Asymptotic Distortion Performance of Source-Channel Diversity over Multihop and Relay Channels

Karim G. Seddik, Andres Kwasinski, and K.J. Ray Liu, *Fellow*, *IEEE*

Abstract—A key challenge in the design of real-time wireless multimedia systems is the presence of fading coupled with strict delay constraints. A very effective answer to this problem is the use of diversity achieving techniques to overcome the fading nature of the wireless channels caused by the mobility of the nodes. The mobility of the nodes gives rise to the need of cooperation among the nodes to enhance the system performance. This paper focuses on comparing systems that exhibit diversity of three forms: source coding diversity, channel coding diversity, and user cooperation diversity implemented through multihop or relay channels with amplify-and-forward or decode-and-forward protocols. Commonly used in multimedia communications, performance is measured in terms of the distortion exponent, which measures the rate of decay of the end-to-end distortion at asymptotically high signal-to-noise ratio (SNR). For the case of repetition coding at the relay nodes, we prove that having more relays is not always beneficial. For the general case of having a large number of relays that can help the source using repetition coding, the optimum number of relay nodes that maximizes the distortion exponent is determined in this paper. This optimum number of relay nodes will depend on the system bandwidth as well as the channel quality. The derived result shows a trade-off between the quality (resolution) of the source encoder and the amount of cooperation (number of relay nodes). Also, the performances of the channel coding diversity-based scheme and the source coding diversity-based scheme are compared. The results show that for both relay and multihop channels, channel coding diversity provides the best performance, followed by the source coding diversity.

Index Terms—Channel coding diversity, distortion exponent, multiple-description source coding, relay channels, source coding diversity.

1 Introduction

One of the most challenging problems in wireless multimedia communications is the need to overcome channel fading. This fading nature of the wireless channel is caused by the mobility of the nodes in the network, which causes random fluctuations in the channel gains. This problem is frequently addressed through diversity techniques, which improve the likelihood of receiving a useful message by transmitting multiple copies of the signal in a way that each is independently affected by channel impairments. Constraints in the mobiles' sizes and powers have produced a new paradigm in diversity-exploiting techniques, where mobile terminals are associated so that they can help each other to ensure successful delivery of multiple copies of a message. The communication channels in this paradigm have received the generic name of *relay channel* [1]. In this paper, we will differentiate between multihop and relay channels. We will consider a multihop channel, where there is no direct path between the source and destination, i.e., the information path between source and destination contains one or more relaying nodes [2]. We will consider a relay channel as that where there is a direct communication path between source and destination as well as one or more paths through relaying nodes [3].

At the signal processing level, several techniques have been proposed for the relays to forward the source's signals. Most notably, the idea of achieving spatial diversity through user cooperation was presented in [4], where the authors introduced the idea of implementing cooperation through various protocols such as the "amplify-and-forward" and the "decode-and-forward" protocols and further studied the outage behavior of user cooperation when using distributed space-time coding in [5].

However, achieving diversity is not exclusive to implementations at the physical layer. As studied in [6], diversity can also be formed when multiple channels are provided to the application layer, where they are exploited through multiple description source encoders. In *Multiple Description Coding*, different descriptions of the source are generated with the property that they can each be individually decoded or, if possible, be jointly decoded to obtain a reconstruction of the source with lower distortion [7]. The achievable rate-distortion performance of multiple description codes was studied in [8]. Aiming at its application in communication systems, multiple description coding had been studied for error resilient source coding applications [9], for communications over networks with packet losses [10], for communications over parallel packet loss channels [11], [12], and as an alternative error control scheme for communication over single physical channel in [13].

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Several works have considered the problem of joint source-channel coding over multiple-input multiple-output (MIMO) channels [14], [15]. The work in [16] considers deriving an upper bound on the distortion exponent of the "single-relay" cooperative channels. Then, Gunduz and Erkip [16] presents some schemes that can achieve the upper bound for some system bandwidth efficiencies but no consideration is given to multiple description encoders or the use of multiple relays that we will consider here. Our work compares the source coding (application layer) diversity to the channel coding (physical layer) diversity, which has not been considered in the literature; the approach based on the diversity-multiplexing trade-off curve [16] cannot be used for this comparison since the diversity-multiplexing trade-off curves for these schemes are not known. Therefore, the proposed method constitutes a more general formulation. Also, we consider the case of having multiple relay nodes, which have not been considered before. This provides a new viewpoint to the problem of how to allocate the bandwidth among the relay nodes and the source encoder. Studies on the transmission of layered source-coded sources over user cooperation channels were presented in [17] for coded cooperation. In [2], the performance of different source-channel diversity achieving schemes over multihop channels was studied.

