# Blind Identification and Equalization for Multiple-Input/Multiple-Output Channels*

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*Abstract*— **To separate and recover multiple signals in data communications, antenna arrays and acoustic sensor arrays, the impulse responses of multiple-input/multiple-output (MIMO) channels have to be identified explicitly or implicitly. This paper deals with the blind identification of MIMO FIR channels based on second-order statistics of the channel outputs and its application in blind channel equalization and signal separation. We first investigate the identifiability of MIMO FIR channels, and obtain a necessary and sufficient condition for MIMO FIR channels to be identifiable up to a unitary ambiguity matrix using second-order statistics. Next, we extend the identification algorithms for single-input/multiple-output (SIMO) FIR channels, such as the algebraic algorithm and the subspace algorithm to the identification of MIMO FIR channels. Since the ambiguity matrix can not be estimated using second-order statistics, we then present a forth-order cumulant based ambiguity matrix estimation algorithm. Finally, we demonstrate effectiveness of the algorithms in this paper by computer simulations.**

## I. Introduction

Adaptive antenna arrays have been recognized as an important technology to be used in wireless communication systems to increase the capacity and improve the quality of communication services. In wireless communication systems, each sensor may receive a superposition of several input signals with linear distortion, which can be modeled as *multiple-input/multiple-output* (MIMO) systems. The MIMO systems are also encountered in other engineering fields including speech processing, sonar array processing, and in the analysis of biological systems.

When the impulse responses of MIMO systems are obtained, linear equalizers or decision feedback equalizers can be used to remove intersymbol interference, suppress co-channel interference or crosstalk, and recover the original signals. However, in most cases, the impulse responses of MIMO systems are unknown, and there is no reference or training signal available. Therefore, blind identification becomes an important technique to estimate the parameters of MIMO systems.

Blind single-input/single-output (SISO) system identification is usually based on the higher-order statistics of the channel output. For the systems with cyclostationary inputs or single-input/multiple-output (SIMO) systems, several recent papers have presented identification methods based on second-order statistics of the channel outputs. Among them, [3], [5], [8], [9] dealt with the blind identifiability and identification of SIMO FIR channels. Tong *et al* [7] first investigated the blind identification of FIR channels and proposed a singular value decomposition (SVD) method to identify FIR channels under a special rank condition. Li and Ding[3] further proved that this special rank condition is equivalent to the identifiability condition of FIR channel based on second-order cyclostationary statistics. Tugnait [9] also studied the identifiability of SIMO FIR channels from the point of view of common zeros of polynomials. Recently, subspace methods[5], [6] have been proposed to identify FIR channels using cyclostationary statistics of the channel output, which exploits the orthogonality of the noise-subspace and signal-subspace and the structure of filtering matrix. Van der Veen, *et al*, first investigated multiple signal separation and recovery in MIMO systems in [10]. They proposed a blind estimation method for multiple signals using decision-directed principle together with signal-subspace property

This paper deals with the blind identification of MIMO FIR channels using second-order statistics. In this paper, we are going to establish a necessary and sufficient condition for MIMO FIR systems to be identified up to an ambiguity unitary matrix using second-order statistics. We also generalize two identification algorithms for SIMO FIR channels to the identification of MIMO FIR channels.

## II. Model Description

The mathematical model of $d$-input/$M$-output MIMO systems can be illustrated as in Figure 1. The $d$ sequences $s_1[n], \cdots, s_d[n]$ are sent through linear channels $h_{ij}[n]$ for $i = 1, \cdots, M$ and $j = 1, \cdots, d$. Hence, the channel output vector $\mathbf{x}[n]$ can be expressed in time-domain as

$$\mathbf{x}[n] = H[n] * \mathbf{s}[n], \tag{1}$$

where we have used the following notations

$$\mathbf{x}[n] \triangleq \begin{pmatrix} x_1[n] \\ \vdots \\ x_M[n] \end{pmatrix}, \tag{2}$$

$$H[n] \triangleq \begin{pmatrix} h_{11}[n] & \dots & h_{1d}[n] \\ \vdots & \vdots & \vdots \\ h_{M1}[n] & \dots & h_{Md}[n] \end{pmatrix}, \tag{3}$$

