Low Complexity Adaptive Beamforming and Power Allocation for OFDM Over Wireless Networks

Masoud Olfat, K. J. Ray Liu
Electrical Engineering Department
University of Maryland
College Park, MD 20742
(molfat, kjrliu@isr.umd.edu)

Farrokh. Rashid-Farrokhi
Wireless Communications Research Dept.
Bell Labs, Lucent Technologies
791 Holmdel-Keyport Rd, Rm. R-111
Holmdel, NJ 07733

Abstract

In this paper, the performance of a multiuser wireless network using OFDM, combined with Power Control and Adaptive Beamforming for uplink transmission is presented. An adaptive power control algorithm is exploited to achieve the desired Signal to Noise and Interference Ratio (SINR) at each subchannel and increase the power efficiency of the mobile transmitter. Therefor, we can achieve a better overall error probability with a fixed total power. A distributed iterative algorithm is used to jointly update the transmission power and the beamformer weights at each subchannel so that it can converge to the optimal solution for both power and beamforming vectors at each subchannel. Another algorithm has been proposed to decrease the complexity resulting from the number of beamformers and FFT blocks. The algorithms use only the interference measured locally by the transmitter. Unlike most of the loading algorithms which optimize the bit distribution and subchannel power allocation for a single transmitter, this approach tries to optimize the power allocation and decrease the interference for the whole network.

1 Introduction

Orthogonal Frequency Division Modulation (OFDM) is a parallel data transmission scheme. OFDM systems employ several techniques such as: frequency and/or time interleaving, time guard band, different coding strategies, simple equalization.

OFDM has several advantages and disadvantages. Among its advantages, high bandwidth efficiency, converting a wideband frequency selective fading channel into a series of narrowband flat fading subchannels, averaging the time domain short term distortion, due to FFT operation at the receiver side, and relatively large block duration, no need for sophisticated equalization method, are considerable.

One problem with OFDM is its poor performance due to the sensitivity of error probability to the subcarrier with the lowest signal to noise ratio. Also, the error probability decreases slowly with increasing signal power [1].

Several methods have been proposed to combat the aforementioned problem. Those methods are basically trying to adjust the bit and power distribution among subchannels according to their performances and are mostly called "loading algorithms" [2], [3], [5] and [8]. However, most of them have considered a single transmitter. In a mobile environment, each user's signal can affect others and this, in turn results in further attenuation. Power control is an appropriate solution to keep the SINR of all subcarriers in a desired level and minimize the interference caused by other users as much as possible, and to achieve better bit error rate. In the receiver side, An adaptive beamformer, further attenuates the interference caused by other users.

This paper is organized as follows: Section 2 introduces the basics of OFDM systems, the problem of power control and its solution. Section 3 along with Section 4 introduces our proposed system and some simulation results and Section 5 concludes the paper.

2 Basic concepts

Consider a data sequence $s_0, s_1, s_2, \ldots, s_{N-1}$, where each s_n represents a symbol of m_n bits. Every N symbols are combined to make a block of data. Each symbol passes through a modulator, which can be QPSK or QAM, resulting a sequence of complex numbers $d_0, d_1, d_2, ...d_{N-1}$. If an Inverse Discrete Fourier Transform (IDFT) operation is performed on this block, the result is another vector of N complex numbers $X_0, X_1, \ldots, X_{N-1}$. The real and imaginary parts of the modulated symbols are separated and the parallel data are converted back to serial. By applying these components to a low pass filter at time intervals T_s (block length) the inphase and the quadrature components of OFDM signal are obtained. These components are upconverted in order to be transmitted through the channel.

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At the receiver, the received signal is down converted to baseband and the inphase and quadrature components of the OFDM signal are extracted.

Assuming perfect block synchronization between the transmitter and the receiver, the latter is able to extract the relevant symbol interval T_s, and sample the components of received signal at the multiples of T_s to obtain the complex samples $\hat{X}_k$ $(k = 0, \ldots, N-1)$. These samples are used to perform a DFT operation and the resultant symbols are being demodulated to estimate symbols $\hat{s}_n$.