Our contribution in this paper is to understand the achievable performance limits of multimedia communication systems combining multiple descriptions coding with user cooperation techniques in multihop and relay channels. Multimedia signals are subject to mainly two factors that affect quality: lossy compression and channel impairments. Also, the transmission of multimedia signals is constrained by the transmission delay, which limits the use of *Automatic Repeat Request* (ARQ) in the presence of channel impairments. Given the presence of fading channels, we will study the system performance at asymptotically large signal-to-noise ratios (SNRs) in terms of the distortion exponent, which measures the rate of decay of the end-to-end distortion at high SNRs.

This paper focuses on studying systems that exhibit diversity of three forms: source coding diversity (when using a dual description encoder), channel coding diversity, and user cooperation diversity (implemented with amplify-and-forward or decode-and-forward user cooperation). The presented analysis derives the distortion exponent for several source-channel diversity achieving schemes. More specifically, we consider the cases where we have not only a single relay but also multiple relays helping the source by *repeating* its information either using the amplify-and-forward or the decode-and-forward protocols. In these cases, we analyze the trade-off between the diversity gain (number of relays) and the quality of the source encoder and we also find the optimum number of relays to help the source to maximize the distortion exponent under the constraint of having a fixed system bandwidth. Depending on the system bandwidth, we find that having more relays is not always beneficial in terms of the distortion exponent. For example, for the relay channel, there exists a system bandwidth threshold below which assigning the system bandwidth for direct transmission, from the source node to the destination node, will result in a higher distortion exponent than the case when some of the system bandwidth is assigned for cooperation. Also, we compare source coding diversity (application-layer diversity through multiple description coding) and channel coding diversity (physical-layer diversity).

The rest of the paper is organized as follows: In Section 2, we present the system model. Section 3 studies different schemes implementing user cooperation over a multihop channel. Section 4 studies different schemes implementing user cooperation over a relay channel. Section 6 summarizes the conclusions for this paper.

2 System Model

We will focus on systems that communicate a source signal over a wireless relay channel. Let the input to the system be a memoryless source. We will assume that communication is performed over a complex, additive white Gaussian noise (AWGN) fading channel. For the simple case of single-input single-output channel, let I denote the maximum mutual information between the channel input and output. For the channel under consideration, $I = \log(1 + |h|^2 SNR)$, where h is the complex channel fading value [18]. Because of the random nature of the fading, I is a random variable. The probability of the channel not being able to support a rate R is called the *outage probability* and is given by $P_0 = \Pr[I < R]$. It will be convenient for us to work with the random function e^I, which has a cumulative distribution function (CDF) F_{e^I} that can be approximated at high SNR as [6]

$$F_{e^I}(t) \approx c\left(\frac{t}{SNR}\right)^p. \tag{1}$$

Both c and p are model-dependant parameters. For example, for the case of Rayleigh fading, we have $p = 1$ and c depends on the channel variance.[1] Other values of p allow to consider other fading channel models, for example, Nakagami fading channels [19].

We consider a communication system consisting of a source, a source encoder, and a channel encoder. Let the input to the system be a memoryless source. The source samples are fed into the source encoder for quantization and compression. The outputs of the source encoder are fed into a channel encoder, which outputs N channel inputs. For K source samples and N channel inputs, we denote by $\beta \triangleq N/K$ the bandwidth expansion factor or processing gain. We assume that K is large enough to average over the statistics of the source but N is not sufficiently large to average over the statistics of the channel, i.e., we assume block-fading wireless channel for which the performance can be characterized in terms of the outage probability.

