On Relay Nodes Deployment for Distributed Detection in Wireless Sensor Networks

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***Abstract*—In this paper, the problem of deploying relay nodes in wireless sensor networks will be considered. A system consisting of a set of sensor nodes communicating to a fusion center, where decisions are made, is considered. Based on our system model assumptions, some sensor nodes will provide "less-informative" measurements to the fusion center about the state of nature; we consider relay nodes deployment in the sensor network instead of the less-informative sensor nodes to forward the measurements of the other, "more-informative" sensor nodes. This introduces a new tradeoff in the system design between the number of measurements sent to the fusion center and the reliability of the more-informative measurements, which is enhanced by deploying more relay nodes in the network. We will analyze the performance of two protocols over Rayleigh flat-fading channels. In Protocol I, each sensor node directly transmits its measurement to the fusion center. In Protocol II, relay nodes will be used instead of the less-informative sensor nodes to forward the measurements of the more-informative sensor nodes. Hence, in Protocol II, the reliability of the more-informative measurements is enhanced at the expense of having fewer measurements sent to the fusion center and this creates the tradeoff between the number of measurements available at the fusion center and the reliability of the measurements.**

I. INTRODUCTION

The study of sensor networks has gained a lot of interest due to their wide range of applications. The applications of sensor networks include monitoring environmental conditions, military applications, health monitoring, and many other applications. A sensor network is composed of a set of sensor nodes used to monitor a certain state of nature. Usually the sensor nodes have limited power, limited computational capacities, and limited memory. Also, sensor nodes are prone to failures and this has raised a lot of challenges for the design of communication protocol for sensor networks [1].

In this paper, the problem of distributed detection with relay nodes deployment in the wireless sensor networks is considered. There exists a plethora of works on distributed detection in sensor network. In [2], the authors considered the problem of how to determine the density of sensor nodes in a linear network where nodes are placed on a line. They study the problem of whether to employ many low-cost, low-power sensors or few high-cost, high-power sensors. In [3], closed-form expressions for the error exponents of the Neyman-Pearson detector are derived for the detection of Gauss-Markov signals corrupted by noise. The work in [4] considered the problem of distributed detection with a rate constraint.

In this paper, the distributed detection problem when relay nodes are deployed in the wireless sensor network to forward sensor nodes' data is considered. In [5], the classical relay channel model based on additive white Gaussian noise (AWGN) channels was presented. In [6] and [7], various node cooperation protocols were proposed and outage probability analyses for these protocols were provided. Considering the application of relaying schemes in sensor networks, [8] has considered the use of relaying to improve the energy-efficiency of the sensor network. The work in [8] also proposed a consensus protocol and analyzed its energy consumption if cooperation is present to improve the energy-efficiency of the sensor network.

Our interest in this paper will be focused on how to deploy relay nodes in the sensor networks. Based on our model assumption, some sensor nodes measurements will provide more information to the fusion center. So some sensors are assumed to be "more-informative" and some sensors are assumed to be "less-informative" to the fusion center[1]. The use of relay nodes, instead of the sensor nodes that are less-informative to the fusion center, to relay measurement for the more-informative sensor nodes will be considered. In other words, assigning the system resources, such as time slots in a TDMA based system, allocated for the less-informative sensor nodes to relay nodes. Clearly, allowing some nodes to relay the measurements of the more-informative sensors will enhance the reliability of these measurements at the expense of sending fewer measurements to the fusion center. There will be a tradeoff between the number of measurements sent to the fusion center and the reliability of these measurements.

II. SYSTEM AND DATA MODELS

In this section, the system model for the wireless sensor network is presented. The sensor network is assumed to have N sensor nodes that are used to monitor a certain phenomenon. The sensor nodes send their sensed measurements to a fusion center to make decisions about the state of nature observed

[1]In the following sections it will become clear what do "more-informative" and "less-informative" mean.

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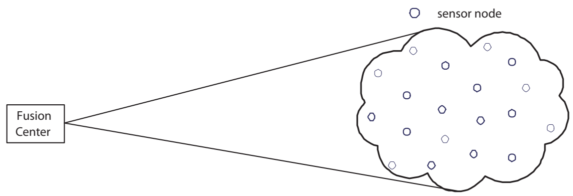

Fig. 1. A Schematic Diagram for the Wireless Sensor Network.

by the sensor network. In this paper, the sensor nodes are assumed to be dumb, i.e., the sensor nodes do not have processing capabilities of the sensed measurements, which can be due to lack of knowledge of the measurement data models under each hypothesis or due to limited processing capabilities of the sensor nodes [9]. In other words, the sensor nodes sense the medium and directly transmit their measurements to the fusion center where decisions are made. The wireless sensor networks is as depicted in Fig. 1. We assume a binary hypotheses detection problem, i.e., the fusion center makes decisions between two hypotheses, namely, H_0 and H_1.

