number of accepted bids at y , respectively. Usually, $\tilde{r}_p(x)$ and $\tilde{r}_s(y)$ can be accurately estimated if a great number of buyers and sellers are participating in the pricing at the same time. However, in our pricing game, only a relatively small number of players are involved in the spectrum sharing at the specific time. The beliefs, namely, $\tilde{r}_p(x)$ and $\tilde{r}_s(y)$ cannot be practically obtained so that we need to further consider using the historical bid/ask information to build up empirical belief values. Considering the characteristics of double auction, we have the following observations: if an ask $\tilde{x} < x$ is rejected, the ask at x will also be rejected; if an ask $\tilde{x} > x$ is accepted, the ask at x will also be accepted; if a bid $\tilde{y} > x$ is made, the ask at x will also be accepted.

Based on the above observations, the players' beliefs can be further defined as follows using the past bid/ask information.

Definition 1: Primary users' beliefs: for each potential ask at x , define

$$\hat{r}_p(x) = \begin{cases} 1 & x = 0 \\ \frac{\sum_{w \geq x} \mu_A(w) + \sum_{w \geq x} \eta(w)}{\sum_{w \geq x} \mu_A(w) + \sum_{w \geq x} \eta(w) + \sum_{w < x} \mu_R(w)} & x \in (0, M) \\ 0 & x \geq M \end{cases} \quad (13)$$

where $\mu_R(w)$ is the number of asks at w that has been rejected, M is a large enough value so that the asks greater than M won't be accepted. Also, it is intuitive that the ask at 0 will be definitely accepted as no cost is introduced.

Definition 2: Secondary users' beliefs: for each potential bid at y , define

$$\hat{r}_s(x) = \begin{cases} 0 & y = 0 \\ \frac{\sum_{w \leq y} \eta_A(w) + \sum_{w \leq y} \mu(w)}{\sum_{w \leq y} \eta_A(w) + \sum_{w \leq y} \mu(w) + \sum_{w > y} \eta_R(w)} & y \in (0, M) \\ 1 & y \geq M \end{cases} \quad (14)$$

where $\eta_R(w)$ is the number of bids at w that has been rejected. And, it is intuitive that the bid at 0 will not be accepted by any primary users.

Noting that it is too costly to build up beliefs on every possible bid or ask price, we can update the beliefs only at some fixed prices and use interpolation to obtain the belief function over the price space. Moreover, only local information is needed for the users updating their beliefs, though public information may accelerate the belief-updating process.

Before using our defined belief functions to assist the strategy decisions, we first look at the Spread Reduction Rule (SRR) of double auction mechanisms. Generally, before the double auction pricing game converges to CE, there may exist a gap between the highest bid and lowest ask, which is called the spread of double auction. The SRR states that any ask that is permissible must be lower than current lowest ask, i.e., outstanding ask [12], and then either each new ask results

TABLE I: Belief-assisted dynamic spectrum allocation

1. Initialize the users' beliefs and bids/asks ◊ The primary users initialize their asks as large values close to M and their beliefs as small positive values less than 1; ◊ The secondary users initialize their bids as small values close to 0 and their beliefs as small positive values less than 1.
2. Belief update based on local information: Update primary and secondary users' beliefs using (13) and (14), respectively
3. Optimal bid/ask update: ◊ Obtain the optimal ask for each primary user by solving (16); ◊ Obtain the optimal bid for each secondary user by solving (17).
4. Update leasing agreement and spectrum pool: ◊ If the outstanding bid is greater than or equal to the outstanding ask, the leasing agreement will be signed between the corresponding users; ◊ Update the spectrum pool by removing the assigned channel.
5. Iteration: If the spectrum pool is not empty, go back to Step 2.