In this paper, we are specifically interested in systems, where the source signal average end-to-end distortion is the figure of merit. Thus, performance will be measured in terms of the expected distortion $E[D] = E[d(\mathbf{s}, \hat{\mathbf{s}})]$, where $d(\mathbf{s}, \hat{\mathbf{s}}) = (1/K)\sum_{k=1}^{K} d(s_k, \hat{s}_k)$ is the average distortion between a sequence $\mathbf{s}$ of K source samples and its corresponding reconstruction $\hat{\mathbf{s}}$ and $d(s_k, \hat{s}_k)$ is the distortion between a single sample s_k and its reconstruction $\hat{s}_k$. We

1. The value of the parameter c will not affect the analysis since we are interested in the distortion exponent, which measures the rate of decay of the end-to-end distortion at high SNRs.

will assume $d(s_k, \hat{s}_k)$ to be the mean-squared distortion measure. Following the fading channels assumption, we will be interested in studying the system behavior at large channel SNRs, where system performances can be compared in terms of the rate of decay of the end-to-end distortion. This figure of merit, called the *distortion exponent* [6], is defined as

$$\Delta \triangleq -\lim_{SNR\to\infty} \frac{\log E[D]}{\log SNR}. \quad (2)$$

We will consider two types of source encoders: a *single description* (SD) and a *dual description* source encoder, i.e., the source encoder generates either one or two coded descriptions of the source.

The performance of the source encoders can be measured through its achievable rate-distortion (R-D) function, which characterizes the relation between source encoding rate and distortion. The R-D function for SD source encoders is frequently considered to be of the form $R = (1/c_2)\log(c_1/D)$, where we are taking the logarithm with base e, and hence, R, the source encoding rate, is measured in nats per channel use. This form of R-D function can approximate or bound a wide range of practical systems such as video coding with an MPEG codec [20], speech using a CELP-type codec [21], or when the high rate approximation holds [6]. Assuming that high-resolution approximation can be applied to the source encoding operation, each of the input samples can be modeled as a memoryless Gaussian source, showing a zero mean, unit-variance Gaussian distribution. In this case, the R-D function can be approximated, without loss of generality, as [18]

$$R = \frac{1}{2\beta}\log\left(\frac{1}{D}\right). \quad (3)$$

For multiple description (MD) source encoders, the R-D region is only known for the dual description source encoders [8]. In dual description encoders, source samples are encoded into two descriptions. Each description can either be decoded independently of the other, when the other is unusable at the receiver, or combined to achieve a reconstruction of the source with lower distortion, when both descriptions are received correctly. This fact is reflected in the corresponding R-D function. Let R_1 and R_2 be the source encoding rates of descriptions 1 and 2, respectively, and $R_{md} = R_1 + R_2$. Let D_1 and D_2 be the reconstructed distortions associated with descriptions 1 and 2, respectively, when each is decoded alone. Let D_0 be the source distortion when both descriptions are combined and jointly decoded. For the same source model and assumptions as in the single description case, R_1 and D_1 and R_2 and D_2 are related through

$$R_1 = \frac{1}{2\beta}\log\left(\frac{1}{D_1}\right), \quad R_2 = \frac{1}{2\beta}\log\left(\frac{1}{D_2}\right). \quad (4)$$

The R-D function when both descriptions can be combined at the source decoder differs depending on whether distortions can be considered low or high [8]. The low distortion scenario corresponds to $D_1 + D_2 - D_0 < 1$ in which case we have

$$\begin{aligned} R_{md} = &\frac{1}{2\beta}\log\left(\frac{1}{D_0}\right) \\ &+ \frac{1}{2\beta}\log\left((1-D_0)^2 \Big/ \Big[(1-D_0)^2 - \big[\sqrt{(1-D_1)(1-D_2)} \right. \\ &\left. - \sqrt{(D_1-D_0)(D_2-D_0)}\big]^2\Big]\right). \end{aligned} \quad (5)$$

All the schemes considered in this work present the same communication statistical conditions to each description. Therefore, it will be reasonable to assume $R_1 = R_2 = R_{md}/2$. Under this condition, it was shown in [6] that the following bounds can be derived from (5):

$$(4D_0D_1)^{-1/(2\beta)} \lessapprox e^{R_{md}} \lessapprox (2D_0D_1)^{-1/(2\beta)}, \quad (6)$$

where the lower bound requires $D_0 \to 0$ and the upper bound requires also $D_1 \to 0$.