The i-th sensor node measurement is x_i, $i = 1, \cdots, N$. The x_i's are assumed to be mutually independent under each hypothesis. The data model under each hypothesis is given by

$$\begin{aligned} H_0 &: \; x_i \sim \mathcal{CN}\left(0, \sigma^2\right) \\ H_1 &: \; x_i \sim \mathcal{CN}\left(m_i, \sigma^2\right), \end{aligned} \tag{1}$$

where σ^2 can be thought of as the measurement noise variance at any sensor node. The notation $x \sim \mathcal{CN}\left(m, \sigma^2\right)$ is used to denote that x is a complex Gaussian random variable with mean m and variance $\sigma^2/2$ per dimension.

In the sequel, the performance of two transmission protocols from the sensor nodes to the fusion center will be compared. In the first protocol, which is denoted by Protocol I, each sensor node directly transmits its measurement to the fusion center without the help of any other node in the network. In the second protocol, which is denoted by Protocol II, relay nodes are used instead of some of the less-informative sensor nodes to forward information of the more-informative sensor nodes. We will derive expressions of the probability of detection error P_e for the previous two protocols. Based on the derived expressions, it can be decided which of the two protocols will result in a better performance in terms of P_e.

A. Protocol I System Model

In Protocol I, each sensor node directly transmits its measurement to the fusion center. Let h_{s_iF} denote the channel gain from the i-th sensor node to the fusion center, which is modeled as zero-mean circularly symmetric complex Gaussian random variable with variance $1/2$ per dimension, i.e., Rayleigh flat-fading is assumed. The channel gains from the sensor nodes to the fusion center are assumed to be independent. The received data at the fusion center due to the i-th sensor node transmission is given by

$$y_{s_iF} = h_{s_iF}\sqrt{P_i}x_i + n_{s_iF}, \tag{2}$$

where P_i is selected to satisfy a power constraint at the sensor node and n_{s_iF} is a receiver additive white Gaussian noise. The term n_{s_iF} is modeled as zero-mean circularly symmetric complex Gaussian random variable with variance $N_0/2$ per dimension.

B. Protocol II System Model

In Protocol II, relay nodes will be deployed in the network instead of the sensor nodes whose measurements do not provide the fusion center with a lot of information about the observed phenomenon. Again, dumb sensor nodes are assumed, which means that a sensor node is not able to process the sensed measurement.

If node j works as a relay for sensor i, then the received signal at the fusion center due to node j transmission is given by

$$y_{jF} = h_{jF}\sqrt{P_i}x_i + n_{jF}, \quad j \in \mathcal{R} \tag{3}$$

where for simplicity of analysis the noise from node i to node j is neglected. Hence, node j transmits a clean version of the measurement of node i to the fusion center[2]. h_{jF} denotes the channel gain from the j-th relay node to the fusion center and is modeled as zero-mean circularly symmetric complex Gaussian random variable with variance $1/2$ per dimension and $\mathcal{R}$ denotes the subset of relay nodes.

III. Performance Analysis

We consider a large sensor network where the number of sensor nodes N is very large, which enables the derivation of asymptotic approximations for the probability of detection error expressions. For simplicity of presentation, the sensor network is assumed to be divided into two subsets of sensor nodes of equal cardinality, namely, $\mathcal{S}$ and $\mathcal{R}$, each has $N/2$ sensor nodes. However, the analysis presented here can be generalized if we have a different partitioning of the sensor nodes. Sensor nodes in subset $\mathcal{S}$ have a mean of m_S and sensor nodes in subset $\mathcal{R}$ have a mean of m_R.

Let $P_{e,I}^{Ray}$ denote the probability of detection error of Protocol I over Rayleigh flat-fading channels. The probability of detection error is defined as $P_{e,I}^{Ray} = \Pr\{\hat{H} \neq H\}$, where H is the true state of nature and $\hat{H}$ is the estimated state of nature at the fusion center.

