Distortion Exponents for Different Source-Channel Diversity Achieving Schemes over Multi-Hop Channels

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Abstract— **The performance limits of multimedia systems combining source (multiple description) coding and channel coding with user cooperation diversity over multi-hop channels is studied. Performance is measured through the distortion exponent, which measures the rate of decay of the end-to-end distortion at asymptotic high SNRs. Two implementations for user cooperation are considered: amplify-and-forward and decode-and-forward. Results comparing different source and channel coding schemes show that optimum channel coding diversity provides the best performance, followed by source coding diversity. The results also show that at low bandwidth expansion factor, source encoding distortion is the main limiting factor. As the bandwidth expansion factor increases, user cooperation diversity is the main limiting factor, thus, the distortion exponent could be improved by increasing the number of relays.**

I. INTRODUCTION

One of the most challenging problems in wireless multimedia communications is the need to overcome channel fading. This problem is frequently addressed through diversity techniques, which improves the likelihood of receiving a useful message by transmitting multiple copies of the signal in a way that each is independently affected by channel impairments. Constrains in the mobiles size and power have produced a new paradigm in diversity-exploiting techniques where mobile terminals are associated so they can help each other to ensure successful delivery of multiple copies of a message. The communication channels in this paradigm have received the generic name of *relay channel* [1]. We will consider a multi-hop channel where there is no direct path between the source and destination; i.e. the information path between source and destination contains one or more relaying nodes. At the signal processing level, several techniques have been proposed for the relays to forward the sources signals. Most notably, the idea of achieving spatial diversity through user cooperation was presented in [2], along with the idea of cooperation through "decode-and-forward". In [3], the authors introduced the idea of implementing cooperation through various protocols

such as the "amplify-and-forward" protocol and further studied the outage behavior of user-cooperation when using distributed space-time coding in [4].

Diversity is not exclusive to implementations at the physical layer. As studied in [5], diversity can also be formed when multiple channels are provided to the application layer, where they are exploited through multiple description source encoders. In *Multiple Description Coding* different descriptions of the source are generated with the property that they can each be individually decoded or, if possible, be jointly decoded to obtain a reconstruction of the source with lower distortion [6]. The achievable rate-distortion performance of multiple description codes was studied in [7].

This paper focus on studying systems that exhibit diversity of three forms: source coding diversity (when using a dual description encoder), channel coding diversity and user-cooperation diversity (implemented through multi-hop channels, with amplify-and-forward or decode-and-forward user cooperation). The presented analysis derives the distortion exponent for several source-channel diversity achieving schemes.

II. SYSTEM MODEL

We will focus on systems that communicate a source signal over a wireless multi-hop. We will assume that communication is performed over a complex, additive white Gaussian noise (AWGN) fading channel. Denoting by I the maximum average mutual information between the channel input and output, for the channel under consideration $I = \log(1+|h|^2 SNR)$, where h is the fading value [8]. Because of the random nature of the fading, I and the ability of the channel to support transmission at some rate are themselves random. The probability of the channel not being able to support a rate R is called the *outage probability* and is given by $P_0 = \Pr[I < R]$. It will be convenient for us to work with the random function e^I, which has a cumulative

distribution function (cdf) F_{e^I} that can be approximated at high SNR as [5]

$$F_{e^I}(t) \approx c \left(\frac{t}{SNR} \right)^p. \tag{1}$$

Both c and p are model-dependant parameters. For the case of Rayleigh fading we have $p = 1$.

We consider a communication system consisting of a source, a source encoder and a channel encoder. The source samples are fed into the source encoder for quantization and compression. The output of the source encoder are fed into a channel encoder which outputs N channel inputs. For K source samples and N channel inputs, we denote by $\beta \triangleq N/K$, the bandwidth expansion factor or processing gain. We assume that K is large enough to average over the statistics of the source but N is not sufficiently large to average over the statistics of the channel, i.e., we assume block fading wireless channel. In this paper we are specifically interested in systems where the source signal average end-to-end distortion is the figure of merit. Thus, performance will be measured in terms of the expected distortion $E[D] = E[d(\mathbf{s}, \hat{\mathbf{s}})]$, where $d(\mathbf{s}, \hat{\mathbf{s}}) = (1/K) \sum_{k=1}^{K} d(s_k, \hat{s}_k)$ is the average distortion between a sequence $\mathbf{s}$ of K samples and its corresponding reconstruction $\hat{\mathbf{s}}$ and $d(s_k, \hat{s}_k)$ is the distortion between a single sample s_k and its reconstruction $\hat{s}_k$. We will assume $d(s_k, \hat{s}_k)$ to be the mean-squared distortion measure. Following the fading channels assumption, we will be interested in studying the system behavior at large channel signal-to-noise ratios (SNRs) where system performances can be compared in terms of the rate of decay of the end-to-end distortion. This figure of merit called the *distortion exponent*, [5], is defined as

