Outage Probability for Two-Way Solar-Powered Relay Networks with Stochastic Scheduling

Wei Li*†, Meng-Lin Ku‡, Yan Chen*, and K. J. Ray Liu*

*Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA
†Department of Information and Communication Engineering, Xi'an Jiaotong University, Xi'an, 710049, China
‡Department of Communication Engineering, National Central University, Taiwan
Email: wli52140@umd.edu, mlku@ce.ncu.edu.tw, {yan, kjrliu}@umd.edu

***Abstract*—An optimal relay transmission policy by exploiting a stochastic energy harvesting (EH) model is proposed for EH two-way relay (TWR) networks, wherein a solar-powered relay with a finite-sized battery adopts an amplify-and-forward protocol for helping relaying signals. The relay transmission power is optimized to minimize the long-term outage probability by considering the causal EH information, battery energy and random channel status. The design framework is formulated as a Markov decision process (MDP), in which a monotonic structure for the long-term reward values and a threshold property for the optimal relay transmission are revealed. Furthermore, an interesting saturation structure of the outage performance is uncovered, which means the expected outage probability eventually approaches to the relay's battery empty probability. Simulation results are demonstrated to verify the theoretical analysis and prove that the proposed optimal policy outperforms other myopic policies.**

I. Introduction

Energy harvesting (EH) communication has recently attracted significant attention due to its effectiveness to solve energy supply problems in wireless networks without tethering to a fixed power grid. In this paradigm, the EH nodes can utilize ambient energy sources, e.g., solar, vibration, electromagnetic radiation, etc., to fulfill data transmissions for an infinite lifetime. However, the uncertain and random nature of the energy sources requires us to revisit power management and transmission scheduling.

Two-way relay (TWR) communications have been recognized as an efficient solution for information exchange between two source nodes via intermediate relay nodes [1]. More recently, there has been a growing interest in studying EH TWR networks, where relay nodes or source nodes can harvest energy from environment. In [2]-[4], power allocation methods for maximizing short-term sum rates in EH TWR networks were investigated by exploiting deterministic EH models, in which the EH information is non-causal and known prior to transmission scheduling. In [2], an EH relay with a data buffer can cache data and utilize flexible scheduling policies. Moreover, the authors in [3] proposed a generalized iterative directional water-filling algorithm for various relaying strategies. An optimization design considering the uncertainty of channel state information was developed in [4]. Furthermore, the authors in [5] investigated the optimal transmission policy for maximizing the long-term sum rates of EH TWR networks with stochastic EH models, wherein the EH information is causal and unknown to transmitters. Besides, the wireless energy transfer in EH TWR networks has been considered in the literature. The authors in [6] proposed a joint design of transceiver and power splitters for the TWR network with energy transfer. Similarly, a problem that the two source nodes can transfer the energy and information simultaneously to the relay in the TWR network was considered in [7]. By far, most of the research works on EH TWR networks focused on the throughput maximization. However, the outage probability performance in EH TWR networks has not been analyzed, especially for stochastic EH models.

Motivated by the aforementioned discussions, we propose an optimal relay transmission policy for the EH TWR network using the data-driven stochastic EH model in [8]. In this network, two source nodes are traditional wireless nodes, while a solar-powered EH relay node is equipped with a finite-sized battery and exploits amplify-and-forward (AF) protocols. Our objective is to minimize the long-term outage probability by adapting the relay transmission power to the causal solar irradiance condition, the battery energy amount and random channel states. The framework is formulated as a Markov decision process (MDP), in which the conditional outage probability was calculated as the reward function. In order to analyze the outage performance, we first propose the monotonic structure of the long-term reward values, and the threshold property of the optimal relay transmission power. Finally, an interesting saturation structure of the outage performance is revealed, in that the expected outage probability eventually approaches to the battery empty probability in sufficiently high signal-to-noise power ratio (SNR) regimes.