Let $\pi_0 = \Pr\{H = H_0\}$ and $\pi_1 = \Pr\{H = H_1\}$ denote the prior probabilities. Without loss of generality, we assume that $\pi_0 = \pi_1 = 1/2$. The variable P_i in (2) is selected such

[2] If the amplify-and-forward protocol is used at the relay nodes and assuming that the distance between the sensor node and the node that relays its measurement is much less than the distance between any sensor node and the fusion center then the noise coming from the sensor node to relay node communication link can be neglected compared to the noise coming from the sensor (relay) node to fusion center link. In the case of amplify-and-forward protocol, the signal to noise ratio (SNR) of sensor-relay-fusion center link will be a scaled harmonic mean of the sensor-relay and relay-fusion center links SNRs, which can be tightly approximated to be the SNR of the relay-fusion center link for the case of relay node very close to the sensor node [10]. This enables us to neglect the noise coming from the sensor-relay communication link.

that the average power of each sensor node equals a power constraint P. Therefore, we have

$$P = \pi_0 P_i \sigma^2 + \pi_1 P_i \left(m_i^2 + \sigma^2\right) \rightarrow P_i = \frac{P}{\sigma^2 + \frac{1}{2} m_i^2}. \quad (4)$$

1) Protocol I Probability of Detection Error: Let $P_{e,I}^{Ray}$ denote the probability of detection error of Protocol I over Rayleigh flat-fading channels. The data model for the received data under each hypothesis is given by

$$\begin{aligned} &H_0 : \ y_{s_iF} \sim \mathcal{CN}\left(0, P_i |h_{s_iF}|^2 \sigma^2 + N_0\right) \\ &H_1 : \ y_{s_iF} \sim \mathcal{CN}\left(\sqrt{P_i} h_{s_iF} m_i, P_i |h_{s_iF}|^2 \sigma^2 + N_0\right), \end{aligned} \quad (5)$$

where each m_i is either m_S or m_R.

With the probability of detection error as a performance measure and assuming perfect channel state information (CSI) at the fusion center, the optimal decision rule at the fusion center is the LR test given by

$$\frac{e^{-\sum_{i=1}^{N} \frac{1}{P_i |h_{s_iF}|^2 \sigma^2 + N_0} \left| y_{s_iF} - \sqrt{P_i} h_{s_iF} m_i \right|^2}}{e^{-\sum_{i=1}^{N} \frac{1}{P_i |h_{s_iF}|^2 \sigma^2 + N_0} \left| y_{s_iF} \right|^2}} \underset{\hat{H}=H_0}{\overset{\hat{H}=H_1}{\gtrless}} 1, \quad (6)$$

where we assumed equal priors, i.e., $\pi_0 = \pi_1 = 1/2$. The decision rule in (6) can be simplified to

$$\begin{aligned} &\sum_{i=1}^{N} \frac{1}{P_i |h_{s_iF}|^2 \sigma^2 + N_0} \left(\sqrt{P_i} y_{s_iF} h_{s_iF}^* m_i^* + \sqrt{P_i} y_{s_iF}^* h_{s_iF} m_i \right) \\ &\underset{\hat{H}=H_0}{\overset{\hat{H}=H_1}{\gtrless}} \sum_{i=1}^{N} \frac{1}{P_i |h_{s_iF}|^2 \sigma^2 + N_0} P_i \left|m_i\right|^2 \left|h_{s_iF}\right|^2. \end{aligned} \quad (7)$$

The probability of detection error expression can be found to be given by

$$\begin{aligned} &P_{e,I}^{Ray} = \\ &E\left\{ Q\left(\frac{1}{2} \sqrt{ \sum_{i \in \mathcal{S}} \frac{P_S \left|h_{s_iF}\right|^2 \left|m_S\right|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + N_0} + \sum_{i \in \mathcal{R}} \frac{P_R \left|h_{s_iF}\right|^2 \left|m_R\right|^2}{P_R \left|h_{s_iF}\right|^2 \sigma^2 + N_0} } \right) \right\}, \end{aligned} \quad (8)$$

where

$$P_S = \frac{P}{\sigma^2 + \frac{1}{2} m_S^2} \quad \text{and} \quad P_R = \frac{P}{\sigma^2 + \frac{1}{2} m_R^2}.$$