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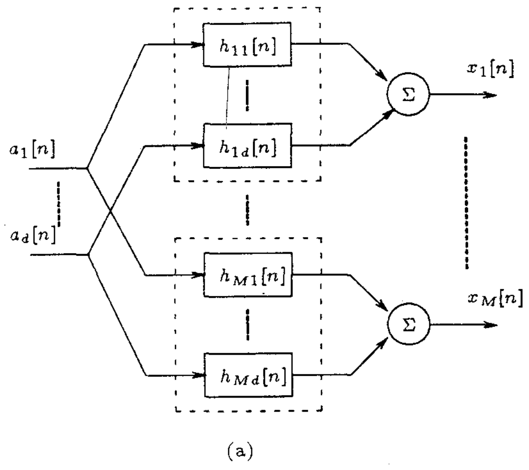


(a)

Fig. 1. MIMO system.

and

$$\mathrm{s}[n] \triangleq \begin{pmatrix} s_1[n] \\ \vdots \\ s_d[n] \end{pmatrix}. \tag{4}$$

Equation (1) can also be written in $Z$-domain as

$$\mathbf{x}(z) = H(z)\mathbf{s}(z), \tag{5}$$

where $\mathbf{x}(z)$, $\mathrm{s}(z)$ and $H(z)$ are the $Z-$transform of $\mathbf{x}[n]$, $\mathrm{s}[n]$, and $H[n]$, respectively. For MIMO FIR channels, $H(z)$ is a ploynomial matrix. The system described above is a single-input/multiple-output (SIMO) system when $d = 1$.

Let the maximum length of $h_{ij}[n]$ be $L$, then the input-output relation of MIMO FIR systems can also be described in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n], \tag{6}$$

where $\mathbf{x}_K[n]$ and $\mathbf{s}_K[n]$ are respectively defined as

$$\mathbf{x}_K[n] \triangleq \begin{pmatrix} \mathbf{x}[n] \\ \vdots \\ \mathbf{x}[n+K-1] \end{pmatrix}, \tag{7}$$

and

$$\mathbf{s}_K[n] \triangleq \begin{pmatrix} \mathrm{s}[n-L+1] \\ \vdots \\ \mathrm{s}[n+K-1] \end{pmatrix}, \tag{8}$$

and $\mathcal{H}_K$ is a $KM \times (L+K-1)d$ block Toeplitz matrix defined as

$$\mathcal{H}_K \triangleq \begin{pmatrix} H[L-1] & \cdots & H[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & H[L-1] & \cdots & H[0] \end{pmatrix}, \tag{9}$$

which is sometimes called *generalized Sylvester matrix*. By means of (6), we are going to develop an algebraic blind identification algorithm for MIMO FIR channels, which can be viewed as the generalization of TXK's algorithm [8] for SIMO FIR channels.

If we define a $KM \times (N-K+1)$ block Toeplitz matrix $\mathcal{X}_K$ by

$$\mathcal{X}_K = (\mathbf{x}_K[0], \mathbf{x}_K[1], \cdots, \mathbf{x}_K[N-K]) \tag{10}$$

and an $(L+K-1)d \times (N-K+1)$ block Toeplitz matrices $\mathcal{S}_K$ by

$$\mathcal{S}_K = (\mathbf{s}_K[0], \mathbf{s}_K[1], \cdots, \mathbf{s}_K[N-K]), \tag{11}$$

then

$$\mathcal{X}_K = \mathcal{H}_K \mathcal{S}_K, \tag{12}$$

From the characteristics of the signal-subspace and noise-subspace of $\mathcal{X}_K$ and the block Toeplitz structure of $\mathcal{H}_K$, we will obtain a subspace identification algorithm, which generalizes the subspace method proposed by Moulines, *et al* in [5].