$$\Delta \triangleq - \lim_{SNR \to \infty} \frac{\log E[D]}{\log SNR}. \tag{2}$$

Let the input to the system be a memoryless source. Each sample is first fed into a source encoder. We will consider two types of source encoders: a *single description* (SD) and a *dual description* source encoder, i.e. the source encoder generates either one or two coded descriptions of the source.

The performance of source encoders can be measured through its achievable rate-distortion (R-D) function, which characterizes the tradeoff between source encoding rate and distortion. The R-D function for SD source encoders is frequently considered to be of the form $R = (1/c_2) \log(c_1/D)$, where R, the source encoding rate, is measured in nats per channel use. This form of R-D function is a good approximation in the high-resolution limit [9]. In this case, the R-D function can be approximated without loss of generality, as [7],

$$R = \frac{1}{2\beta} \log \left(\frac{1}{D} \right). \tag{3}$$

For multiple description (MD) source encoders, the R-D region is only known for the dual description source

encoders [7]. In dual description encoders, source samples are encoded into two descriptions. Each description can either be decoded independently of the other, when the other is unusable at the receiver, or combined to achieve a reconstruction of the source with a lower distortion, D_0, when both descriptions are received correctly. This fact is reflected in the corresponding R-D function. Let R_1 and R_2 be the source encoding rates of descriptions 1 and 2, respectively, and $R_{md} = R_1 + R_2$. All the schemes we will consider in this work present the same communication conditions for each description. Therefore, it will be reasonable to assume $R_1 = R_2 = R_{md}/2$. Under this condition, it was shown in [5] that the following bounds can be derived

$$(4D_0 D_1)^{-1/(2\beta)} \lesssim e^{R_{md}} \lesssim (2D_0 D_1)^{-1/(2\beta)}, \tag{4}$$

where the lower bound requires $D_0 \to 0$ and the upper bound requires also $D_1 \to 0$.

In the case of the high distortion scenario, $D_1 + D_2 - D_0 > 1$, the R-D function equals

$$R_{md} = \frac{1}{2\beta} \log \left(\frac{1}{D_0} \right). \tag{5}$$

III. MULTI-HOP AMPLIFY-AND-FORWARD PROTOCOL

In this section, we will consider the analysis for multi-hop amplify-and-forward schemes with different channel and source coding diversity achieving schemes.

A. Single Relay

The system under consideration consists of a source, a relay, and a destination. Transmission of a message is done in two phases. In phase 1, the source sends its information to the relay node. The received signal at the relay node is given by

$$y_{r_1} = h_{s,r_1} \sqrt{P} x_s + n_{s,r_1}, \tag{6}$$

where h_{s,r_1} is the channel gain between the source and the relay node, P is the source transmit power where $E[\|x_s\|^2] = 1$, n_{s,r_1} is the noise at the relay node modeled as zero mean circularly symmetric complex Gaussian noise with variance $N_0/2$ per dimension, and x_s is the transmitted source symbol. In phase 2, the relay normalizes the received signal by the factor $\alpha_1 \le \sqrt{\frac{P}{P|h_{s,r_1}|^2 + N_0}}$ [3] and retransmits to the destination. The received signal at the destination is given by

$$\begin{aligned} y_d &= h_{r_1,d} \alpha_1 y_{r_1} + n_{r_1,d} \\ &= h_{r_1,d} \alpha_1 h_{s,r_1} \sqrt{P} x_s + h_{r_1,d} \alpha_1 n_{s,r_1} + n_{r_1,d}, \end{aligned} \tag{7}$$

where $n_{r_1,d}$ is the noise at the destination node and is modeled as zero mean circularly symmetric complex Gaussian noise with variance $N_0/2$ per dimension. The mutual information is maximized when $\alpha_1 = \sqrt{\frac{P}{P|h_{s,r_1}|^2 + N_0}}$, i.e.,

satisfying the power constraint with equality. The mutual information in this case was found to be [3]

$$I(x_s, y_d) = \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}\right), \tag{8}$$

where $SNR = P/N_0$. At high SNR, we have

$$I(x_s, y_d) \approx \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) \tag{9}$$

$$\approx \log\left(\frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right).$$