The expectation in (8) is taken over the channel statistics. Finding a closed-form expression for the expectation in (8) is very difficult even for the simple case of having $N = 2$. This motivates us to consider a large sensor network in which the number of sensor nodes is very large which enables the calculation of asymptotic approximation for the probability of detection error. For such a large network, define the random variable u as

$$u = \sum_{i=1}^{N} \frac{P_i \left|h_{s_iF}\right|^2 \left|m_i\right|^2}{P_i \left|h_{s_iF}\right|^2 \sigma^2 + N_0}, \quad (9)$$

which is the summation inside the argument of the Q-function of (8). The random variable u is the summation of $N/2$ i.i.d. random variables (the expression in (9) is the summation of $N/2$ i.i.d., where each random variable is the sum of an element from the subset $\mathcal{S}$ and an element from the subset $\mathcal{R}$), which can be approximated to be a Gaussian random variable. This results from using the central limit theory (CLT) [11]. The probability of detection error is now given by

$$P_{e,I}^{Ray} = E\left\{ Q\left(\frac{1}{2} \sqrt{u} \right) \right\}. \quad (10)$$

To get the approximation for the expression in (10), we need to calculate the mean and the variance of the random variable u. The mean of u can be found as follows

$$m_u = \frac{N}{2} \cdot E\left\{ \frac{P_S \left|h_{s_iF}\right|^2 \left|m_S\right|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + N_0} + \frac{P_R \left|h_{s_jF}\right|^2 \left|m_R\right|^2}{P_R \left|h_{s_jF}\right|^2 \sigma^2 + N_0} \right\} \quad (11)$$

for some $i \in \mathcal{S}$ and $j \in \mathcal{R}$. Define the random variable $h = \left|h_{s_iF}\right|^2$ for some i. Under our model assumptions, h follows, for any i, an exponential distribution with a probability density function (pdf) given by $P(h) = e^{-h}, \ h \geq 0$. The mean of the random variable u can be found to be

$$\begin{aligned} m_u = &\frac{N}{2\sigma^2} \left(\left|m_S\right|^2 + \left|m_R\right|^2 - \frac{N_0 \left|m_S\right|^2}{P_S \sigma^2} e^{\frac{N_0}{P_S \sigma^2}} \mathbf{\Gamma}\left(0, \frac{N_0}{P_S \sigma^2}\right) \right. \\ &\left. - \frac{N_0 \left|m_R\right|^2}{P_R \sigma^2} e^{\frac{N_0}{P_R \sigma^2}} \mathbf{\Gamma}\left(0, \frac{N_0}{P_R \sigma^2}\right) \right), \end{aligned} \quad (12)$$

where $\mathbf{\Gamma}(.,.)$ is the incomplete Gamma function defined as [12]

$$\mathbf{\Gamma}(a, \mu) = \int_{\mu}^{\infty} t^{a-1} e^{-t} dt, \quad \mu > 0. \quad (13)$$

Let δ_u^2 denote the variance of the random variable u. The variance of u can be calculated to be (details omitted due to space limitations)

$$\begin{aligned} \delta_u^2 = \frac{N}{2} \Bigg(&\frac{\left|m_S\right|^4}{\sigma^4} \Bigg[\frac{N_0}{P_S \sigma^2} - \frac{N_0^2}{P_S^2 \sigma^4} e^{\frac{N_0}{P_S \sigma^2}} \mathbf{\Gamma}\left(0, \frac{N_0}{P_S \sigma^2}\right) \\ &- \frac{N_0^2}{P_S^2 \sigma^4} e^{\frac{2N_0}{P_S \sigma^2}} \left(\mathbf{\Gamma}\left(0, \frac{N_0}{P_S \sigma^2}\right) \right)^2 \Bigg] \\ &+ \frac{\left|m_R\right|^4}{\sigma^4} \Bigg[\frac{N_0}{P_R \sigma^2} - \frac{N_0^2}{P_R^2 \sigma^4} e^{\frac{N_0}{P_R \sigma^2}} \mathbf{\Gamma}\left(0, \frac{N_0}{P_R \sigma^2}\right) \\ &- \frac{N_0^2}{P_R^2 \sigma^4} e^{\frac{2N_0}{P_R \sigma^2}} \left(\mathbf{\Gamma}\left(0, \frac{N_0}{P_R \sigma^2}\right) \right)^2 \Bigg] \Bigg). \end{aligned} \quad (14)$$

Using the Gaussian approximation for the random variable u, the probability of detection error for large N can be approximated as

$$P_{e,I}^{Ray} = E\left\{ Q\left(\frac{1}{2} \sqrt{u} \right) \right\} \approx \frac{1}{\pi} \int_{\theta=0}^{\frac{\pi}{2}} e^{-\left(\frac{m_u}{8 \sin^2 \theta} + \frac{\delta_u^2}{128 \sin^4 \theta} \right)} d\theta, \quad (15)$$

where we have used the special property of the Q-function as $Q(u) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{u^2}{2 \sin^2 \theta}} d\theta$ [13]. The integration in equation

(15) can be easily computed using any numerical integration algorithm. Equation (15) provides an approximation for the probability of detection error of Protocol I over Rayleigh flat-fading channels.