II. Energy Harvesting Two-way Relay Network

We consider an EH TWR network, wherein two traditional wireless source nodes, A and B, exchange information

This work was supported by the China Scholarship Council (CSC), the National Natural Science Foundation of China (NSFC) under Grant No.61461136001, and the Science and Technology Program of Shaanxi Province under Grant No.2011K06-10.

978-1-4799-7591-4/15/$31.00 ©2015 IEEE

simultaneously via an EH relay node, R, which can harvest energy from the solar and is equipped with a rechargeable finite-sized battery. Each node is operated in a half-duplex mode and equipped with a single antenna. It is assumed that there is no direct link between the two source nodes, and the wireless channels are reciprocal, quasi-static and Rayleigh flat fading. The two independent and identically distributed (i.i.d.) random variables, h_{ar} and h_{br}, with complex Gaussian distribution $\mathcal{CN}(0,1)$, represent the channel coefficients of the wireless links between the source nodes and R, respectively. Let $\gamma_1 = |h_{ar}|^2$ and $\gamma_2 = |h_{br}|^2$ denote the instantaneous channel gain with exponential distribution and unit mean. The relay has the perfect channel state information (CSI) of these two links. Further, it is assumed that the two source A and B have the same transmission power P, while the transmission power of R is given by P_r. The transmission duration includes two phases, a multiple access (MA) phase and a broadcast (BC) phase, and the relay utilizes amplify-and-forward (AF) protocol to broadcast the received signal to A and B.

Define R_1 and R_2 as the achievable data rates of the A-B link and the B-A link, respectively. From [1], the achievable rate pair (R_1, R_2) can be expressed as

$$R_1 \le \frac{1}{2} I_1 = \frac{1}{2}\log\left[1 + \frac{\gamma_1\gamma_2\eta\eta_r}{\gamma_1\eta + \gamma_2(\eta+\eta_r)+1}\right]; \quad (1)$$

$$R_2 \le \frac{1}{2} I_2 = \frac{1}{2}\log\left[1 + \frac{\gamma_1\gamma_2\eta\eta_r}{\gamma_1(\eta+\eta_r) + \gamma_2\eta+1}\right], \quad (2)$$

where $\eta = P \,/\, N_0$ and $\eta_r = P_r \,/\, N_0$, with N_0 being the additive white Gaussian noise (AWGN) power at each node. Therefore, the two outage events can be defined as $\mathcal{E}_{out,1} = \left\{\frac{1}{2}I_1 < R_{th1}\right\}$ and $\mathcal{E}_{out,2} = \left\{\frac{1}{2}I_2 < R_{th2}\right\}$, where R_{th1} and R_{th2} denote the target rates for A and B, respectively. Thus, the outage probability of the TWR network is given by [9]

$$P_{out} = \Pr\left\{\mathcal{E}_{out,1} \cup \mathcal{E}_{out,2}\right\}. \quad (3)$$

III. Markov Decision Process

The design of the relay transmission policy is influenced by a couple of dynamic and stochastic factors, such as the solar irradiance conditions, the energy amount in the battery and the channel conditions among the three nodes. Our objective is to find the optimal relay transmission power that can minimize the long-term outage probability of the TWR network. To achieve this, the design framework is formulated as an MDP. The fundamental components in the MDP model include states, actions and reward functions, and the detailed descriptions of all these elements are introduced as follows.

A. Relay Action. Let $\mathcal{W} = \{0, 1, \cdots, N_p - 1\}$ denote an action set of the relay transmission power. If the transmission action is set as $W = w \in \mathcal{W}$, the relay transmission power P_r is equal to wP_u during the BC phase, where P_u is the basic transmission power corresponding to one energy quantum E_u during a half policy period $T \,/\, 2$, i.e., $E_u = P_u T \,/\, 2$.