## III. Identifiability using Second-oreder Statistics

Since the rank of generalized Sylvester matrix $\mathcal{H}_K$ plays an important role in the blind identification of MIMO FIR channels, we will reveal the relationship between its singularity and reducibility of polynomial matrix $H(z)$. First, we give the definition of the MIMO FIR channel identification based on second-order statistics of the channel outputs.

### A. The Definition of MIMO Channel Identification

In most cases, blind channel (system) identification algorithms can only estimate the channels up to some ambiguity.

For the blind channel identification and equalization of single-input/single-output (SISO) channels, the delay and the phase of the gain of the impulse response can not be estimated since $s[n]$ and $e^{j\theta}s[n-n_d]$ for any integer $n_d$ and $\theta \in [-\pi, \pi]$ share the same (second- and higher-order) statistics.

For the blind identification of SIMO FIR channels using second-order statistics of the channel outputs, the algorithms presented in [5], [8] can identify the impulse response up to a constant factor. In fact, this constant factor is unable to be identified by means of second-order statistics of the channel output.

For MIMO FIR systems, if the system inputs $s_i[n]$ are independent identically distributed (i.i.d.) for different $i$ and $n$, then $\mathrm{s}[n]$ and $\mathbf{u}\mathrm{s}[n-n_d]$ for any integer $n_d$ and $d \times d$ unitary matrix $\mathbf{u}$ have the same second-order statistics. Hence, we can at most identify the impulse responses of MIMO FIR channels up to a delay and an ambiguity unitary matrix. Therefore, an MIMO FIR system is said to be *identified by means of second-order statistics of the channel outputs*, if we can find an $\widehat{H}[n]$ such that

$$\widehat{H}[n] = H[n-n_d]\mathbf{u}^H. \tag{13}$$

for all integer $n$, some integer $n_d$, and unitary matrix $\mathbf{u}$. Hence, the *mean square error* (MSE) of identified channel should be defined as

$$MSE \triangleq \min_{n_d, \mathbf{u}} \sum_n \| H[n] - \widehat{H}[n+n_d]\mathbf{u} \|_F^2, \tag{14}$$

where $\| \,.\, \|_F$ denotes the Frobenius norm.

### B. A Necessary and Sufficient MIMO FIR Channel Identification Condition

Not all MIMO FIR channels can be identified using second-order statistics of the channel outputs. For SIMO FIR channels, which is a special case of MIMO FIR channels when $d = 1$, a necessary and sufficient condition for them to be identifiable by using second-order statistics of the channel output is that there is no common zero among subchannels [3], [8], [9]. However, for general MIMO FIR channels, necessary and sufficient identification condition is unknown yet. Here, we establish a necessary and sufficient condition for $\mathcal{H}_K$ to be of full (column) rank, which, as we will show, is also a necessary and sufficient condition for MIMO FIR channels to be identifiable by using second-order statistics of the channel outputs.

First we give a relevant definition. An $M \times d$ ($M > d$) polynomial matrix $H(z)$ is said to be *irreducible* if there is no $d \times d$ polynomial matrix $R(z)$, with non-constant determination, such that $H(z) = \tilde{H}(z)R(z)$, where $\tilde{H}(z)$ is an $M \times d$ polynomial matrix.

Using the properties of the matrix polynomials and filtering matrices, we are able to establish the following identifiability theorem.