2) Protocol II Probability of Detection Error: In this section, we will compute an approximate expression for the probability of detection error of Protocol II over Rayleigh flat-fading channels.

In Protocol II, each sensor from the subset $\mathcal{S}$ will be assigned a relay node to forward its measurement. In this case, sensor nodes from the subset $\mathcal{R}$ are not used and their resources are assigned to relay nodes. Let $\mathcal{L}$ denote the subset of relay nodes where $|\mathcal{L}| = N/2$. This enables the definition of the set $\mathcal{O}$ of size $N/2$ such that $\mathcal{O} = \{(i,j): \ i \in \mathcal{S}, \ j \in \mathcal{L}, \text{ node } j \text{ works as a relay for sensor } i\}$.

Now, we start the probability of detection error analysis at the fusion center. Define the 2×1 received data vector $\mathbf{y}_{(i,j)} = [y_{s_iF}, y_{jF}]^T$ and the mean vector $\mathbf{m}_{(i,j)} = [\sqrt{P_i} h_{s_iF} m_i, \sqrt{P_i} h_{jF} m_i]^T, \ (i,j) \in \mathcal{O}$. In Protocol II, the components of the vector $\mathbf{y}_{(i,j)}$ are correlated since the measurement of sensor i will be transmitted by relay node j. Therefore, the probability density function of the vector $\mathbf{y}_{(i,j)}$ under each hypothesis is given by

$$\begin{aligned} H_0 &: \ \mathbf{y}_{(i,j)} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{C}_{(i,j)}\right) \\ H_1 &: \ \mathbf{y}_{(i,j)} \sim \mathcal{N}\left(\mathbf{m}_{(i,j)}, \mathbf{C}_{(i,j)}\right), \end{aligned} \tag{16}$$

where

$$\mathbf{C}_{(i,j)} = \begin{pmatrix} P_i \left|h_{s_iF}\right|^2 \sigma^2 + N_0 & P_i h_{s_iF} h_{jF}^* \sigma^2 \\ P_i h_{s_iF}^* h_{jF} \sigma^2 & P_i \left|h_{jF}\right|^2 \sigma^2 + N_0 \end{pmatrix} \tag{17}$$

is the auto-covariance matrix of the vector $\mathbf{y}_{(i,j)}$ and is the same under both hypotheses. Note that under our data model assumption of having independent measurements at the sensor nodes the vectors $\mathbf{y}_{(i,j)}$ and $\mathbf{y}_{(k,l)}$, for (i,j) and $(k,l) \in \mathcal{O}$, are mutually independent for $(i,j) \neq (k,l)$.

Using the probability of detection error as a performance measure, the optimal decision rule at the fusion center is the LR test which can be simplified to

$$\begin{aligned} &\sum_{(i,j)\in\mathcal{O}} \left(\mathbf{m}_{(i,j)}^{\mathcal{H}} \mathbf{C}_{(i,j)}^{-1} \mathbf{y}_{(i,j)} + \mathbf{y}_{(i,j)}^{\mathcal{H}} \mathbf{C}_{(i,j)}^{-1} \mathbf{m}_{(i,j)} \right) \\ &\qquad \underset{\hat{H}=H_0}{\overset{\hat{H}=H_1}{\gtrless}} \sum_{(i,j)\in\mathcal{O}} \mathbf{m}_{(i,j)}^{\mathcal{H}} \mathbf{C}_{(i,j)}^{-1} \mathbf{m}_{(i,j)}, \end{aligned} \tag{18}$$

where $(\cdot)^{\mathcal{H}}$ denotes the Hermitian transpose.

The probability of detection error of Protocol II can now be given as

$$\begin{aligned} P_{e,II}^{Ray} &= E\left\{ Q\left(\frac{1}{2} \sqrt{\sum_{(i,j)\in\mathcal{O}} \mathbf{m}_{(i,j)}^{\mathcal{H}} \mathbf{C}_{(i,j)}^{-1} \mathbf{m}_{(i,j)}} \right) \right\} \\ &= E\left\{ Q\left(\frac{1}{2} \sqrt{\sum_{(i,j)\in\mathcal{O}} \frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0}} \right) \right\}, \end{aligned} \tag{19}$$

where $P_i = P_S$ for all i since $i \in \mathcal{S}$.