The transmitter introduces a cyclic extension guard interval T_G (larger than the delay spread of the channel) before the low pass filter to preserve the orthogonality of the subchannels at the presence of InterSymbol Interference (ISI) caused by channel distortion and multipath delays. So, if the receiver neglects the received signal outside the time interval T_s, the effect of ISI is avoided.

Several loading algorithms have been proposed to adjust the rate and power at each subchannel. For example Chow and Coiffi [2] proposed a method in which they distribute the bits according to the capacity of the subchannels. They have used the concept of "SNR gap approximation Γ", where $10\log\Gamma$ is the SNR gap between the subchannel capacity and the bandwidth efficiency of the real modulation block. In each iteration, they find the number of bits assigned to each subchannel and round it to the maximum integer. The criteria here, is to minimize the overall error probability.

Fisher [5] exploits the fact that the signal power and the rate at each subchannel are related. He tries to fix the data rate and transmitted signal power at each subchannel and transmit at the lowest possible error rate.

The objective of power control in wireless networks is to minimize the transmitted power when at the same time the target error probabilities are met. To do this, we should keep each mobile's SINR above a threshold called minimum protection ratio. We denote the link gain between the jth mobile and ith base station by G_{ji}, and the jth mobile transmitted power by P_j. Note that, G_{ji} encompasses the effect of shadow fading, attenuation due to distance and the frequency shift. The SINR at the ith receiver is given by

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ji}P_j + N_i}, \quad (1)$$

where N_i is the thermal noise at the ith base station. Our objective is to maintain the transmitted power as low as possible and at the same time, keep the SINRs above the threshold. Using $\Gamma_i = \gamma_i$ and the Perron- Frobenius theorem[6], the ith mobile power is updated by

$$P_i^{n+1} = \frac{\gamma_i}{G_{ii}}(\sum_{j\neq i} G_{ji}P_j^n + N_i) \quad i = 1, \ldots, M \quad (2)$$

The right hand side in (2) is a function of the interference at the ith mobile (the quantity inside the parenthesis), the link gain G_{ii}, and the target SINR. All of these parameters can be measured locally by the base station and transmitted through a feedback channel to the correspondent mobile.

3 The Proposed System

In a multicarrier system, the error probability of the whole system is affected significantly by the subchannel with the highest attenuation. Therefore in the case of frequency selective fading channel, the performance of the whole system in terms of error probability will improve slowly by increasing the transmitted power. So, in order to get a minimum overall error probability, the optimum procedure, in a fixed total power policy, is to have a uniform error probability for all of the subchannels [1].

However, in a cellular environment, there are cochannel interferences between different mobiles trying to use the same subchannel. In this situation the amount of interference at each subchannel for each mobile is directly related to the power of other mobiles using the same subchannel, and so is the SINR. Therefore, the individual loading algorithms do not come up with the optimum power and bit distribution. In this paper, our objective is to optimize the power distribution at each subchannel for all of the mobiles, so that

1-The SINR is fixed at all of the subchannels for all of the mobiles, and therefore the error probability decreases faster with growing SINR compared to that of unbalanced SINRs.

2-The total power used to achieve the aforementioned objective is minimized.

The basic idea behind this, is to allocate less power to the subchannels with better performances, and more power to the subchannels with low SINR.

We assume that there is no interchannel interferences and so the subchannels are independent. We perform the power control algorithm mentioned in previous section for each subchannel separately.

Fig. 1 depicts the proposed system for the mobiles using OFDM scheme. First, we assume a single antenna OFDM receiver at the base stations.

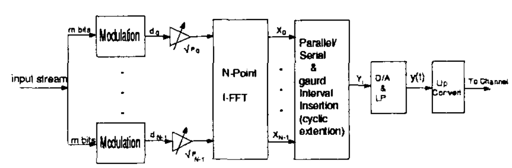


Figure 1. OFDM Transmitter using power control.

By applying the adaptive power control algorithm, we guarantee that the ratio of the desired signal power to the combination of interference and noise at lth subchannel is at least a prespecified value γ_l.