**Theorem 1:** *For an MIMO FIR channel with length $L$ and $H[L-1]$ being of full (column) rank, the necessary and sufficient condition for such a channel to be identifiable up to an ambiguity unitary matrix by using second-order statistics of the channel outputs is that $H(z)$ is irreducible.*

From Theorem 1, we can conclude the *identifiability condition* of MIMO FIR channels as the following:

An MIMO FIR channel is said to satisfy the *identifiability condition* if

(i). $H[L-1]$ is of full (column) rank with $L$ being the length of the MIMO FIR channel, and

(ii). $H(z)$ is irreducible, or equivalently, $H(z)$ is of full (column) rank for all $z \in \mathcal{C}$.

When $d = 1$, an MIMO system becomes a SIMO system. In this case, Theorem 1 reduces to a special case and can be stated as: An SIMO FIR channel is identifiable using second-order statistics of the channel outputs if and only if $H[L-1]$ is nonzero and there is no common zero among the $M$ subchannels, which has been presented in [3].

## IV. Blind Identification Algorithms Based on Second-Order Statistics

In this section, we will develop blind identification algorithms for MIMO FIR channels. They generalize the algebraic algorithm proposed in [8] and subspace algorithm proposed in [5] to the identification of MIMO FIR channels.

### A. Algebraic Identification

Assume that the channel input vector $\mathbf{s}[n]$ is i.i.d. with zero-mean and unit-variance, that is

$$R_s[m] \triangleq E\{\mathbf{s}[n]\mathbf{s}^H[n+m]\} = \delta[m]I_d, \tag{15}$$

where $\delta[m]$ is the Kronecker delta function and $I_d$ is a $d \times d$ identity matrix. From (6) and (15), we have

$$R_{x_K}[0] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n]\} = \mathcal{H}_K\mathcal{H}_K^H, \tag{16}$$

and

$$R_{x_K}[1] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n+1]\} = \mathcal{H}_K J \mathcal{H}_K^H, \tag{17}$$

where $J$ is an $(L+K-1)d \times (L+K-1)d$ matrix defined as

$$J \triangleq \begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ I_d & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & I_d & \mathbf{0} \end{pmatrix}. \tag{18}$$

Using the algebraic property of $R_{x_K}[0]$ and $R_{x_K}[1]$, the MIMO channels can be identified up to an ambiguity unitary matrix, which is stated as follows.

**Theorem 2:** *For digital communication systems with MIMO FIR channels satisfying the identifiability condition, if the channel input vector $\mathbf{s}[n]$ is i.i.d. with zero-mean and unit-variance, then, based on $R_{x_K}[0]$ and $R_{x_K}[1]$ for any $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$, the impulse response matrices of MIMO FIR channels $H[n]$ for $n = 0, \cdots, L-1$ can be uniquely determined up to an ambiguity unitary matrix.*

The above theorem demonstrates the blind identifiability of MIMO FIR channels based on second-order statistics of the channel outputs for those MIMO FIR channels satisfying the identifiability condition.

Using Theorem 2, we can develop the following *algebraic identification algorithm*:

**Step 1.** Estimate $R_{x_K}[m]$ for $m = 0,\ 1$ by

$$\widehat{R}_{x_K}[m] = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{x}_K[n]\mathbf{x}_K^H[n+m]. \tag{19}$$

**Step 2.** Find $R$ by

$$R = F R_{x_K}[1] F^H, \tag{20}$$

where

$$F = \Sigma^{-1}U_s^H, \tag{21}$$

$$\Sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_{(L+K-1)d}), \tag{22}$$

$$U_s = [\mathbf{u}_1, \cdots, \mathbf{u}_{(L+K-1)d}], \tag{23}$$

with $\sigma_i^2$ being the nonzero eigenvalues of $R_{x_K}[0]$ corresponding to the eigenvectors $\mathbf{u}_i$ for $i = 1, \cdots, (L+M-1)d$.

**Step 3.** Estimate $\mathcal{H}_K$ by

$$\widehat{\mathcal{H}}_K = U_s\Sigma Q, \tag{24}$$

where

$$Q = (\mathbf{r}, R\mathbf{r}, \cdots, R^{L+K-2}\mathbf{r}), \tag{25}$$

with $\mathbf{r} = (\mathbf{r}_1, \cdots, \mathbf{r}_d)$ and $\mathbf{r}_i$ for $i = 1, \cdots, d$ being the left singularvectors of $R$ corresponding to the only $d$ zero singular-values.