Equation (9) indicates that the two-hop amplify-and-forward channel appears as a link with signal-to-noise ratio that is the scaled harmonic mean of the source-relay and relay-destination channels signal-to-noise ratios[1]. To calculate the distortion exponent let $Z_1 = |h_{s,r_1}|^2 SNR$ and $Z_2 = |h_{r_1,d}|^2 SNR$. Assuming symmetry between the source-relay and relay-destination channels, we have

$$F_{Z_1}(t) \approx c\left(\frac{t}{SNR}\right)^p$$
$$F_{Z_2}(t) \approx c\left(\frac{t}{SNR}\right)^p, \tag{10}$$

where $F_{Z_1}(.)$ and $F_{Z_2}(.)$ are the cdf of Z_1 and Z_2, respectively. The scaled harmonic mean of two nonnegative random variables can be upper and lower bounded as

$$\frac{1}{2}\min(Z_1, Z_2) \leq \frac{Z_1 Z_2}{Z_1 + Z_2} \leq \min(Z_1, Z_2). \tag{11}$$

While the lower bound is achieved if and only if $Z_1 = Z_2$, $Z_1 = 0$ or $Z_2 = 0$, the upper bound is achieved if and only if $Z_1 = 0$ or $Z_2 = 0$. Define the random variable $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$. From (11) we have

$$\Pr\left[\min(Z_1, Z_2) < t\right] \leq \Pr\left[Z < t\right] \leq \Pr\left[\min(Z_1, Z_2) < 2t\right]. \tag{12}$$

Then we have

$$\Pr\left[\min(Z_1, Z_2) < t\right] = 2F_{Z_1}(t) - (F_{Z_1}(t))^2$$
$$\approx 2c\left(\frac{t}{SNR}\right)^p - c^2\left(\frac{t}{SNR}\right)^{2p}$$
$$\approx c_1\left(\frac{t}{SNR}\right)^p, \tag{13}$$

where $c_1 = 2c$. Similarly, we have

$$\Pr\left[\min(Z_1, Z_2) < 2t\right] \approx c_2\left(\frac{t}{SNR}\right)^p, \tag{14}$$

where $c_2 = 2^{p+1}c$. From (13) and (14) we get

$$c_1\left(\frac{t}{SNR}\right)^p \lesssim F_Z(t) \lesssim c_2\left(\frac{t}{SNR}\right)^p, \tag{15}$$

[1]The scaling factor is 1/2 since the harmonic mean of two numbers, X_1 and X_2, is $\frac{2X_1 X_2}{X_1 + X_2}$.

where $F_Z(t)$ is the cdf of the random variable Z. The minimum expected end-to-end distortion can now be computed as

$$E[D] = \min_D\Big\{\Pr\left[I(x_s, y_d) < R(D)\right] + D\cdot$$
$$\Pr\left[I(x_s, y_d) \geq R(D)\right]\Big\}, \tag{16}$$

where D is the source encoder distortion and R is the source encoding rate. Note that (16) implicitly assumes that in the case of an outage the missing source data is concealed by replacing the missing source samples with their expected value (equal to zero) and we assume unit variance source (i.e., the source distortion under outage event equals 1). Using the bounds in (15) the minimum expected distortion can be upper and lower bounded as

$$\min_D\left\{c_1\left(\frac{\exp(R(D))}{SNR}\right)^p + \left[1 - c_2\left(\frac{\exp(R(D))}{SNR}\right)^p\right]D\right\}$$
$$\lesssim E[D] \lesssim$$
$$\min_D\left\{c_2\left(\frac{\exp(R(D))}{SNR}\right)^p + \left[1 - c_1\left(\frac{\exp(R(D))}{SNR}\right)^p\right]D\right\}. \tag{17}$$

