Sequential Multi-Channel Access Game in Distributed Cognitive Radio Networks

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***Abstract*—In distributed cognitive radio networks, due to the negative network externality, rational secondary users tend to avoid accessing same vacant primary channels with others. Moreover, they usually make channel access decisions in a sequential manner to avoid collisions. The characteristic of negative network externality and the structure of sequential decision making make the multi-channel access problem challenging, which has not been well studied by existing literatures. In this paper, we propose a multi-channel access game, which not only considers the negative network externality, but also takes into account their sequential decision making structure. We design a recursive best response algorithm to find the subgame perfect Nash equilibria. Finally, we conduct simulations to validate the effectiveness and efficiency of the proposed methods.**

I. Introduction

Recently, cognitive radio technology is considered as an effective approach to mitigate the problem of crowded electromagnetic radio spectrum [1]. In a cognitive radio network, the unlicensed users, called as Secondary Users (SUs), can opportunistically utilize the spectrum resources of licensed users, called as Primary Users (PUs) under the constraint of without harmful interference to the PUs [2]. Spectrum access issue is one of the most important problem in cognitive radio networks. Lots of multi-channel spectrum access methods based on different mathematical models have been proposed, e.g., Markov decision process (MDP) based approaches [3]-[4] and game theoretic approaches [5]-[7]. Moreover, the joint spectrum sensing and channel access problems were studied in [8]-[10]. However, most of the existing works have focused on a simultaneous channel access scenario and the sequential channel access in distributed cognitive radio networks has not been well investigated.

When making channel access decision, each SU not only should consider the channel quality, but also take into account other SUs' channel access decisions since the more SUs access the same channel, the less throughput each SU can obtain. Such a phenomenon is known as negative network externality [11], i.e., the negative influence of other users' behaviors on one user's reward, due to which each user tends to avoid making the same decision with others to maximize his/her own utility. Moreover, in a fully distributed cognitive radio network, SUs usually need to make decision sequentially to avoid collision, which makes the multiple SUs' multi-channel access problem even more challenging. Although this is an important and practical issue, there is few work considering both negative network externality and sequentially decision making structure.

In our previous works [12]-[13], we proposed Chinese Restaurant Game to address the sequential decision making with negative network externality. However, the underlying assumption of Chinese Restaurant Game is each SU can only access one primary channel at each time slot [14]-[15], due to which it cannot be directly applied to the multi-channel access problem. To tackle the challenge, in this paper, we propose a multi-channel access game for distributed cognitive radio network by considering both negative network externality and SU's sequential decision making structure. A recursive best response algorithm is designed for SUs to distributely find the Nash equilibrium. We compare our algorithm with myopic, learning and random algorithms under with/ without resource constraint scenarios, where the simulation results show that the proposed best response algorithm perform the best.

The rest of this paper is organized as follows. Firstly, we introduce the proposed multi-channel access game formulation in Section II. Then, we analyze the game in Section III and show the simulation results in Section IV. Finally, the conclusions are drawn in Section V.

II. Multi-Channel Access Game Formulation

We consider a primary network with M independent primary channels denoted by $\{\mathrm{Ch}_1, \mathrm{Ch}_2, ..., \mathrm{Ch}_M\}$. The primary channel state is denoted as $\boldsymbol{\theta} = \{\theta_1, \theta_2, ..., \theta_M\}$, where $\theta_j \in \{\mathcal{H}_-, \mathcal{H}_+\}$ represents the state of channel Ch_j, $\mathcal{H}_-$ means the channel is idle and $\mathcal{H}_+$ means not idle. There are N SUs, labeled by $\{1, 2, ..., N\}$, each of which can simultaneously access multiple vacant primary channels during one time slot. All SUs are considered as rational users in the sense that each SU makes multi-channel access decision with the objective of maximizing his/her own expected reward. Here, we introduce the concept of belief to describe SUs' uncertainty about the current channel state, denoted by

$$\mathbf{p} = \big\{p_j = \mathrm{Po}(\theta_j = \mathcal{H}_-),\ j = 1, 2, .., M\big\}, \tag{1}$$

where p_j represents SUs' estimation of the probability that channel Ch_j is in idle state, which can be obtained by perform spectrum sensing.