*B. Subspace Identification*

The algebraic algorithm developed in the previous section exploits the structure of $R_{x_K}[0]$ and $R_{x_K}[1]$. The crucial assumption there is that the channel inputs are i.i.d.. Here, we will develop a subspace algorithm for MIMO FIR channel identification, which does not require the i.i.d. assumption of input symbols.

In what follows, we will assume that $\mathcal{S}_K$ is of full row-rank, which means that the vector sequence $\mathbf{s}[n]$ is *persistently exciting of order* $K$ [1]. It relaxes the i.i.d. assumption for input vector sequence.

**Theorem 3:** *For digital communication systems with MIMO FIR channels satisfying the identifiability condition, let the channel input vector sequence* $\mathbf{s}[n]$ *is persistently exciting of order* $K$. *For any* $K \geq \lceil \frac{(L-1)d}{M-d} \rceil + 1$, *if there are a Sylvester matrix* $\tilde{\mathcal{H}}$ *and a matrix* $\tilde{\mathcal{S}}$ *with the same dimension as* $\mathcal{H}_K$ *and* $\mathcal{S}_K$ *respectively, such that* $\mathcal{X}_K = \tilde{\mathcal{H}}\tilde{\mathcal{S}}$, *then* $\tilde{\mathcal{H}} = \mathcal{H}_K(\mathbf{p} \otimes I_{L+K-1})$ *for some* $d \times d$ *nonsingular ambiguity matrix* $\mathbf{p}$. *Futhermore, if* $\tilde{\mathcal{H}}$ *satisfies the constraints (16), then* $\mathbf{p}$ *is unitary.*

Compared with the algebraic identification theorem, subspace identification theorem makes full use of the Sylvester structure of the channel matrix $\mathcal{H}_K$. Without using the i.i.d property of the channel input, the channel can be identified up to a $d \times d$ constant ambiguity matrix $\mathbf{p}$. Theorem 3 is in fact the extension of Theorem 2 in [5] to the identification of MIMO FIR channels.

Using the similar procedures to those in [5], a subspace algorithm has been developed in [11] for the identification of MIMO FIR channels satisfying the identifiability condition. Theorem 3 establishes the theoretical foundation for the subspace algorithm.

The algebraic algorithm exploits the algebraic property of the correlation matrix of the channel output. Since this algebraic property relies on the i.i.d assumption of the channel input, the algebraic algorithm needs more symbols to estimate channels. On the other hand, the subspace algorithm uses the Sylvester structure of channel matrix based on the exciting persistence of the channel input vector sequence, hence, it requires less symbols. This fact will be confirmed by computer simulation results in next section.

## V. Equalization and Multiple Signals Recovery

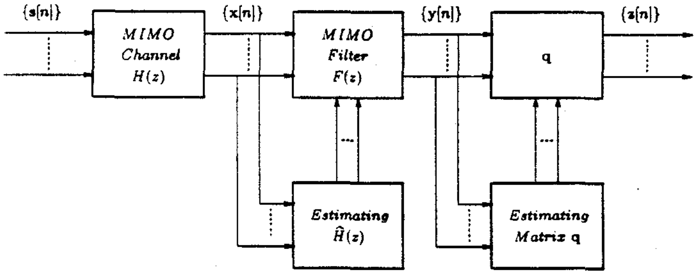


Fig. 2. MIMO channel equalizer.