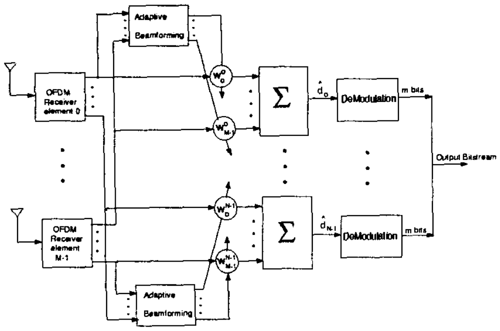


Figure 2. Antenna Array in an OFDM Receiver

In Fig. 2 , we have considered multiple antennas at the receiver. The joint beamforming and power control procedure is performed at each subchannel separately, assuming perfect orthogonalization.

The adaptive beamforming vector is calculated to minimize the total energy, when the gain toward the desired signal is constant.

We denote the power allocated to mobile i at subchannel k as P_i^k, a sample of a white Gaussian noise at subchannel k by n^k and its Fourier transform by $\hat{n}^k$ and the array response for the signal coming from mobile i to base j by $\mathbf{a}_{ij}$.

The received vector at the jth base station and kth subchannel (assuming negligible delay spread from different paths) is given by

$$\underline{\mathbf{x}}_j^k = \frac{1}{\sqrt{N}} \sum_{i=0}^{M-1} (\sum_{l=0}^{N-1} \underbrace{\sqrt{P_i^l G_{ij}^l} d_i^l e^{-j2\pi lk/N}}_{t_{ij}^l}) \underline{\mathbf{a}}_{ij} + \underline{\mathbf{n}}^k. \quad (3)$$

The resultant lth output of the FFT block at each antenna is the following vector

$$\underline{\hat{\mathbf{d}}}_j^l = \sum_{i=0}^{M-1} t_{ij}^l \underline{\mathbf{a}}_{ij} + \underline{\hat{\mathbf{n}}}^l. \quad l = 0, \ldots, N-1 \qquad (4)$$

Assuming that the noise samples are zero mean, independent Gaussian variables, and the transmitted symbols are independent and have average energy equal to 1, the output energy will be

$$\epsilon^l = \underline{\mathbf{w}}_j^{l^H} E[\underline{\hat{\mathbf{d}}}_j^l \underline{\hat{\mathbf{d}}}_j^{l^H}] \underline{\mathbf{w}}_j^l =$$

$$[\sum_{i \neq j} (P_i^l G_{ij}^l \underbrace{\underline{\mathbf{w}}_j^{l^H} \underline{\mathbf{a}}_{ij} \underline{\mathbf{a}}_{ij}^H \underline{\mathbf{w}}_j^l}_{u_{ij}^l}) + \frac{N_0}{2} \|\underline{\mathbf{w}}_j^l\|^2] + P_j^l G_{jj}^l u_{jj}^l,$$

in which the term inside the bracket is the interference plus noise and the second term is the power of the signal coming from desired direction.

The signal to noise ratio at the output of the beamformer is given by

$$\Gamma_j^l = \frac{P_j^l G_{jj}^l u_{ij}^l}{\sum_{i=0_{i \neq j}}^{M-1} (P_i^l G_{ij}^l u_{ij}^l) + \frac{N_0}{2} \|\underline{\mathbf{w}}_j^l\|^2}. \qquad (5)$$

The problem of beamforming will be

$$\underline{\mathbf{w}}_j^l = \arg\min \{\sum_{i=0_{i \neq j}}^{M-1} (P_i^l G_{ij}^l u_{ij}^l) + \frac{N_0}{2} \|\underline{\mathbf{w}}_j^l\|^2]\},$$
$$subject\ to\ \underline{\mathbf{w}}_j^{l^H} \underline{\mathbf{a}}_{ij} = 1. \qquad (6)$$

Using a Lagrange multiplier [7] the optimum weight vectors are obtained by

$$\underline{\mathbf{w}}_j^l = \frac{\mathbf{R}_j^{l^{-1}} \underline{\mathbf{a}}_{ij}}{\underline{\mathbf{a}}_{ij}^H \mathbf{R}_j^{l^{-1}} \underline{\mathbf{a}}_{ij}}. \quad l = 0, \ldots, N-1 \qquad (7)$$

when

$$\mathbf{R}_j^l = \sum_{i=0}^{M-1} (P_i^l G_{ij}^l \underline{\mathbf{a}}_{ij} \underline{\mathbf{a}}_{ij}^H) + \frac{N_0}{2} \mathbf{I} \qquad (8)$$

is the correlation matrix at subchannel l at base station j.