For sufficiently large SNRs ,we have

$$\min_D\left\{c_1\left(\frac{\exp(R(D))}{SNR}\right)^p + D\right\} \lesssim E[D] \lesssim$$
$$\min_D\left\{c_2\left(\frac{\exp(R(D))}{SNR}\right)^p + D\right\}. \tag{18}$$

From (3), $\exp(R(D)) = D^{\frac{-1}{2\beta_m}}$, where $\beta_m = N_m/K$ as illustrated in Fig. 1, which leads to

$$\min_D c_1\frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D \lesssim E[D] \lesssim \min_D c_2\frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D. \tag{19}$$

Differentiating the lower bound and setting equal to zero we get the optimizing distortion

$$D^* = \left(\frac{2\beta_m}{c_1 p}\right)^{\frac{-2\beta_m}{2\beta_m + p}} SNR^{\frac{-2\beta_m p}{2\beta_m + p}}. \tag{20}$$

Substituting from (20) into (19) we get

$$C_{LB}\, SNR^{\frac{-2\beta_m p}{2\beta_m + p}} \lesssim E[D] \lesssim C_{UB}\, SNR^{\frac{-2\beta_m p}{2\beta_m + p}}, \tag{21}$$

where C_{LB} and C_{UB} are terms that are independent of the SNR.

The distortion exponent is now given by the following theorem.

Theorem 1: The distortion exponent of the two-hop single relay amplify-and-forward protocol is

$$\Delta_{SH-1R-AMP} = \frac{2p\beta_m}{p + 2\beta_m}, \tag{22}$$

where $\beta_m = N_m/K$, and N_m is the number of the source channel uses. (refer to Fig. 1)

Fig. 1.   Two-hop single relay system' time frame structure.

Fig. 2.   Two-hop M relays system' time frame structure.

In the sequel, we will use

$$F_Z(t) \approx \acute{c} \left(\frac{t}{SNR} \right)^p, \qquad (23)$$

where Z is the scaled harmonic mean of the source-relay and relay-destination signal-to-noise ratios and $\acute{c}$ is a constant. Although the last relation does not follow directly from (15) we use it for simplicity of presentation. The analysis is not affected by this substitution as we can always apply the analysis presented here by forming upper and lower bounds on the expected distortion and this will yield the same distortion exponent. We consider now a system consisting of a source, M relay nodes and a destination as shown in Fig. 2. The M relay nodes amplify the received signals from the source and then retransmit to the destination. The destination selects the signal of the highest quality (highest SNR) to recover the source signal. The distortion exponent of this system is given by the following theorem. (proof is omitted due to space limitations)

Theorem 2: The distortion exponent of the two-hop M relays selection channel coding diversity amplify-and-forward protocol is

$$\Delta_{SH-MR-AMP} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m}. \qquad (24)$$

The distortion exponent shows a tradeoff between the diversity and the source encoder performance. Increasing the number of relay nodes increases the diversity of the system at the expense of using lower rate source encoder (higher distortion under no outage). To get the optimal number of relays, M_{opt}, note that the distortion exponent in (24) can be easily shown to be concave in the number of relays. Differentiating and setting equal to zero, we get

$$\frac{\partial}{\partial M} \Delta_{SH-MR-AMP} = 0 \longrightarrow M_{opt} = 2\sqrt{\frac{\beta_m}{p}}. \qquad (25)$$

If M_{opt} in (25) is an integer number then it is the optimal number of relays. If M_{opt} in (25) is not an integer, substitute in (24) with the largest integer that is less than M_{opt} and the smallest integer that is greater than M_{opt} and choose the one that yields the higher distortion exponent as the optimum number of relay nodes. From the result in (25) it is clear that the number of relays decreases, for a fixed β_m, as p increases. For higher channel quality (higher p) the system performance is limited by the distortion introduced by the source encoder in the absence of outage. Then, as p increases, the optimum number of relays decreases to allow for the use of a better source encoder with lower source encoding distortion. In this scenario, the system is said to be a quality limited system because the dominant
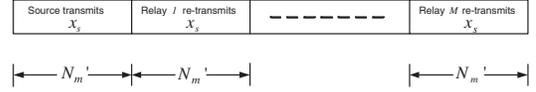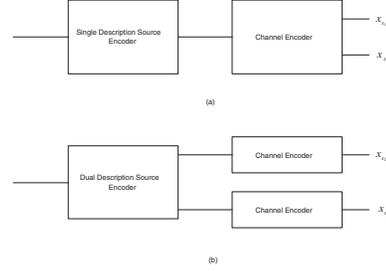