It is very difficult to get a closed-form expression for $P_{e,II}^{Ray}$ in (19). Again, we make the assumption of large sensor network to get an approximate expression for the probability of detection error in this case. To get that expression, define the random variable w as

$$w = \sum_{(i,j)\in\mathcal{O}} \frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0}, \tag{20}$$

which is the summation in the argument of the Q-function in (19). The probability of detection error is now given by

$$P_{e,II}^{Ray} = Q\left(\frac{1}{2}\sqrt{w}\right). \tag{21}$$

The random variable w is the summation of $N/2$ i.i.d. random variables that can be approximated for large N to be a Gaussian random variable by applying the CLT. To get the approximate expression for the probability of detection error we need to calculate the mean and the variance of w. The mean m_w of w is given by

$$m_w = \frac{N}{2} \cdot E\left\{ \frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0} \right\} \tag{22}$$

for some $(i,j) \in \mathcal{O}$. Define $h = \left|h_{s_iF}\right|^2$ and $t = \left|h_{jF}\right|^2$. The random variables h and t are independent, exponential random variables. Hence, m_w can be calculated to be

$$m_w = \frac{N |m_S|^2}{2\sigma^2} \left(1 - \frac{N_0}{P_S\sigma^2} e^{\frac{N_0}{P_S\sigma^2}} \int_{t=o}^{\infty} \mathbf{\Gamma}\left(0, t + \frac{N_0}{P_S\sigma^2}\right) dt \right), \tag{23}$$

where the last integral can be efficiently evaluated using any numerical integration algorithm.

The variance δ_w^2 of the random variable w can be calculated as

$$\begin{aligned} \delta_w^2 = \frac{N}{2} \Bigg[& E\left\{ \left(\frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0} \right)^2 \right\} \\ & - \left(E\left\{ \frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0} \right\} \right)^2 \Bigg] \end{aligned} \tag{24}$$

for some $(i,j) \in \mathcal{O}$. To evaluate the expectations in (24), we need to calculate the expectation

$$\begin{aligned} & E\left\{ \left(\frac{P_S \left|h_{s_iF}\right|^2 |m_S|^2 + P_S \left|h_{jF}\right|^2 |m_S|^2}{P_S \left|h_{s_iF}\right|^2 \sigma^2 + P_S \left|h_{jF}\right|^2 \sigma^2 + N_0} \right)^2 \right\} \\ = & \frac{|m_S|^2}{\sigma^2} \Bigg(1 - \left(\frac{N_0}{P_S\sigma^2}\right)^2 e^{\frac{N_0}{P_S\sigma^2}} \mathbf{\Gamma}\left(0, \frac{N_0}{P_S\sigma^2}\right) \\ & - \frac{N_0}{P_S\sigma^2} \left(2 + \frac{N_0}{P_S\sigma^2}\right) e^{\frac{N_0}{P_S\sigma^2}} \int_0^{\infty} \mathbf{\Gamma}\left(0, t + \frac{N_0}{P_S\sigma^2}\right) dt \Bigg). \end{aligned} \tag{25}$$

From (23) and (25) the value of δ_w^2 can be calculated.

Following a similar analysis to the one presented in the previous section, we can get an approximate expression for the probability of detection error as

$$P_{e,II}^{Ray} \approx \frac{1}{\pi} \int_{\theta=0}^{\frac{\pi}{2}} e^{-\left(\frac{m_w}{8\sin^2\theta} + \frac{\delta_w^2}{128\sin^4\theta}\right)} \, d\theta. \tag{26}$$

To compare the performances of the two protocols, the values of the approximate expressions for the probability of detection error given in (15) and (26) are used to decide which of the two protocols performs better in terms of P_e.

Returning back to the exact error expressions given in (8) and (19), we have the following inequality

$$E\left\{Q\left(\frac{1}{2}\sqrt{\sum_{i=1}^{N} \frac{P_S\left|h_{s_iF}\right|^2\left|m_S\right|^2}{P_S\left|h_{s_iF}\right|^2\sigma^2+N_0}}\right)\right\} <$$
$$E\left\{Q\left(\frac{1}{2}\sqrt{\sum_{(i,j)\in\mathcal{O}} \frac{P_S\left|h_{s_iF}\right|^2\left|m_S\right|^2+P_S\left|h_{jF}\right|^2\left|m_S\right|^2}{P_S\left|h_{s_iF}\right|^2\sigma^2+P_S\left|h_{jF}\right|^2\sigma^2+N_0}}\right)\right\}$$
$$< E\left\{Q\left(\frac{1}{2}\sqrt{\sum_{i\in\mathcal{S}} \frac{P_S\left|h_{s_iF}\right|^2\left|m_S\right|^2}{P_S\left|h_{s_iF}\right|^2\sigma^2+N_0}}\right)\right\}, \tag{27}$$