The following algorithm achieves the jointly optimal power allocations and adaptive beamforming at each subchannel:

1-At subchannel l, using (7) the weight vector $\underline{\mathbf{w}}_j^l(n+1)$ is computed at each receiver j such that the cochannel interference is minimized. To evaluate the correlation matrix, we use $P_i^l(n)$.

2-The updated power, $P_j^l(n+1)$, is then obtained by

$$P_j^l(n+1) = \frac{\gamma_l}{G_{jj}^l} [\sum_{i=0_{i \neq j}}^{M-1} (G_{ij}^l P_j^l(n) u_{ij}^l(n)) + \frac{N_0}{2} \|\underline{\mathbf{w}}_j^l\|^2 (n)]$$

By looking at (7) and a simple calculation, we can see that if α is the denominator of (7), $P_j^l(n+1)$ can be evaluated by $P_j^l(n+1) = \frac{\Gamma_l}{\alpha G_{jj}^l}$, too.

It has been shown that the above algorithm converges to the optimal power allocation and beamforming vectors and the solution is unique [4].

In Fig. 2, to be able to consider the subchannels independently, we had to apply the weight vectors after the OFDM receiver. Having L antennas at each beamformer and N subchannels, the complexity of each receiver would be in order of $LN \log N + NL^3$. So, an algorithm which decreases the complexity is desirable. Fig. 3 shows a proposed system in which we have reduced the number OFDM receivers and beamformers to 1. Since the weight vectors pass through the FFT block, the subchannels are not independent anymore, which means we are no longer able to consider the joint beamforming and power control at different subchannels separately. In this case we have actually moved the beamforming from frequency domain to time domain. Apparently, this system is much less complex than that of Fig. 2. Using the same parameters, the complexity of Fig. 3 is $N \log N + L^3$. For N equals 128 and L equals 4, this number will be 960 compared to 11776 for Fig. 2.

Let's consider Fig. 3 as the receiver for the j'th base station. Using the same notations, the pth output of the FFT block will be

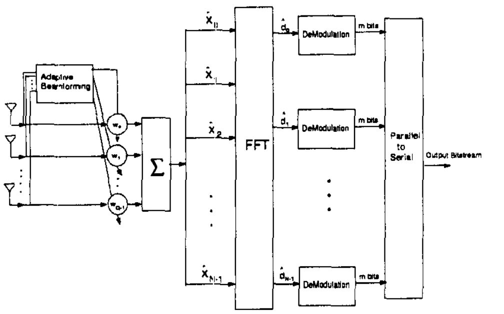


Figure 3. *Low complexity OFDM receiver with one beamformer.*

$$z^p = \sum_{i=0}^{M-1} t_i^p \underbrace{\sum_{q=0}^{Q-1} a_i^q w_q^*}_{v_i^*} + \sum_{q=0}^{Q-1} w_q^* \hat{n}_q^p = \underline{\mathbf{v}}^H \underline{\mathbf{t}}^p + \underline{\mathbf{w}}^H \underline{\hat{\mathbf{n}}}^p.$$

Using the fact that the energy of the input symbols are 1, the energy at subchannel p is $e^p = E[z^p z^{p^*}]$

$$e^p = \underline{\mathbf{v}}^H diag\,(P_i^p G_i^p)\,\underline{\mathbf{v}} + \frac{N_0}{2}\,|\underline{\mathbf{w}}|^2$$
$$= \sum_{i=0}^{M-1} P_i^p G_i^p\,|v_i|^2 + \frac{N_0}{2}\,|\underline{\mathbf{w}}|^2.$$