Once the impulse responses of MIMO channels are known, the some optimum filter can be applied to recover the multiple signals. As indicated in Section III, using second-order statistics, the MIMO FIR channels can be identified up to an ambiguity matrix $\mathbf{q}_o$, that is, we can find $\widehat{H}[n]$, such that,

$$\widehat{H}[n] = H[n]\mathbf{q}_o. \tag{26}$$

Without loss of generality, we assume that $\mathbf{q}_o$ is a unitary matrix. As shown in Figure 2, if an MIMO filter with $Z$-transform

$$F(z) = (\widehat{H}^H(z)\widehat{H}(z))^{-1}\widehat{H}^H(z) \tag{27}$$

is applied, then the $Z$-transform of the filter output $\mathbf{y}[n]$ is

$$\begin{aligned} \mathbf{y}(z) &= F(z)\mathbf{x}(z) \\ &= \mathbf{q}_o^{-1}\mathbf{s}(z). \end{aligned} \tag{28}$$

To remove the effect of the ambiguity matrix, we perform a linear transform $\mathbf{q} = [q_{ij}]$ to the MIMO filter output $\mathbf{y}[n]$ for each time $n$. To recover $s_i[n]$ for $i = 1, \cdots, d$ up to a scaler, $\mathbf{q}$ must satisfy

$$\mathbf{q}\mathbf{q}_o = PD, \tag{29}$$

where $P$ is a $d \times d$ permutation matrix and $D$ is a diagonal matrix defined as

$$D \triangleq \mathrm{diag}\{e^{j\theta_1}, \cdots, e^{j\theta_d}\}, \tag{30}$$

with $\theta_i \in [-\pi, \ \pi]$ for $i = 1, \cdots, d$. The following theorem will give an ambiguity matrix estimation algorithm based on up to forth-order cumulants of the MIMO filter output $\mathbf{y}[n]$.

**Theorem 4:** *Assume that*

*(i). channel input* $s_i[n]$*'s satisfy*

$$E\{s_i[n]\} = 0, \quad E\{s_i[n]^2\} = 0 \tag{31}$$

*and* $s_i[n]$ *and* $s_j[n]$ *are independent for all* $i \neq j$, *and*

*(ii).* $\mathbf{y}[n]$ *satisfies Equation (28) with* $\mathbf{q}_o$ *being unitary.*

*Then, if a unitary matrix* $\mathbf{q}$ *minimizes* $\sum_{ij\ i\neq j} |R_{ij}|$, *it will satisfy (29), where*

$$\begin{aligned} R_{ij} &\triangleq Cum(z_i, z_i^*, z_j, z_j^*) \\ &= \sum_{k_1,k_2,k_3,k_4} q_{ik_1}q_{ik_2}^*q_{jk_3}q_{jk_4}^* Cum(y_{k_1}, y_{k_2}^*, y_{k_3}, y_{k_4}^*), \end{aligned} \tag{32}$$

*with* $Cum(x_1, x_2, x_3, x_4)$ *being the cumulant of random variables* $x_1$, $x_2$, $x_3$ *and* $x_4$ *defined as*

$$\begin{aligned} & Cum(x_1, x_2, x_3, x_4) \\ \triangleq\ & E\{x_1x_2x_3x_4\} - E\{x_1x_2\}E\{x_3x_4\} \\ =\ & -E\{x_1x_3\}E\{x_2x_4\} - E\{x_1x_4\}E\{x_2x_3\}. \end{aligned} \tag{33}$$

## VI. Computer Simulation Results and Conclusions

A Monte Carlo simulation example has been conducted to illustrate MIMO FIR channel identification algorithms with application to QAM digital communication systems.