in an agreed transaction or it becomes the new outstanding ask. A similar argument can be applied to bids. By defining current outstanding ask and bid as ox and oy , respectively, we let $\bar{r}_p(x) = \hat{r}_p(x) \cdot I_{[0,ox]}(x)$ for each x and $\bar{r}_s(y) = \hat{r}_s(y) \cdot I_{[oy,M]}(y)$ for each y , which are modified belief function considering the SRR. Note that $I_{(a,b)}(x)$ is defined as

$$I_{(a,b)}(x) = \begin{cases} 1 & \text{if } x \in (a, b); \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

By using the belief function $\bar{r}_p(x)$, the payoff maximization of selling the i th primary user's j th channel can be written as

$$\max_{x \in (oy, ox)} E[U_{p_i}(x, j)], \quad (16)$$

where $U_{p_i}(x, j)$ represents the payoff introduced by allocating the j th channel when the ask is x , and then $E[U_{p_i}(x, j)] = (x - c_i^j) \cdot \bar{r}_p(x)$. Similarly, as for the secondary user s_i , the payoff maximization of leasing the j th channel in the spectrum pool can be written as

$$\max_{y \in (oy, ox)} E[U_{s_i}(y, j)], \quad (17)$$

where $U_{s_i}(y, j)$ represents the payoff introduced by leasing the j th channel in the spectrum pool when the bid is y , and then $E[U_{s_i}(y, j)] = (v_i^j - y) \cdot \bar{r}_s(y)$. Therefore, by solving the optimization problem for each primary and secondary user using (16) and (17), respectively, primary and secondary users can make the optimal decision of spectrum allocation at every stage conditional on dynamic spectrum demand and supply. Based on the above discussions, we illustrate our belief-assisted dynamic pricing algorithm for spectrum allocation in Table I.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed belief-assisted dynamic spectrum sharing approach in wireless networks. Considering a wireless network covering 100×100 area, we simulate J primary users by randomly placing them in the network. These primary users can be the base stations serving for different wireless network operators or different access points in a mesh network. Here we assume the primary users' locations are fixed and their unused channels

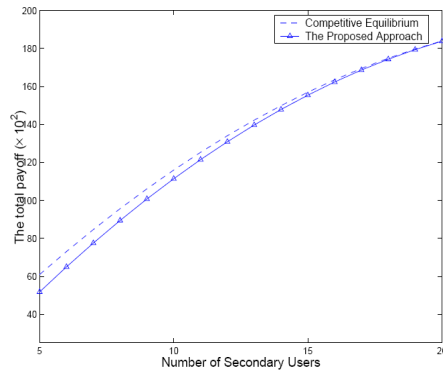


Fig. 2: Comparison of the total payoff for the proposed scheme and theoretical Competitive Equilibrium.

are available to the secondary users within the distance of 50. Then, we randomly deploy K secondary users in the network, which are assumed to be mobile devices. The mobility of the secondary users is modeled using a simplified random waypoint model [16], where we assume the “thinking time” at each waypoint is close to the effective duration of one channel-leasing agreement, the waypoints are uniformly distributed within the distance of 10, and the traveling time is much smaller than the “thinking time”. Let the cost of an available channel in the spectrum pool be uniformly distributed in $[10, 30]$, the reward payoff of leasing one channel be uniformly distributed in $[20, 40]$. If a channel is not available to some secondary users, let the corresponding reward payoffs of this channel be 0. Note that $J = 5$ and 10^3 pricing stages have been simulated. Let $n_i = 4$, $\forall i \in \{1, 2, \dots, J\}$ and $\gamma = 0.99$. In our simulation, the local bid/ask information within the transmission range of each node is used for belief construction and update.

In Figure 2, we compare the total payoff of all users of our proposed approach with that of the theoretical CE outcomes for different number of secondary users. It can be seen from this figure that the performance loss of our approach is very limited compared to that of the theoretical optimal solutions. Moreover, when the number of secondary users increases, our approach is able to approach the optimal CE. It is because that the belief function reflects the spectrum demand and supply more accurately when more users are involved in spectrum sharing.