This is the energy at a particular subchannel which depends on the weight vector of the beamformer. Of course, it is not possible to minimize the energy at all of the subchannels simultaneously, thus we have to define a metric that is a positive combination of all e^p's ($p = 0, ..., N-1$). One such a metric is the sum of the squares of these elements, but since each e^p is actually an energy quantity, we simply need to minimize the sum of the energies:

$$\sum_{p=0}^{N-1} e^p = \underline{\mathbf{v}}^H diag\left(\sum_{p=0}^{N-1} P_i^p G_i^p\right)\underline{\mathbf{v}} + \frac{NN_0}{2}\,|\underline{\mathbf{w}}^2| =$$
$$\sum_{i=0}^{M-1} |v_i|^2 \sum_{p=0}^{N-1} P_i^p G_i^p + \frac{NN_0}{2}\,|\underline{\mathbf{w}}|^2.$$

Since the constraint for different subchannels are the same and does not depend on p, our problem will be

$$\underline{\mathbf{w}} = \arg\min\{\textstyle\sum_{i=0}^{M-1} |\underline{\mathbf{w}}^H \underline{\mathbf{a}}_{ij}|^2 \sum_{p=0}^{N-1} P_i^p G_{ij}^p + \frac{NN_0}{2}\,|\underline{\mathbf{w}}|^2\},$$
$$subject\ to\ \underline{\mathbf{w}}^H \underline{\mathbf{a}}_{jj} = 1, \qquad (9)$$

which is very similar to a normal beamforming process, except that the correlation matrix is substituted by

$$\mathbf{R} = \sum_{i=0}^{M-1} \underline{\mathbf{a}}_{ij}\underline{\mathbf{a}}_{ij}^H \sum_{p=0}^{N-1} P_i^p G_{ij}^p + \frac{NN_0}{2} I.$$

and the MVDR solution is given by (7)

setup	antenna	power	policy	SINR
1	single	adaptive	-	fixed
2	single	uniform	-	variable
3	multiple	adaptive	joint	fixed
4	multiple	uniform	-	variable
5	multiple	adaptive	tandem	fixed
6	multiple	adaptive	LC-joint	fixed
7	multiple	uniform	LC	variable
8	multiple	adaptive	LC-tandem	fixed

Table 1. simulation System setups

4 Simulation Results

We have used a 36 base stations wireless network (one mobile for each), 64 subchannels QPSK OFDM systems, a Gaussian white noise with 1MHz noise bandwidth ($4X10^{-15}$ variance) and a range of 0dB to 20dB for SINR and 12Mbit data file.

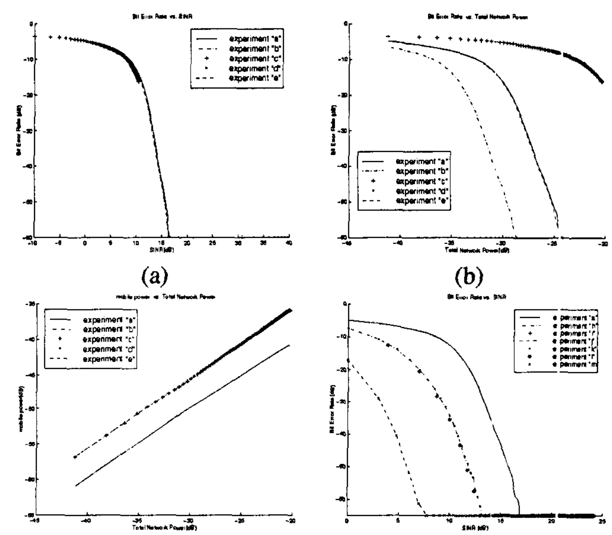
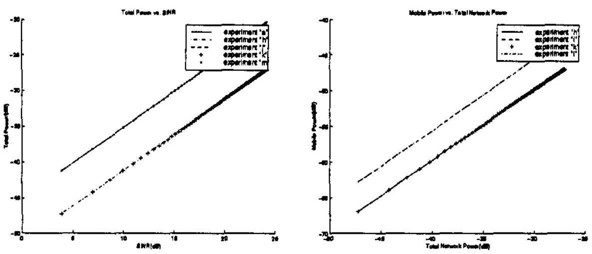


Figure 4. (a) The bit error rate (dB) versus desired SINR (dB). (b)The bit error rate (dB) versus total network power (dB) .(c) The mobile power (dB) versus total network power (dB) for system setup 1(d) The bit error rate (dB) versus SINR (dB) comparing system setups.