which means that

$$P_{e,I}^{Ray}\left(\left|m_R\right| = \left|m_S\right|\right) < P_{e,II}^{Ray} < P_{e,I}^{Ray}\left(m_R = 0\right). \tag{28}$$

Equation (28) tells the story. For the extreme case of having $m_R = 0$, Protocol II results in a better performance if compared to Protocol I. In this case, the measurements from sensors that have a zero-mean measurement convey no information to the fusion center. Therefore, in this case it is better to use relay nodes, instead of sensor nodes with zero-mean measurements, to forward information for the other more-informative sensor nodes. For the other case of having $|m_R| = |m_S|$, the measurements coming from the different sensor nodes are of equal importance to the fusion sensors. As such, Protocol I performs better than Protocol II as what can be seen from (28). Between these two extremes, and depending on the value of $|m_R|$ and other system parameters, Protocol I may preform better than Protocol II and vice versa.

IV. Simulation Results

In this section, we present some simulation results. In all simulations we will normalize the power at each sensor node to be $P = 1$ and $m_S = 1$, which is the mean of the more-informative sensor nodes under hypothesis H_1.

We simulate a two-sensor network over Rayleigh flat-fading channels. Fig. 2 shows the probability of detection error versus P/N_0 for the case of having a measurement noise of variance $\sigma^2 = 0.1$. From Fig. 2, it is clear that Protocol I always performs better than Protocol II for the case of having $m_R = 1$ as explained before. From Fig. 2, we can see that Protocol II is always better than Protocol I for the case of having $m_R = 0$. For any value of m_R that is between 0 and 1, deciding which protocol will perform better depends on other system parameters such as the measurement noise and communication noise variances. In Fig. 2 and as P/N_0 increases we can see that Protocol II saturates to a probability of detection error level that equals the error level of Protocol I for the case $m_R = 0$. As P/N_0 increases the system performance will be limited by the measurement noise and hence, having a relay node to forward the measurement of the first sensor will not improve the system performance (in this case, the received signals from the sensor and the relay nodes will be almost the same and hence, there will no gain for Protocol II over the case of having $m_R = 0$). In this case of very high P/N_0, it is better to have the second sensor sending its measurement to the fusion center instead of using a relay to forward the measurement of the first sensor.

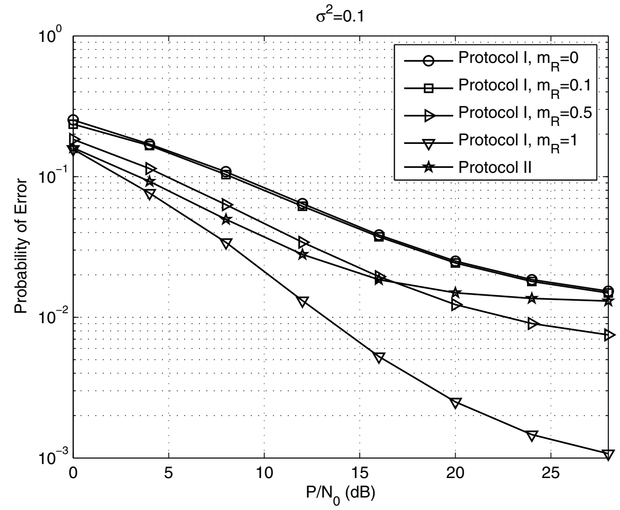

Fig. 2. The probability of detection error versus P/N_0 (dB) for a two-sensor network over wireless fading channels for the case of having a measurement noise of variance $\sigma^2 = 0.1$.