All SUs make decisions on which channels to access to and broadcast their decisions to others via the control channel. In order to avoid the collision, we assume that SUs sequentially

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make and broadcast their decisions according to a certain predefined order. As we will see later, SUs' behaviors and utilities highly depend on the decision order. For the sake of fairness, we consider that the decisions order is randomized and thus different at different time slots. As rational users, SUs should take into account all possible factors to maximize their expected payoffs, including channel state, i.e., channel is vacant or occupied, as well as previous and subsequent SUs' decisions, i.e., the negative network externality. In this paper, we formulate this multi-channel access problem as a non-cooperative game and derive the best response for each SU. Let us denote the strategy of the i-th SU $\mathbf{d}_i = \big(d_{i,1}, d_{i,2}, ..., d_{i,M}\big)'$, where $d_{i,j} = 1$ represents that the i-th SU accesses channel Ch_j and otherwise we have $d_{i,j} = 0$. In such a case, the strategy profile of all SUs can be denoted by a $M \times N$ matrix as:

$$\mathbf{D} = (\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_N) = \begin{bmatrix} d_{1,1} & d_{2,1} & \cdots & d_{N,1} \\ d_{1,2} & d_{2,2} & \cdots & d_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ d_{1,M} & d_{2,M} & \cdots & d_{N,M} \end{bmatrix}. \quad (2)$$

Each SU can expect his/her transmission data rate by the belief of the channel state and the number of SUs who will share the same channel with him/her. Assuming SUs share the primary channel through TDMA, we can define the expected utility function of the i-th SU accessing channel Ch_j as

$$U_{i,j} = \frac{p_j g_{i,j}}{N_j} - c_j, \quad (3)$$

where p_j is SUs' belief of the state of channel Ch_j, $g_{i,j}$ is the i-th SU's gain of accessing channel Ch_j which depends on the channel gain, N_j is the total number of SUs sharing channel Ch_j and c_j is the cost of accessing channel Ch_j. From (3), we can see that the more accurate the belief, the better the expected utility SUs can obtain. Moreover, the utility function is a decreasing function in terms of N_j, which can be regarded as the characteristic of negative network externality since the more SUs access channel Ch_j, the lower utility each SU can obtain. Based on the utility function, we can define each SU's best response, which maximizes each SU's utility, as follows

$$\mathbf{d}_i^{(t)*} = \arg\max_{\mathbf{d}_i \in \{0,1\}^M} \sum_{j=1}^{M} d_{i,j} \cdot U_{i,j}. \quad (4)$$

Note that due to hardware limitation and/or power constraint, SUs may not be able to access all channels at one time slot, i.e., there may be a resource constraint. In this paper, we assume that each SU can at most simultaneously access L channels at each time slot. In such a case, SUs' decisions are subject to the following constraints:

$$\sum_{j=1}^{M} d_{i,j} \le L, \quad \forall\ i = 1, 2, ..., N, \quad (5)$$

III. Sequential Multi-Channel Access Game

In this section, we study the sequential multi-channel access game. According to the aforementioned game formulation, let us first define $\mathbf{n}_{-i} = \{n_{-i,1}, n_{-i,2}, ..., n_{-i,M}\}$ with

$$n_{-i,j} = \sum_{\zeta \ne i} d_{\zeta,j} \quad (6)$$

being the number of SUs accessing channel Ch_j except the i-th SU . Let $\mathbf{p} = \{p_1, p_2, ..., p_M\}$, where $p_j = \mathrm{P}(\theta_j = \mathcal{H}_0)$ is SUs' belief regarding the state of channel Ch_j. Given $\mathbf{p}$ and $\mathbf{n}_{-i}$, according to (4), the i-th SU's best channels access decision can be found by the following optimization problem.

$$\mathbf{d}_i^* = \mathrm{BR}_i(\mathbf{p}, \mathbf{n}_{-i}) = \arg\max_{\mathbf{d}_i \in \{0,1\}^M} \sum_{j=1}^{M} d_{i,j} \cdot U_{i,j}, \quad (7)$$
$$\text{s.t.} \sum_{j=1}^{M} d_{i,j} \le L < M,$$

where $U_{i,j}$ is given in (3). From (7), we can see that the i-th SU's decision on whether to access channel Ch_j is coupled with the decisions on all other channels, and thus (7) cannot be decomposed into M subproblems. Nevertheless, we can still find the best response of each SU by comparing all possible combinations of L channels.