B. System States. Let $\mathcal{S} = \mathcal{Q}_e \times \mathcal{H}_{ar} \times \mathcal{H}_{br} \times \mathcal{Q}_b$ be a four-tuple state space, where $\times$ denotes the Cartesian product, $\mathcal{Q}_e = \{0, 1, \cdots, N_e - 1\}$ represents a solar power state set, $\mathcal{H}_{ar} = \{0, 1, \cdots, N_c - 1\}$ and $\mathcal{H}_{br} = \{0, 1, \cdots, N_c - 1\}$ are channel state sets of h_{ar} and h_{br}, respectively, and $\mathcal{Q}_b = \{0, 1, \cdots, N_b - 1\}$ denotes a finite battery state set. Meanwhile, define a random variable $S = (Q_e, H_{ar}, H_{br}, Q_b) \in \mathcal{S}$ as the system stochastic state of the MDP, and it remains steady during one policy management period T.

(a) Solar power State: An N_e-state stochastic EH model in [8] is utilized to mimic the harvested solar power conditions. This data-driven EH model is a Gaussian mixture hidden Markov chain, and different states represent distinct solar irradiance intensity. The dynamic of the states is determined by transition probabilities $P(Q_e = e' | Q_e = e), \forall e, e' \in \mathcal{Q}_e$ [8].

If the solar power state is given by $Q_e = e \in \mathcal{Q}_e$, the harvested solar power per unit area, P_h, is a continuous random variable with Gaussian distribution $\mathcal{N}(\mu_e, \rho_e)$. Moreover, the harvested solar energy during one policy period T can be calculated as $E_h = P_h T \Omega \eta$, where Ω is the solar panel area size, and η denotes the energy conversion efficiency. Further, a quantization model is utilized to deal with the harvested energy, which is stored in the battery and consumed for data transmission in unit of one basic energy quantum E_u. The probability of the number of harvested energy quanta conditioned on the e^{th} solar state, $P(Q = q | Q_e = e)$ for $q \in \{0, 1, \cdots, \infty\}$, is theoretically derived in [8].

(b) Battery State: If the relay battery state is $Q_b = b \in \mathcal{Q}_b$, the number of available energy quanta in the relay battery is given by b, i.e., the available energy is bE_u. We exploit the *harvest-store-use* protocol, which means the harvested energy in the current policy period is first stored in the battery, and then consumed in the next policy period [10]. Let b' and b denote the next and current battery states, respectively. The battery state transition can be given by $b' = b - w + q$, where q represents the number of harvested energy quanta in the current policy period. Further, the battery state transition probability at the e^{th} solar power state is given by $P_w(Q_b = b' | Q_b = b, Q_e = e)$ [8].

(c) Channel States: The instantaneous channel gains, γ_1 and γ_2, are quantized into N_c levels using a finite number of thresholds, given by $\Gamma = \{0 = \Gamma_0, \Gamma_1, \cdots, \Gamma_{N_c} = \infty\}$. The i^{th} channel state represents the channel gain belongs to the interval $[\Gamma_i, \Gamma_{i+1})$. We assume that the channel gain is quasi-static in one policy management period T. Moreover, the channel state transition is formulated by a finite-state Markov chain, and the transition probability is defined in [11].

C. MDP State Transition: Since the solar irradiance condition and the fading channel status are independent, the system state transition probability from $s = (e, h, g, b)$ to $s' = (e', h', g', b')$ between two adjacent policy periods with respect to the relay power action w is given by

$$\begin{aligned} P_w(s'|s) = &P(Q_e = e' | Q_e = e) \cdot P(H_{ar} = h' | H_{ar} = h) \quad (4) \\ &\cdot P(H_{br} = g' | H_{br} = g) \cdot P_w(Q_b = b' | Q_b = b, Q_e = e) \end{aligned}$$

D. Reward Function. The conditional outage probability at a fixed system state $s = (e, h, g, b) \in \mathcal{S}$ for a relay power action $w \in \mathcal{W}$ is utilized as the reward function, defined as

$$R_w(s) \triangleq P_{out}(w, h, g) = \Pr\{\mathcal{E}_{out,1} \cup \mathcal{E}_{out,2} | P_r = wP_u, H_{ar} = h, H_{br} = g\} \quad (5)$$

According to (1)-(3) and the definition of channel states, the conditional outage probabilities can be explicitly calculated.