Several experiments have been performed for each value of SINR. Different path gains are used for different mobiles and also for different subchannels to reflect the effect of frequency selective fading channels, but at each subchannel, flat fading has been considered. Table 1 shows the system setups for different experiments. Note that by "tandem" we mean, first performing adaptive power control and then beamforming at each receiver without updating them jointly, also LC stands for Low Complex which refers to Fig. 3. In experiment a we have used setup 1 and a typical base station. Using system setup 2, the SINR at all of the base stations have been evaluated and a typical base station, the worst, the best and the average base station are used in experiments b, c,d and e, respectively, where the same total power as that of a is divided uniformly among different mobiles and all of the subchannels. Experiments h, i, j, k, l and m use a typical base station along with setups 3 through 8 respectively.



(a) (b)

Figure 5. (a) The Total Power (dB) versus SINR (dB) comparing system setups. (b) Mobile power (dB) versus total network power (dB) comparing setup 3,4, 6 and 7

As it is observable from Fig. 4.a, in the single antenna case, the bit error rate versus SINR for different experiments follows the same pattern. The reason is that we are using the same modulation scheme for all of the subchannels and all of the mobiles. By fixing the modulation scheme, the performance (BER) of a link is determined by its SINR. Fig. 4.b shows that using a fixed power policy, some base stations can achieve much lower bit error rate than that of case a. However, compared to the adaptive power allocation scheme, the performance of some other cases is much lower. The point is, by power allocation we specify a lower bound for SINR at all of the subchannels and so all of the subchannels at all of the mobiles have SINR in the vicinity of the desired value while the total transmitted power is minimized. If we divide this power uniformly, it is natural that different subchannels at different mobiles, based on their channel responses, perform differently. By power control, all of the subchannels perform at the same level of performance. Also, it is clear from Fig. 4.c that those mobiles which perform better in terms of BER in a fixed power policy, consume more power than the adaptive policy and so the overall performance is depreciated. Fig. 4.d shows the BER versus SINR for the cases using multiple antenna elements at the receiver. The joint policy performs better than single antenna, but lower than the tandem policy, with the cost of much higher total power for the network(Fig. 5.a). So the overall performance of the joint policy, in terms of total network power versus a prespecified SINR is optimized. Fig. 5.b shows that having the same total power, the mobile power is lower in adaptive power allocation scheme. Through simulation, It was observed that when SINR approaches to 19dB, no fixed power allocation is feasible. The joint power control and beamforming scheme converges faster than the single antenna case, in terms of number of mobiles and the distance between the mobile and the base station (base dimension). In other words the frequency reuse at each subchannel has increased significantly. Fig. 4.d to 5.b compare the performance of the receivers of Fig. 2 and 3 Clearly, the former has lower bit error rate, but withe higher complexity. The performance of Fig. 3 is very close to 2 while its complexity is significantly lower.

5 Conclusion

An iterative and adaptive joint power control and beamforming algorithm proposed in [4] has been used to fix the signal to noise and interference ratio and minimize the cochannel interference at each subchannel of an OFDM system in a network of mobiles. We have used the total mobile power and the total transmitted power in the whole network as another measures of performance. The proposed systems try to optimize the tradeoff between the transmitted power and bit error rate for the whole network, not for a single transmitter. We have shown by using adaptive antenna at an OFDM base station, the effects of interference and white noise diminishes more significantly. This also speeds up the convergence of the iterative algorithm. By moving the beamformer from frequency domain to time domain we have been able to reduce the complexity of the receiver significantly.

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