Let $\mathbf{\Phi} = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_H\}$ denote the set of all combinations of l $(1 \le l \le L)$ channels out of M channels, where $H = \sum_{l=1}^{L} \mathrm{C}_M^l = \sum_{l=1}^{L} \frac{M!}{l!(M-l)!}$ and $\boldsymbol{\phi}_h = (\phi_{h,1}, \phi_{h,2}, ..., \phi_{h,M})'$ is one possible combination with $\phi_{h,j}$ representing whether channel Ch_j is selected to access to, e.g.,

$$\boldsymbol{\phi}_h = (\underbrace{1, 1, ..., 1}_{l}, \underbrace{0, 0, ..., 0}_{M-l})' \quad (8)$$

means the SU accesses channel $\mathrm{Ch}_1, \mathrm{Ch}_2, ..., \mathrm{Ch}_l$ $(1 \le l \le L)$. In other words, $\mathbf{\Phi}$ is the candidate strategy set of each SU with resource constraint L.

Let us define the i-th SU's observation of previous SUs' channel access decisions as

$$\mathbf{n}_i = \{n_{i,1}, n_{i,2}, ..., n_{i,M}\}, \quad (9)$$

where $n_{i,j} = \sum_{k=1}^{i-1} d_{k,j}$ is the number of SUs deciding to access channel Ch_j before the i-th SU. Let $\mathbf{m}_i$ denote the subsequent SUs' decisions after the i-th SU, we have its recursive form as

$$\mathbf{m}_i = \mathbf{m}_{i+1} + \mathbf{d}_{i+1}. \quad (10)$$

Then, let

$$\mathbf{m}_i|_{\mathbf{d}_i = \boldsymbol{\phi}_h} = \{m_{i,1}|_{\mathbf{d}_i = \boldsymbol{\phi}_h}, m_{i,2}|_{\mathbf{d}_i = \boldsymbol{\phi}_h}, ..., m_{i,M}|_{\mathbf{d}_i = \boldsymbol{\phi}_h}\}, \quad (11)$$

with $m_{i,j}|_{\mathbf{d}_i = \boldsymbol{\phi}_h}$ being the predicted number of subsequent SUs who will access channel Ch_j under the condition of $\mathbf{d}_i = \boldsymbol{\phi}_h$, where $\mathbf{d}_i = (d_{i,1}, d_{i,1}, ..., d_{i,M})'$ and $\boldsymbol{\phi}_h \in \mathbf{\Phi}$. In such a case, the predicted number of SUs accessing each primary channel excluding the i-th SU is

$$\hat{\mathbf{n}}_{-i}|_{\mathbf{d}_i = \boldsymbol{\phi}_h} = \mathbf{n}_i + \mathbf{m}_i|_{\mathbf{d}_i = \boldsymbol{\phi}_h}. \quad (12)$$