Theorem 1: For the given target rate pair (R_{th1}, R_{th2}), the conditional outage probability of the TWR network utilizing the AF protocol associated with the state $s = (e, h, g, b)$ and relay action w can be expressed as in (6).

Specifically, when the relay remains silent, the reward function is equal to one. While the SNR is sufficiently high, i.e., N_0 approaches to zero, it suffices to spend only one energy quantum for achieving zero outage probability under fixed target rates and channel states. Thus, it is given by

$$R_{w=0}(s) = 1, \lim_{N_0 \to 0} R_{w \geq 1}(s) = 0. \quad (7)$$

E. Optimization of Relay Transmission Policy. Define the policy $\pi(s) : \mathcal{S} \to \mathcal{W}$ as the action that indicates the relay transmission power with respect to a given system state. The goal of the MDP is to find the optimal policy π^* that can optimize the expected discounted long-term reward as

$$V_\pi(s_0) = \mathbb{E}_\pi\left[\sum\nolimits_{k=0}^{\infty} \lambda^k R_{\pi(s_k)}(s_k)\right], \pi(s_k) \in \mathcal{W}, \quad (8)$$

where s_0 is the initial state, and $0 \leq \lambda < 1$ is a discount factor that guarantees the convergence. The optimal policy for minimizing (8) can be found through the Bellman equation, which can be efficiently implemented by the well-known value iterations as follows [12]

$$V_w^{(i+1)}(s) = R_w(s) + \lambda \sum\nolimits_{s' \in S} P_w(s'|s) V^{(i)}(s'); \quad (9)$$

$$V^{(i+1)}(s) = \min_{w \in \mathcal{W}} \left(V_w^{(i+1)}(s)\right), \quad (10)$$

where i is the iteration number. The value iteration alternates between (9) and (10) until a stopping criterion is satisfied.

IV. Performance Analysis of Outage Probability

The performance of the expected outage probability for the proposed optimal policy in high SNR regimes is analyzed in this section. First, we will introduce a monotonic structure of the expected long-term reward.

Lemma 1: For any fixed system state $s = (e, h, g, b > 0) \in \mathcal{S}$ in the i^{th} value iteration, the expected long-term reward is non-increasing in the battery state, and the differential value of the expected long-term rewards between two adjacent battery states is not larger than one, i.e., $1 \geq V^{(i)}(e, h, g, b-1) - V^{(i)}(e, h, g, b) \geq 0, \forall b \in \mathcal{Q}_b \backslash \{0\}$. Therefore, the optimal policy π^* is also satisfied with the above special structure, i.e., $1 \geq V_{\pi^*}(e, h, g, b-1) - V_{\pi^*}(e, h, g, b) \geq 0$, $\forall b \in \mathcal{Q}_b \backslash \{0\}$.

Based on the above special structure, the following threshold property of the optimal policy is proposed.

Lemma 2: For any fixed system state $s = (e, h, g, b > 0) \in \mathcal{S}$ with the non-empty battery, in sufficiently high SNRs, i.e., N_0 approaches to zero, the optimal relay power action w^* is equal to one.

In the following, we prove that there exists a saturation structure of the expected outage probability.

Theorem 2: In sufficiently high SNRs, the expected outage probability for the proposed optimal policy π^* is equal to the battery empty probability.

Proof: The expected outage probability can be calculated by taking expectation on the reward function with respect to the optimal policy π^* as follows

$$\bar{R} = \sum\nolimits_{s \in \mathcal{S}} [p_{\pi^*}(e,h,g,b=0) \times R_{w^*}(e,h,g,b=0) + p_{\pi^*}(e,h,g,b \geq 1) \times R_{w^*}(e,h,g,b \geq 1)] \quad (11)$$

where $p_{\pi^*}(s)$ denotes the system steady state probability with respect to the optimal policy π^*, and can be computed by the balance equation and transition probability in [8].