Fig. 3.   Two relays source and channel encoding for (a) optimal channel coding diversity (b) source coding diversity.

phenomena in the end-to-end distortion is source encoding distortion and not outage. Similarly as β_m increases (higher bandwidth), for a fixed p, the performance will be limited by the outage event rather than the source encoding distortion. As β_m increases, the optimum number of relays increases to achieve better outage performance. In this case, the system is said to be an outage limited system.

B. Optimal Channel Coding Diversity with 2 Relays

We consider now a system comprising a source, two relays and a destination. The two blocks, x_{s_1} and x_{s_2}, are constructed as shown in Fig. 3(a). The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 4. From (9), it is straight forward to show that the mutual information is

$$I \approx \log \left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR} \right)$$
$$+ \log \left(1 + \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR} \right), \qquad (26)$$

where x_{s_1} and x_{s_2} are independent zero mean circularly symmetric complex Gaussian random variables with variance $1/2$ per dimension.

We can show that the distortion exponent of this system is given by the following theorem.

Theorem 3: The distortion exponent of the two-hop two-relay optimal channel coding diversity amplify-and-forward system is

$$\Delta_{SH-2R-OPTCH-AMP} = \frac{2p\beta_m}{p + \beta_m}. \qquad (27)$$

Proof *From [5], the distortion exponent for the optimal channel coding diversity over two parallel channels can be written as*

$$\Delta_{SH-2R-OPTCH-AMP} = \frac{4p\beta_m''}{p + 2\beta_m''}, \qquad (28)$$

Using (23) and (26) and considering $\beta_m'' = N_m''/K$ where N_m'' is the number of source channel uses for the x_{s_1} (x_{s_2})

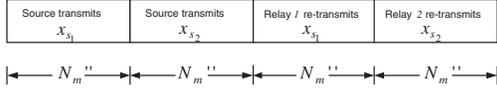| Source transmits x_{s_1}	Source transmits x_{s_2}	Relay 1 re-transmits x_{s_1}	Relay 2 re-transmits x_{s_2}
$\longleftarrow N_m'' \longrightarrow$	$\longleftarrow N_m'' \longrightarrow$	$\longleftarrow N_m'' \longrightarrow$	$\longleftarrow N_m'' \longrightarrow$

Fig. 4. Two-hop 2 relays optimal channel coding diversity (source coding diversity) system' time frame structure.

block (refer to Fig. 4) we get for our system the same distortion exponent as (28). For fair comparison with the previous schemes we should have $2N_m = 4N_m''$, *which means that* $\beta_m'' = \frac{1}{2}\beta_m$. *Finally, substituting this relation in (28) yields (27).* ∎

C. Source Coding Diversity with 2 Relays

We consider again a system with one source, two relays and one destination nodes. The source transmits two blocks x_{s_1} and x_{s_2} constructed as shown in Fig. 3(b). Each block represents one of the two descriptions generated by the dual descriptions source encoder. In this case, the two blocks are broken up before the channel encoder, that is each description is fed to a different channel encoder. The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 4. The distortion exponent of this system is given by the following theorem.

Theorem 4: The distortion exponent of the two-hop 2 relays source coding diversity amplify-and-forward protocol is

$$\Delta_{SH-2R-SRC-AMP} = \max\left[\frac{4p\beta_m}{3p+2\beta_m}, \frac{2p\beta_m}{p+2\beta_m}\right].$$
(29)

Proof *From [5], the distortion exponent for the source coding diversity over two parallel channels can be written as*

$$\Delta_{SH-2R-SRC-AMP} = \max\left[\frac{8p\beta_m''}{3p+4\beta_m''}, \frac{4p\beta_m''}{p+4\beta_m''}\right],$$
(30)