Algorithm 1 BR_MCA$(\mathbf{p}, \mathbf{n}_i, i)$

if SU $i == N$ **then**
  //********For the** N**-th SU********//
  for $j = 1$ to M **do**
    $U_{i,j} = \frac{p_j g_{i,j}}{n_{N,j}+1} - c_j$
  end for
  $\mathbf{j} = \{j_1, j_2, ..., j_L\} \leftarrow \arg\max^L_{j\in\{1,2,...,M\}} \{U_{i,j}\}$
  for $j = 1$ to M **do**
    if $(U_{i,j} > 0)\&\&(j \in \mathbf{j})$ **then**
      $d_{N,j} \leftarrow 1$
    else
      $d_{N,j} \leftarrow 0$
    end if
  end for
  $\mathbf{m}_N = \mathbf{0}$
else
  //********For the** $\{1, 2, ..., N-1\}$**-th SU********//
  //****Predicting****//
  for $\boldsymbol{\phi}_h = \boldsymbol{\phi}_1$ to $\boldsymbol{\phi}_H$ **do**
    $(\mathbf{d}_{i+1}, \mathbf{m}_{i+1}) \leftarrow$ BR_MCA$(\mathbf{p}, \mathbf{n}_i + \boldsymbol{\phi}_h, i+1)$
    $\mathbf{m}_i \leftarrow \mathbf{m}_{i+1} + \mathbf{d}_{i+1}$
    $U_i(\boldsymbol{\phi}_h) = \sum_{j=1}^{M} \frac{p_j g_{i,j}}{n_{i,j}+m_{i,j}+\phi_{h,j}} - c_j$
  end for
  //****Making decision****//
  $\boldsymbol{\phi}_h^* \leftarrow \arg\max_{\boldsymbol{\phi}_h\in\boldsymbol{\Phi}}\{U_i(\boldsymbol{\phi}_h)\}$
  $(\mathbf{d}_{i+1}, \mathbf{m}_{i+1}) \leftarrow$ BR_MCA$(\mathbf{p}, \mathbf{n}_i + \boldsymbol{\phi}_h^*, i+1)$
  $\mathbf{d}_i \leftarrow \boldsymbol{\phi}_h^*$
  $\mathbf{m}_i \leftarrow \mathbf{m}_{i+1} + \mathbf{d}_{i+1}$
end if
return $(\mathbf{d}_i, \mathbf{m}_i)$

According to above definitions, we can write the i-th SU's expected utility by accessing channel Ch_j when $\mathbf{d}_i = \boldsymbol{\phi}_h$ as

$$U_{i,j}|_{\mathbf{d}_i=\boldsymbol{\phi}_h} = \frac{p_j g_j}{n_{i,j} + m_{i,j}|_{\mathbf{d}_i=\boldsymbol{\phi}_h} + \phi_{h,j}} - c_j, \qquad (13)$$

Then, the total expected utility the i-th SU can obtain with $\mathbf{d}_i = \boldsymbol{\phi}_h$ is the sum of $U_{i,j}|_{\mathbf{d}_i=\boldsymbol{\phi}_h}$ over all M channels, i.e.,

$$U_i|_{\mathbf{d}_i=\boldsymbol{\phi}_h} = \sum_{j=1}^{M} U_{i,j}|_{\mathbf{d}_i=\boldsymbol{\phi}_h}. \qquad (14)$$

In such a case, we can find the optimal $\boldsymbol{\phi}_h^*$ which maximizes the i-th SU's expected utility $U_i|_{\mathbf{d}_i=\boldsymbol{\phi}_h}$ as follows

$$\boldsymbol{\phi}_h^* = \arg\max_{\boldsymbol{\phi}_h\in\boldsymbol{\Phi}} \{U_i|_{\mathbf{d}_i=\boldsymbol{\phi}_h}\}. \qquad (15)$$

To obtain the best response in (15), each SU needs to calculate the expected utilities defined in (13), which requires to predict $m_{i,j}|_{\mathbf{d}_i=\boldsymbol{\phi}_h}$, i.e., the number of SUs who access channel Ch_j after the i-th SU. In such a case, the i-th SU needs to predict the decisions of all SUs from $i+1$ to N. When it comes to the N-th SU, since he/she knows exactly the decisions of all the previous SUs, he/she can find the best response without making any prediction, i.e., $m_{N,j} = 0$. Based on such an intuition, given current belief $\mathbf{p} = \{p_1, p_2, ..., p_M\}$ and current observation $\mathbf{n}_i = \{n_{i,1}, n_{i,2}, ..., n_{i,M}\}$, we design a recursive best response algorithm BR_MCA$(\mathbf{p}, \mathbf{n}_i, i)$ for solving the multi-channel access game in Algorithm 1, where MCA means multi-channel access. As we can see, the N-th SU only needs to compare the expected utilities of accessing all M channels respectively and choose L or less than L channels with highest positive expected utilities. Note that $\max^L$ means finding the highest L values. For other SUs, each one needs to first recursively predict the subsequent SUs' decisions, and then make his/her own decision based on the prediction and current observations.