In the case of the high distortion scenario, $D_1 + D_2 - D_0 > 1$, the R-D function equals

$$R_{md} = \frac{1}{2\beta}\log\left(\frac{1}{D_0}\right). \quad (7)$$

The channel-encoded message is then sent from the source node to a destination node with or without user cooperation. In a setup with user cooperation, relay nodes are associated with the source node to achieve user cooperation diversity. Communication in a cooperative setup with one relay node takes place in two phases. In phase 1, the source node sends information to its destination node. This transmission can be overheard by the relay because of the broadcast nature of wireless communications. In phase 2, the relay node cooperates by forwarding to the destination the information received from its associated source node. At the destination node, both signals received from the source and the relay are combined and detected, thus, creating a virtual spatial diversity setup. For each additional relay used during the transmission, a new phase, similar to phase 2, needs to be added to allow transmission from the new relay. For fair comparison of the different schemes considered in this paper and because of this multiphase transmission, we need to fix the total number of channel uses for a source block of size K and to change the bandwidth expansion factor accordingly for each scheme.

We will consider two techniques that implement user cooperation, amplify-and-forward and decode-and-forward, each differing in the processing done at the relay [4]. In amplify-and-forward, the relay retransmits the source's signal without further processing other than power amplification. In decode-and-forward, the relay first decodes the message from the source. If the decoded message has no error, the relay reencodes it and transmits a copy. If the relay fails to decode the message, it idles until the next is received.

3 Multihop Channels

In this section, we consider the distortion exponents of multihop networks using amplify-and-forward and decode-and-forward user cooperation protocols. The multihop channel is a channel where there is no direct path between the source and destination, i.e., the information path

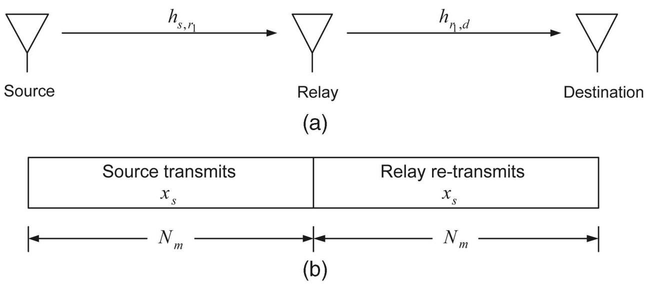

Fig. 1. Two-hop single-relay system: (a) system model, (b) time frame structure.

between source and destination contains one or more relaying nodes. Without loss of generality, we consider the two-hop case. The analysis can be extended to scenarios with larger number of hops.

3.1 Two-Hop Amplify-and-Forward Protocol

In this section, we will consider the analysis for two-hop amplify-and-forward schemes with different channel and source coding diversity achieving schemes. We derive the distortion exponent for the two-hop single-relay channel with an SD source encoder and extend the result to the case of M relays with repetition channel coding diversity. The result shows a trade-off between the number of relays (user cooperation diversity) and the quality of the source encoder. We also derive the distortion exponent when using the multiple description coding. Since we consider the case of dual description source encoders, we derive the distortion exponent for the case of having two relays helping the source. In addition, we consider channel coding diversity with two relay nodes so as to be able to compare the results with the source coding diversity scheme.

3.1.1 Single Relay

The system under consideration consists of a source, a relay, and a destination as shown in Fig. 1. Transmission of a message is done in two phases. In phase 1, the source sends its information to the relay node. The received signal at the relay node is given by

$$y_{s,r_1} = h_{s,r_1}\sqrt{P}x_s + n_{s,r_1}, \tag{8}$$

where h_{s,r_1} is the channel gain between the source and the relay node, x_s is the transmitted source symbol with $E[\|x_s\|^2] = 1$, P is the source transmit power, and n_{s,r_1} is the noise at the relay node modeled, as zero-mean circularly symmetric complex Gaussian random variable with variance $N_0/2$ per dimension. In phase 2, the relay scales the received signal by the factor [4]

$$\alpha_1 \le \sqrt{\frac{P}{P|h_{s,r_1}|^2 + N_0}}$$

and retransmits to the destination. The received signal at the destination is given by

$$\begin{aligned} y_{r_1,d} &= h_{r_1,d}\alpha_1 y_{r_1} + n_{r_1,d} \\ &= h_{r_1,d}\alpha_1 h_{s,r_1}\sqrt{P}x_s + h_{r_1,d}\alpha_1 n_{s,r_1} + n_{r_1,d}, \end{aligned} \tag{9}$$