Using (23) and (26) and considering $\beta_m'' = N_m''/K$ *(refer to Fig. 4) we get for our system the same distortion exponent as (30). For fair comparison with the previous schemes,* $2N_m = 4N_m''$; *which leads to* $\beta_m'' = \frac{1}{2}\beta_m$. *Substituting this equality in (30) completes the proof.* ∎

IV. MULTI-HOP DECODE-AND-FORWARD PROTOCOL

In this section, we will analyze schemes using multi-hop decode-and-forward user cooperation under different channel and source coding diversity schemes. In these cases, the relay nodes decode the received source symbols. Only those relay nodes that had correctly decoded the source symbols will proceed to retransmit them to the destination node. An outage occurs when either the source-relay or the relay-destination channel are in outage, as discussed in Section II. That is, the quality of the source-relay-destination link is limited by the minimum of the source-relay and relay-destination channels. For the single relay case we can formulate the outage as

$$P_{outage} = \Pr\left[\min(I(x_s, y_{r_1}), I(x_{r_1}, y_d)) < R(D)\right], \quad (31)$$

where x_{r_1} is the transmitted signal from the relay node. Note that in those schemes using decode-and-forward the quality (mutual information) of any source-relay-destination link is limited by the minimum of the source-relay and relay-destination links SNRs. On the other hand, for two-hop amplify-and-forward schemes, the performance is limited by the scaled harmonic mean of the source-relay and the relay-destination links SNRs which is strictly (if both links are not absent) less than the minimum of the two links SNRs. Hence, the multi-hop amplify-and-forward protocol has a higher outage probability (lower quality) than the multi-hop decode-and-forward protocol. That is, in terms of outage probability, the multi-hop decode-and-forward protocol outperforms the multi-hop amplify-and-forward protocol. The above argument is also applicable under different performance measures (for example, if the performance measure was symbol error rate). From our presentation so far it is clear that the distortion exponents for multi-hop decode-and-forward schemes are the same as their corresponding multi-hop amplify-and-forward schemes for the repetition channel coding diversity and source coding diversity cases. For example, for the two-hop single relay decode-and-forward scheme, the minimum expected distortion is given by the lower bound in (21), which has the same distortion exponent as the two-hop single relay amplify-and-forward scheme. We collect these results in the following theorem.

Theorem 5: The distortion exponent of the multi-hop decode-and-forward schemes are:

- for the two-hop single relay

$$\Delta_{SH-1R-DEC} = \frac{2p\beta_m}{p+2\beta_m}, \quad (32)$$

- for the two-hop M relays selection channel coding diversity

$$\Delta_{SH-MR-DEC} = \frac{4Mp\beta_m}{M(M+1)p+4\beta_m}, \quad (33)$$

- for the two-hop 2 relays source coding diversity

$$\Delta_{SH-2R-SRCDEC} = \max\left[\frac{4p\beta_m}{3p+2\beta_m}, \frac{2p\beta_m}{p+2\beta_m}\right].$$
(34)

A. Optimal Channel Coding Diversity with 2 Relays

We consider now the use of optimal channel coding with two-relay decode-and-forward protocols. In this case, the relay will perform joint decoding of the two blocks x_{s_1} and x_{s_2} as illustrated in Fig. 5, which means that when any relay decodes correctly it could forward both x_{s_1} and x_{s_2}. Allowing the first relay to forward only x_{s_1} if it has decoded correctly will cause a degradation in the performance if the second relay decoded erroneously. Hence, if the first relay decoded correctly and the second did not, it is better (in terms of outage probability) for the
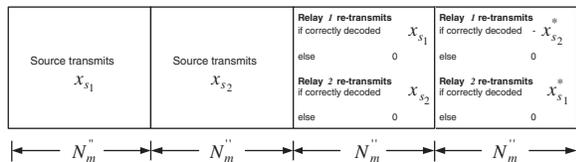
Fig. 5. Two-hop 2 relays decode-and-forward optimal channel coding diversity system' time frame structure.

first relay to forward both x_{s_1} and x_{s_2}. Clearly, a similar argument could be applied to the operation of the second relay. Also, when both relays decode correctly, allowing the second relay to transmit also x_{s_1} and x_{s_2} will cause a loss of diversity. To gain both advantages (lower outage probability when only one relay decodes correctly and diversity when both correctly decode) we propose to use a space-time transmission scheme. In our case we choose the Alamouti scheme [10], with the time frame structure as shown in Fig. 5. Then, the distortion exponent of this system is given by the following theorem. (proof is omitted due to space limitations)