According to the harvest-store-use protocol, the relay keeps silent when the battery is empty. Meanwhile, by applying Lemma 2, the optimal action w^* is equal to one in sufficiently high SNRs when the battery state $b \geq 1$. From (7), the expected reward in high SNRs can be expressed as

$$\lim_{N_0 \to 0} \bar{R} = \sum_{e=0}^{N_e-1} \sum_{h=0}^{N_c-1} \sum_{g=0}^{N_c-1} p_{\pi^*}(e,h,g,b=0) = P_{\pi^*}(b=0), \quad (12)$$

which means the expected outage probability converges to the battery empty probability in high SNRs. ■

V. Simulation Results

The outage probability of the proposed optimal policy based on the stochastic EH model in [8] is evaluated by computer simulations. The analytical results are computed according to (11), while the simulation results are obtained

$$P_{out}(w,h,g) \begin{cases} = 1, & (\gamma_{th1} \geq \Gamma_{h+1}) \text{ or } (\gamma_{\text{th2}} \geq \Gamma_{\text{h+1}}) \text{ or } (\gamma_{\text{th3}} \geq \Gamma_{\text{g+1}}) \text{ or } (\gamma_{\text{th4}} \geq \Gamma_{\text{g+1}}); \\ = 0, & (\gamma_{th1} \leq \Gamma_h) \text{ and } (\gamma_{\text{th2}} \leq \Gamma_{\text{h}}) \text{ and } (\gamma_{\text{th3}} \leq \Gamma_{\text{g}}) \text{ and } (\gamma_{\text{th4}} \leq \Gamma_{\text{g}}); \\ \approx 1 - \frac{e^{-\max(\gamma_{th1},\gamma_{th2})} - e^{-\Gamma_{h+1}}}{e^{-\Gamma_h} - e^{-\Gamma_{h+1}}} \cdot \frac{e^{-\max(\gamma_{th3},\gamma_{th4})} - e^{-\Gamma_{g+1}}}{e^{-\Gamma_g} - e^{-\Gamma_{g+1}}}, & \text{otherwise;} \end{cases} \quad (6)$$

$$\gamma_{th1} = \frac{(P + wP_u) N_0}{P \cdot wP_u}\left(2^{2R_{th1}} - 1\right), \gamma_{th2} = \frac{N_0}{wP_u}\left(2^{2R_{th2}} - 1\right), \gamma_{th3} = \frac{N_0}{wP_u}\left(2^{2R_{th1}} - 1\right), \gamma_{th4} = \frac{(P + wP_u) N_0}{P \cdot wP_u}\left(2^{2R_{th2}} - 1\right).$$

by using the Monte-Carlo method. The policy period is set as $T = 300s$. The numbers of the solar power states, channel states and battery states are four, six and twelve, respectively. The solar energy conversion efficiency is $\eta = 20\%$. The channel is quantized as $\Gamma = \{0, 0.3, 0.6, 1.0, 2.0, 3.0, \infty\}$, and the channel gain is generated using Jakes' model [13] with the normalized Doppler frequency $f_D = 0.05$. The battery state is initialized randomly. We assume that the target rates R_{th1} and R_{th2} are identical and equal to $R_{th}/2$. In the value iteration algorithm, the discount factor is set as $\lambda = 0.99$. A normalized SNR is defined with respect to the transmission power of $1\,mW$.