Fig. 3 shows the probability of detection error versus P/σ^2 for the case of having $P/N_0 = 10$ dB. In Fig. 3, Protocol II is always better than Protocol I for the case of having $m_R = 0$ as expected. Also, Protocol I is better than Protocol II for the case of having $m_R = 1$. As P/σ^2 becomes very large the performance of Protocol II approaches that of Protocol I with $m_R = 1$. In this case of very high P/σ^2 the system performance will be limited by the communication noise rather than the measurement noise. In this case the signal from the relay node will appear as a new measurement with mean equals 1 under hypothesis H_1 and this is why the performance of Protocol II approaches the performance of Protocol I with $m_R = 1$. Note that As P/σ^2 becomes very large the performance of Protocol I with any $|m_R| > 0$ approaches the same error value as that of Protocol I with $m_R = 1$. The reason for that is because we assume all nodes to have the same power for transmission. At very high P/σ^2, scaling the measurement by a factor to meet the power constraint, and because we have a very low level of measurement noise, will make the signals transmitted from all of the sensor nodes to

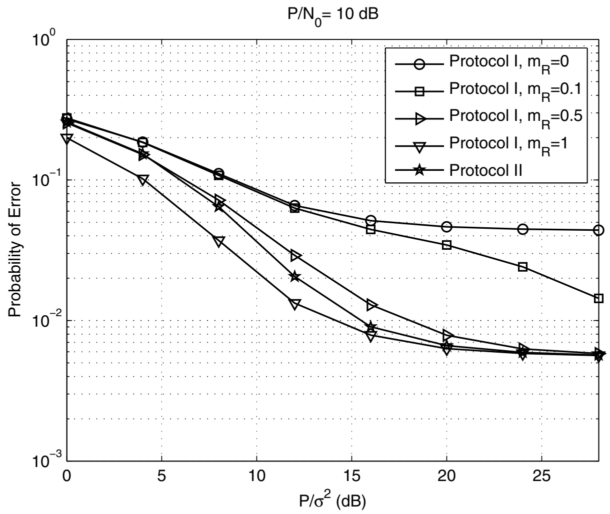

Fig. 3. The probability of detection error versus P/σ^2 (dB) for a two-sensor network over wireless fading channels for the case of having a communication signal-to-noise ratio of variance $P/N_0 = 10$ dB.

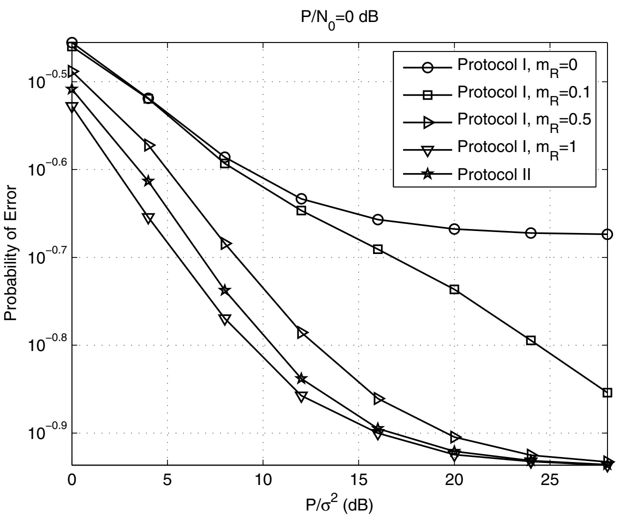

Fig. 4. The probability of detection error versus P/σ^2 (dB) for a two-sensor network over wireless fading channels for the case of having a communication signal-to-noise ratio of variance $P/N_0 = 0$ dB.

be almost the same.

Finally, Fig. 4 shows the probability of detection error versus P/σ^2 for the case of having $P/N_0 = 0$ dB. Again, the observations that were made for Fig. 3 also applies for Fig. 4. As P/σ^2 tends to infinity, the performance of Protocol II approaches that of Protocol I with $m_R = 1$.

V. Conclusions

In this paper, we have considered the problem of distributed detection over wireless fading channels with the deployment of relay nodes. We have considered a system model where some sensor nodes convey more information about the state of nature to the fusion center than some other sensor nodes. We have considered the performance of two protocols, Protocol I where each sensor directly transmits its measurement to the fusion center and Protocol II where relay nodes are used instead of the sensor nodes that are less-informative to the fusion center to forward the measurements of the other more-informative sensor nodes. We compare the performances of the two protocols using the probability of detection error as a performance measure. By comparing the performance of the two protocols, we can see that a tradeoff exists between the number of measurements sent to the fusion center and the reliability of the more-informative measurements. Protocol I provides the fusion center with more measurements and Protocol II has the advantage of increased reliability of the more-informative measurements. In general, if all of the sensor measurements are of equal importance then it is always better for each sensor to send its measurement to the fusion center rather than to use relay nodes. By deriving probability of detection error expressions we can compare the two protocols performance at any system operating parameters to decide which of the two protocols performs better.

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