We have modified the simulation example in [10] for QAM digital communication systems. In our simulation, the digital signals $s_i[n]$'s are independent of each other for any different $n$ or $i$, and they are uniformly distributed over $\{\pm 1 \pm j\}$. The digital signals are modulated by raised-cosin shaping filters

TABLE I

THE PARAMETERS OF THE MULTIPATH CHANNEL

| Sources | i | $\alpha_i$ | $\tau_i$ | $p_i$ |
|---|---|---|---|---|
| 1st | 1 | $-10°$ | 0.0 | $1.0e^{-2.72j}$ |
| | 2 | $-2°$ | 0.3 | $0.8e^{0.61j}$ |
| | 3 | $-120°$ | 1.2 | $0.4e^{0.93j}$ |
| | 4 | $160°$ | 2.1 | $0.4e^{1.64j}$ |
| 2nd | 1 | $10°$ | 0.5 | $1.0e^{-3.07j}$ |
| | 2 | $15°$ | 0.9 | $0.9e^{1.17j}$ |
| | 3 | $-40°$ | 1.5 | $0.5e^{-1.06j}$ |
| | 4 | $150°$ | 2.8 | $0.3e^{1.83j}$ |

with roll-off factor 0.35 and truncated to a length of six symbol periods. The modulated signals are received by 2 sensors spaced by half wavelength. The simulated channel consists of four paths per signal and each path is specified by angle-of-arrival $\alpha_i$, delay $\tau_i$, and complex gain $p_i$ as shown in Table I. From Table I, the maximum channel length is $L = 9$. The channel noise is complex white Gaussian with zero-mean and variance determined by the signal-to-noise ratio (SNR).

The signal from each sensor is 5-time oversampled, resulting $M = 10$ in the MIMO channel model shown as in Figure 1. The algebraic and subspace algorithms are used respectively to estimate the impulse responses of the multiple-input/multiple-output (MIMO) FIR channel. Although the maximum length of the MIMO FIR channel is $L = 9$, our simulation indicates that the estimation obtains the optimum performance when $\hat{L} = 8$. The estimation performance is measured by the mean-square-error (MSE) defined in (14) in Section III. Figure 2 and 3 demonstrate the MSE's of both algebraic algorithm and subspace algorithm via SNR and the number of symbols respectively. 100 independent trials have been conducted under the same simulation scenario to obtain these MSE's. From these figures, when $SNR \geq 40dB$ and the number of samples $N \geq 500$, the estimation can attain satisfying performance. As indicated in Section IV, the subspace algorithm has better performance than the algebraic algorithm.

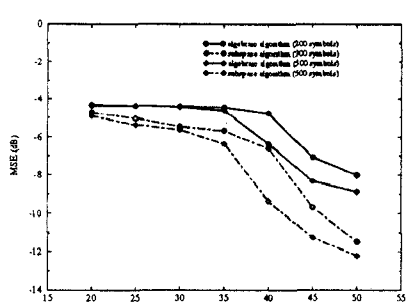

Fig. 3. The MSE of estimated channel impulse responses via SNR based on 100 ensemble trials.

We have investigated the blind identification of MIMO FIR channels based on the second-order statistics of the channel outputs. A necessary and sufficient condition for MIMO FIR channels to be identified up to an ambiguity matrix has been established. An algebraic identification algorithm and a subspace identification algorithm have been developed. Using these algorithms, the MIMO FIR channels satisfying the identifiability condition can be estimated up to an ambiguity matrix. Since the ambiguity matrix can not be identified using second-order statistics, we also presented an ambiguity matrix estimation method using forth-order cumulant. With this ambiguity matrix estimation method, the MIMO channel identification algorithms developed in this paper can be applied to array processing in mobile radio communication systems to increase the communication capacity.

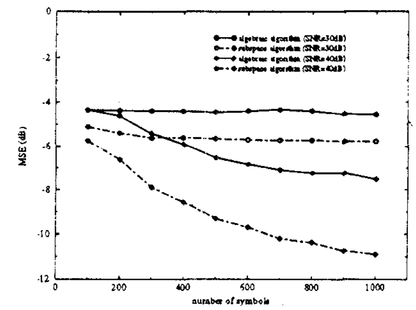

Fig. 4. The MSE of estimated channel impulse responses via the number of symbols used in the estimation based on 100 ensemble trials.

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