Fig. 1 shows the outage probability of our proposed optimal policy for different solar panel size Ω and source nodes' power P. There is a minor gap between the analysis results and simulation results when SNR is small, while the curves become almost identical in high SNRs. This is because the conditional outage probability derived in Theorem 1 is an approximate value. It can be observed that there exists the saturation structure, i.e., the outage probability is gradually saturated and eventually close to the battery empty probability (the dashed line without markers) with respect to the optimal policy in sufficiently high SNRs, instead of going to zero. This phenomenon coincides with Theorem 2. It can be seen that the saturation outage probability in high SNR regimes, i.e., the battery empty probability, becomes smaller when the solar panel size Ω gets larger. The reason can be explained as follows. Since a bigger solar panel size Ω means there is more energy harvested within one policy period, the battery empty probability $P(b = 0)$ can be decreased by increasing Ω. Besides, the instantaneous throughput can be improved with the increase of the transmission power of the source nodes P, yielding a better outage performance.

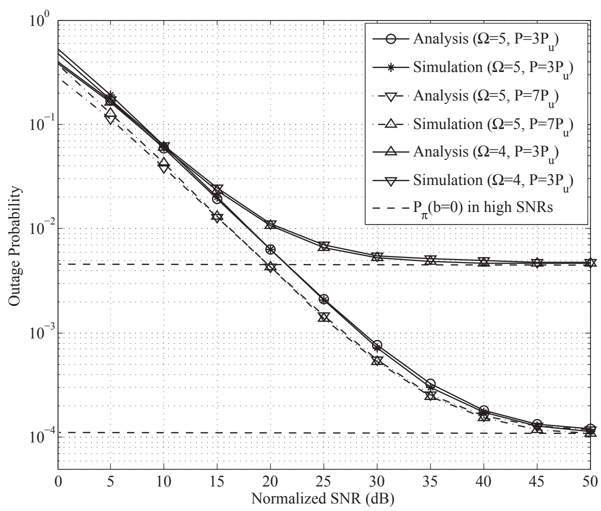

Fig. 1. Outage probability for solar panel size Ω and source nodes' transmission power P ($R_{th} = 4bit/s/Hz$, $P_u = 35\ mW$, Unit of $\Omega:\ cm^2$)

Fig. 2 compares the outage probabilities of our proposed optimal policy and two myopic policies for different target sum rates R_{th}. For these two myopic policies, the relay transmission power is set without concern for the channel state and the battery state transition probabilities. Instead, the relay transmits signals as long as the battery is non-empty. In Myopic policy I, the largest available energy in the battery is consumed by the relay for one transmission period. Regarding with Myopic policy II, the relay attempts to exploit the lowest power, i.e., the basic transmission power P_u. It can be seen that the outage performance of our proposed optimal policy is superior to those of the two myopic policies. The outage probabilities of these three policies are all saturated in sufficiently high SNR regimes, and the saturation outage probabilities correspond to their own battery empty probabilities in sufficiently high SNRs. Since the proposed optimal policy is equivalent to Myopic policy II in high SNR regimes according to Lemma 2, the saturation outage probabilities of these two policies are identical. Regarding with Myopic policy I, since the largest available energy in the battery is consumed at once, its battery empty probability is much larger than that of our proposed optimal policy. In other words, the proposed optimal policy outperforms Myopic policy I in high SNR regimes. Besides, when the target sum rate R_{th} gets smaller, the outage probability becomes better.

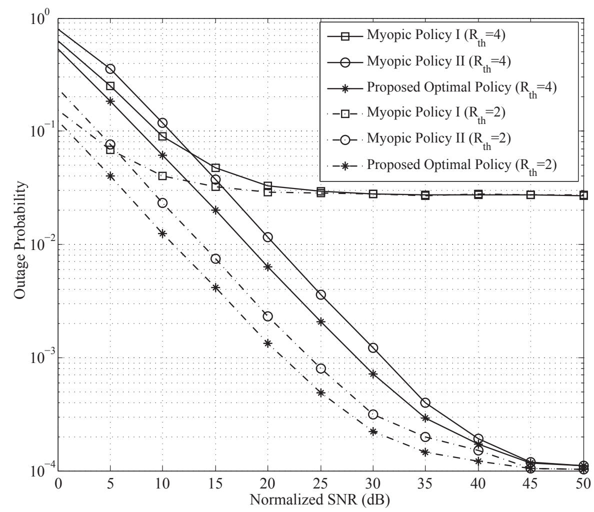

Fig. 2. Outage probabilities of the proposed optimal policy and myopic policies ($\Omega = 5\,cm^2$, $P_u = 35\ mW$, $P = 3P_u$, Unit of R_{th}: $bit/s/Hz$)