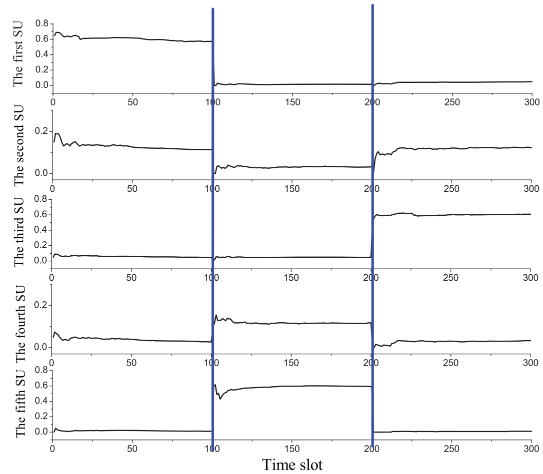

Fig. 1. Each SU's utility in homogenous case without resource constraint.

IV. Simulation Results

In this section, we conduct simulation to verify the performance of the proposed recursive best response algorithm. We simulate a primary network with five independent primary channels $\{\mathrm{Ch}_1, \mathrm{Ch}_2, \mathrm{Ch}_3, \mathrm{Ch}_4, \mathrm{Ch}_5\}$. SUs' detection probability and false alarm probability are set as $P_d = 0.9$ and $P_f = 0.1$, respectively. In the following, we will first simulate the proposed game without resource constraint, i.e., $L \geq M$, where SUs should access all the primary channels that can give them positive expected utility. Then, we simulated the proposed game with resource constraint, i.e., $L < M$.

A. Multi-Channel Access Game without Resource Constraint

In this subsection, we evaluate the performance of the proposed best response algorithm for multi-channel access game without resource constraint. For the homogenous case, we set all SUs' gain function as $g_{i,j} = 1$. At each time slot, we let SUs sequentially make decisions based on their estimated state of each channel according to Algorithm 1. In the simulation, we dynamically adjust the order of decision making to ensure the fairness, and set the SU number as $N = 5$ to specifically show each SU's utility. In Fig. 1, we show all SUs' utilities along with the simulation time, where the order of decision making changes every 100 time slots. In the first 100 time slots, during which the order of decision making is $1 \to 2 \to 3 \to 4 \to 5$, we can see that the first SU obtains the highest utility while the last SU obtains the lowest utility since he/she can only access 1 primary channel. In the second 100 time slots, we reverse the decision making order as $5 \to 4 \to 3 \to 2 \to 1$, which leads to that the first SU receives the lowest utility. Therefore, by periodically changing the order of decision making, we can expect that the utilities of all SUs will tend to be the same after a period of time.

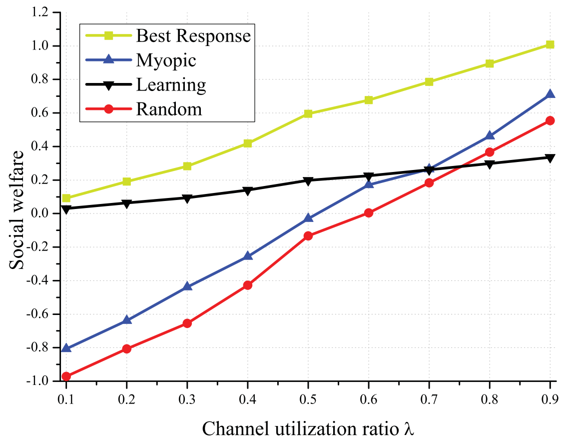

Fig. 2. Social welfare comparison without resource constraint.