Theorem 6: The distortion exponent of the two-hop 2 relays optimal channel coding diversity decode-and-forward protocol is

$$\Delta_{SH-2R-OPTCH-DEC} = \frac{2p\beta_m}{p + \beta_m}. \quad (35)$$

Figure 6 compares the distortion exponent for the various systems as a function of β_m for the multi-hop channel. From Figure 6 it is clear that the optimal channel coding diversity gives better distortion exponent than the source coding diversity. A similar observation was made in [5] for the case of parallel channels. Note that as β_m increases, the factor that limits the distortion exponent performance is the diversity (number of relays nodes). In this case (high β_m), the system is said to be an outage limited system as the outage probability, rather than the quality of the source encoder, is the main limiting factor in the end-to-end distortion. Figure 6 shows that in this scenario, the distortion exponent performance is improved by increasing diversity by increasing the number of relays. At low β_m the system is said to be quality limited as the quality of the source encoder (distortion under no outage), rather than the outage probability, is the main limiting factor in the end-to-end distortion. In this case, the gain from using a better source encoder, that has a higher resolution, is more significant than the gain from increasing the number of relay nodes. Figure 6 shows that in this scenario, the distortion exponent performance is improved by using only a single relay node allowing for the use of a higher resolution source encoder.

V. CONCLUSION

The presented study focused on analyzing the achievable performance limits, which was measured in terms of the distortion exponent. Our results show that for the multi-hop channels, optimum channel coding diversity provides
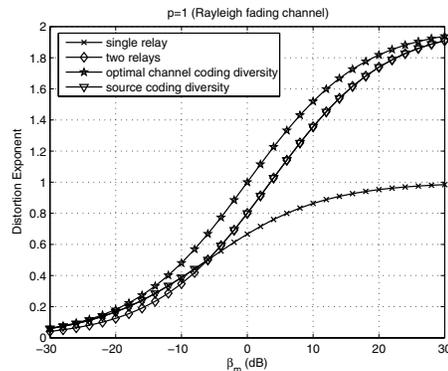

Fig. 6. Distortion exponents for two-hop amplify-and-forward (decode-and-forward) protocol.

better performance, followed by source coding diversity. We showed that as the bandwidth expansion factor increases, the distortion exponent is improved by increasing the number of relays because user cooperation diversity is the main limiting factor. In these cases, the system is said to be an outage limited system. Therefore, it is better to cooperate with more relays in this case which results in minimizing the outage probability and, consequently, minimizing the end-to-end distortion.

REFERENCES

[1] G. Kramer, M. Gatspar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Info. Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity- parts I and II," *IEEE Trans. Comm.*, vol. 51, no. 11, pp. 1927–1948, Nov. 2003.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Info. Theory*, vol. 50, no. 12, pp. 3062–3080, December 2004.

[4] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Info. Theory*, vol. 49, no. 10, pp. 2415–2525, October 2003.

[5] J. N. Laneman, E. Martinian, G. W. Wornell, and J. G. Apostolopoulos, "Source-channel diversity for parallel channels," *IEEE Trans. Info. Theory*, vol. 51, no. 10, pp. 3518–3539, Oct. 2005.

[6] L. Ozarov, "On a source coding problem with two channels and three receivers," *Bell Sys. Tech. Journal*, vol. 59, no. 10, pp. 1909–1921, Dec. 1980.

[7] A. A. El Gamal and T. M. Cover, "Achievable rates for multiple descriptions," *IEEE Trans. Info. Theory*, vol. 28, no. 6, pp. 851–857, November 1982.

[8] T. Cover and J. Thomas, *Elements of Information Theory*, John Wiley Inc., 1991.

[9] B. Hochwald and K. Zeger, "Tradeoff between source and channel coding," *IEEE Trans. Info. Theory*, vol. 43, no. 5, pp. 1412–1424, September 1997.

[10] S. M. Alamouti, "A simple transmit diversity technique for wireless communiucations," *IEEE Journ. Sel. Areas in Comm.*, vol. 16, no. 8, pp. 1451–1458, October 1998.