Now we study the overhead of our pricing approach. Here we measure the pricing overhead by showing the average number of bids and asks for each stage. In Figure 3, the overhead of our pricing approach is compared to that of the traditional continuous double auction when the same total payoff is achieved. Assume the minimal bid/ask step δ of the continuous double auction to be 0.01. It can be seen from the figure that our approach substantially decreases the pricing communication overhead. Note that when decreasing the overhead, our proposed approach may introduce extra complexity to update the beliefs.

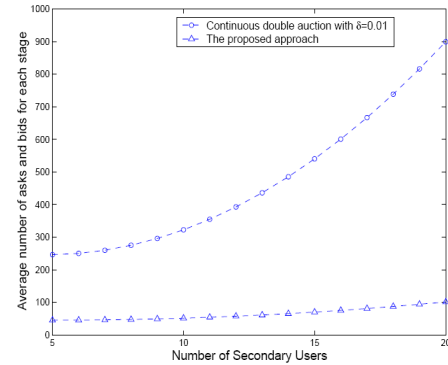


Fig. 3: Comparison of the overhead between the proposed scheme and continuous double auction scheme.

VI. CONCLUSIONS

In this paper, we have studied dynamic pricing for efficient spectrum allocation in wireless networks with selfish users. We model the dynamic spectrum allocation as a multi-stage game and propose a belief-assisted dynamic pricing approach to maximize the users’ payoffs while providing them the participating incentives via double auction rules. Simulation results show that the proposed scheme can approach the optimal spectrum efficiency by only using limited pricing overhead.

REFERENCES

- [1] FCC, “Spectrum policy task force report,” *FCC Document ET Docket No. 02-135*, November 2002.
- [2] FCC, “Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies: notice of proposed rule making and order,” *FCC Document ET Docket No. 03-108*, December 2003.
- [3] R. J. Berger, “Open spectrum: a path to ubiquitous connectivity,” *FCC ACM Queue* 1, 3, May 2003.
- [4] J. M. Peha, “Approaches to spectrum sharing,” *IEEE Communications Magazine*, vol. 43, pp. 10–12, February 2005.
- [5] M. M. Buddhikot, “Dimsumnet: new directions in wireless networking using coordinated dynamic spectrum access,” in *Proc. of IEEE WoW-MoM’05*, 2005.
- [6] C. Peng, H. Zheng, and B. Y. Zhao, “Utilization and fairness in spectrum assignment for opportunistic spectrum access,” to appear in *Mobile Networks and Applications (MONET)*, 2006.
- [7] L. Cao and H. Zheng, “Distributed spectrum allocation via local bargaining,” in *Proc. of IEEE DySpan*, 2005.
- [8] R. Etkin, A. Parekh, and D. Tse, “Spectrum sharing for unlicensed bands,” in *Proc. of IEEE DySpan*, 2005.
- [9] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*, The MIT Press, Cambridge, Massachusetts, 1994.
- [10] D. Fudenberg and J. Tirole, *Game Theory*, The MIT Press, Cambridge, Massachusetts, 1991.
- [11] V. Krishna, *Auction Theory*, Academic Press, 2002.
- [12] S. Gjerstad and J. Dickhaut, “Price formation in double auctions,” *Games and Economic Behavior*, vol. 22, pp. 1–29, 1998.
- [13] L. Hurwicz, R. Radner, and S. Reiter, “A stochastic decentralized resource allocation process: Part i,” *Econometrica*, vol. 43, pp. 363–393, 1975.
- [14] D. Bertsekas, *Dynamic Programming and Optimal Control*, vol. 1,2, Athena Scientific, Belmont, MA, Second edition, 2001.
- [15] Z. Ji, W. Yu, and K. J. R. Liu, “An optimal dynamic pricing framework for autonomous mobile ad hoc networks,” in *Proc. of IEEE INFOCOM’06*, 2006.
- [16] D. B. Johnson and D. A. Maltz, “Dynamic source routing in ad hoc wireless networks, mobile computing,” *IEEE Transactions on Mobile Computing*, pp. 153–181, 2000.