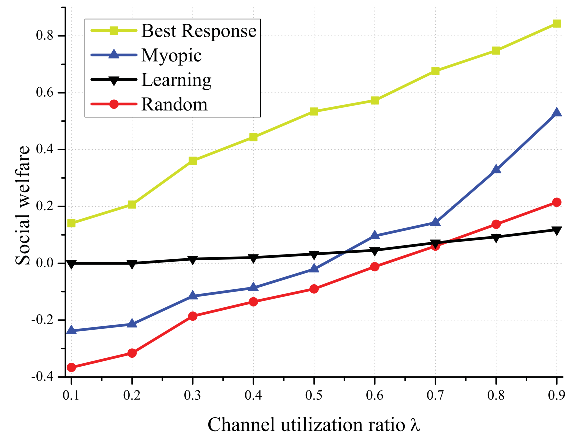

Fig. 3. Social welfare comparison with resource constraint.

For the heterogenous case, we randomize each SU's gain function $g_{i,j}$ between 0 and 1. In this simulation, we compare the performance in terms of SUs' social welfare, which is defined as the total utilities of all SUs, among different kinds of algorithms listed as follows:

- **Best Response**: The proposed recursive best response algorithm in Algorithm 1.
- **Myopic**: At each time slot, the i-th SU selects channels only according to his/her current observation $\mathbf{n}_i = \{n_{i,j}, \forall j\}$ without channel sensing.
- **Learning**: At each time slot, each SU selects channels only according to current belief p_j without considering the negative network externality.
- **Random**: Each SU randomly accesses channels.

For the myopic and learning strategies, the i-th SU's expected utility of accessing channel Ch_j can be calculated by

$$U_{i,j}^{\mathrm{m}} = p_j \frac{g_{i,j}}{n_{i,j} + d_{i,j}} - c_j, \quad (16)$$

$$U_{i,j}^{\mathrm{l}} = p_j \frac{g_{i,j}}{d_{i,j}} - c_j. \quad (17)$$

With these expected utilities, both myopic and learning algorithm can be derived by (4). We can see that the myopic strategy is without social learning and the learning strategy is without consideration of negative network externality. In the simulation, in order to verify the influence of channel utilization ratio on SUs' social welfare, we set the utilization ratios of all five primary channels as the same and adjust λ from 0.1 to 0.9, i.e., from very busy primary channel to very idle primary channel.

Fig. 2 shows the performance comparison result, where the x-axis is the channel utilization ratio λ and y-axis is the social welfare averaged over hundred of time slots. From the figure, we can see with the increase of λ, the social welfare keeps increasing for all algorithms and our best response algorithm performs the best. When $\lambda \leq 0.7$, the learning algorithm performs better than myopic and random algorithms. This is because, when PUs occupy the channel with a relatively high probability, by adopting learning algorithm, although SUs do not consider the negative network externality, they can accurately estimate the channel state of each time slot and avoid to access the time slots when PUs are active. Moreover, when $\lambda \geq 0.8$, we can see that the learning algorithm performs the worst, which is because when the primary channels are very idle, considering other SUs' decisions, i.e., negative network externality, plays a more important role than channel state learning.

B. Multi-Channel Access Game with Resource Constraint

In this subsection, we evaluate the performance of the proposed best response algorithm for multi-channel access game with resource constraint $L = 3$. Similar to the heterogenous case of the without-resource-constraint scenario, we randomize each SU's gain $g_{i,j}$ within $[0, 1]$ and compare the performance of our proposed best response algorithm, with myopic, learning and random algorithms in terms of SUs' social welfare. For the myopic, learning and random algorithms, same resource constraint is adopted, i.e., each SU can at most access 3 channels. Fig. 3 shows the performance comparison result, from which we can see the phenomenon is similar to the case without resource constraint where our best response algorithm performs the best.

V. Conclusion

In this paper, we proposed and studied the multiple SUs' multi-channel access game. A recursive best response algorithm is designed for SUs to distributely find the Nash equilibrium. Simulation results show that our proposed best response algorithms outperform myopic, learning and random algorithms in terms of social welfare.

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