VI. Conclusion

The optimal relay transmission policy for minimizing the long-term outage probability in the EH TWR network is proposed. We make use of stochastic EH models to describe the solar irradiance condition, and design the optimal policy. First, we find out the monotonic structure of the long-term reward and the threshold property of the optimal relay power. Furthermore, an interesting saturation structure is uncovered to predict the performance limit of the outage probability in sufficiently high SNRs.

REFERENCES

[1] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Achievable Rate Regions and Performance Comparison of Half Duplex Bi-Directional Relaying Protocols," *IEEE Trans. Wireless Commun.*, vol. 57, no. 10, pp. 6405 – 6418, Oct. 2011.

[2] B. Varan and A. Yener, "The Energy Harvesting Two-Way Decode-and-Forward Relay Channel with Stochastic Data Arrival," in *Proc. of 2013 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Dec. 2013, pp. 371–374.

[3] K. Tutuncuoglu, B. Varan, and A. Yener, "Optimum Transmission Policies for Energy Harvesting Two-way Relay Channels," in *Proc. of 2013 International Conference on Communication (ICC)*, Budapest, Hungary, June 2013, pp. 586–590.

[4] I. Ahmed, A. Ikhlef, D. W. K. Ng, and R. Schober, "Optimal Resource Allocation for Energy Harvesting Two-Way Relay Systems with Channel Uncertainty," in *Proc. of 2013 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Dec. 2013, pp. 345–348.

[5] W. Li, M.-L. Ku, Y. Chen, and K. J. R. Liu, "On the Achievable Sum Rate for Two-Way Relay Networks with Stochastic Energy Harvesting," in *Proc. of 2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Atlanta, GA, USA, Dec. 2014, pp. 133–137.

[6] Z. Wen, S. Wang, C. Fan, and W. Xiang, "Joint Transceiver and Power Splitter Design Over Two-Way Relaying Channel With Lattice Codes and Energy Harvesting," *IEEE Commun. Lett.*, vol. 18, no. 11, pp. 2039–2042, Nov. 2014.

[7] Z. Fang, X. Yuan, and X. Wang, "Distributed Energy Beamforming for Simultaneous Wireless Information and Power Transfer in the Two-Way Relay Channel," *IEEE Signal Processing Lett.*, vol. 22, no. 6, pp. 656–660, 2015.

[8] M.-L. Ku, Y. Chen, and K. J. R. Liu, "Data-Driven Stochastic Transmission Policies for Energy Harvesting Sensor Communications," *IEEE J. Select. Areas Commun.*, vol. PP, no. 99, p. 1, Jan. 2015.

[9] X. Lin, M. Tao, Y. Xu, and R. Wang, "Outage Probability and Finite-SNR DiversityCMultiplexing Tradeoff for Two-Way Relay Fading Channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, pp. 3123 – 3136, Sep. 2013.

[10] N. Michelusi, L. Badia, and M. Zorzi, "Optimal Transmission Policies for Energy Harvesting Devices with Limited State-of-Charge Knowledge," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 3969–3982, Nov. 2014.

[11] H. S. Wang and N. Moayeri, "Finite-state Markov channel-a useful model for radio communication channels," *IEEE Trans. Wireless Commun.*, vol. 44, no. 1, pp. 163–171, Feb. 1995.

[12] M. Puterman, *Markov Decision Process-Discrete Stochastic Dynamic Programming*. John Wiley and Sons, 1994.

[13] W. C. Jakes, *Microwave Mobile Communication*. New York: Wiley, 1974.