where $n_{r_1,d}$ is the noise at the destination node and is modeled as zero-mean circularly symmetric complex Gaussian random variable with variance $N_0/2$ per dimension. Mutual information is maximized when x_s, the transmitted source symbol, is distributed as a circularly symmetric complex Gaussian random variable with zero mean and variance $1/2$ per dimension [18]. Consequently, the mutual information is maximized when

$$\alpha_1 = \sqrt{\frac{P}{P|h_{s,r_1}|^2 + N_0}},$$

i.e., satisfying the power constraint with equality [4]. The mutual information in this case was found to be

$$I(x_s, y_d) = \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}\right), \tag{10}$$

where $SNR = P/N_0$. At high SNR, we have

$$\begin{aligned} I(x_s, y_d) &\approx \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) \\ &\approx \log\left(\frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right). \end{aligned} \tag{11}$$

Equation (11) indicates that the two-hop amplify-and-forward channel appears as a link with signal-to-noise ratio that is a scaled harmonic mean of the source-relay and relay-destination channels signal-to-noise ratios.[2] To calculate the distortion exponent, let $Z_1 = |h_{s,r_1}|^2 SNR$ and $Z_2 = |h_{r_1,d}|^2 SNR$. Assuming symmetry between the source-relay and relay-destination channels, we have

$$F_{Z_1}(t) \approx c\left(\frac{t}{SNR}\right)^p, \quad F_{Z_2}(t) \approx c\left(\frac{t}{SNR}\right)^p, \tag{12}$$

where $F_{Z_1}(.)$ and $F_{Z_2}(.)$ are the CDFs of Z_1 and Z_2, respectively. The scaled harmonic mean of two nonnegative random variables can be upper- and lower-bounded as

$$\frac{1}{2}\min(Z_1, Z_2) \le \frac{Z_1 Z_2}{Z_1 + Z_2} \le \min(Z_1, Z_2), \tag{13}$$

where the lower bound is achieved if and only if $Z_1 = Z_2$, $Z_1 = 0$, or $Z_2 = 0$ and the upper bound is achieved if and only if $Z_1 = 0$ or $Z_2 = 0$.

Define the random variable $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$. From (13), we have

$$\Pr[\min(Z_1, Z_2) < t] \le \Pr[Z < t] \le \Pr[\min(Z_1, Z_2) < 2t]. \tag{14}$$

Then, we have

$$\begin{aligned} \Pr[\min(Z_1, Z_2) < t] &= 2F_{Z_1}(t) - (F_{Z_1}(t))^2 \\ &\approx 2c\left(\frac{t}{SNR}\right)^p - c^2\left(\frac{t}{SNR}\right)^{2p} \\ &\approx c_1\left(\frac{t}{SNR}\right)^p, \end{aligned} \tag{15}$$

where $c_1 = 2c$. Similarly, we have

2. The scaling factor is $1/2$ since the harmonic mean of two numbers, X_1 and X_2, is $\frac{2X_1X_2}{X_1+X_2}$.

$$\Pr[\min\,(Z_1, Z_2) < 2t] \approx c_2\left(\frac{t}{SNR}\right)^p, \tag{16}$$

where $c_2 = 2^{p+1}c$. From (15) and (16), we get

$$c_1\left(\frac{t}{SNR}\right)^p \lessapprox F_Z(t) \lessapprox c_2\left(\frac{t}{SNR}\right)^p, \tag{17}$$

where $F_Z(t)$ is the CDF of the random variable Z.

The minimum expected distortion can now be computed as

$$\begin{aligned} E[D] &\approx \min_D\{\Pr[Z < \exp(R(D))] + \Pr[Z \geq \exp(R(D))] \cdot D\} \\ &= \min_D\{F_Z(\exp(R(D))) + [1 - F_Z(\exp(R(D)))] \cdot D\}. \end{aligned} \tag{18}$$

where D is the source encoder distortion and R is the source encoding rate. Note that (18) implicitly assumes that in the case of an outage, the missing source data are concealed by replacing the missing source samples with their expected value (equal to zero). Since we assume unit variance source, the source distortion under outage event equals 1.

It is noteworthy that in (18), we formulate the problem in a more general way than the formulation considered in [22], [23], where the channel coding rate is restricted to the form $R(D) = r\log(SNR)$, where r is the multiplexing gain. The formulation in [22], [23] then makes use of the diversity-multiplexing trade-off (DMT) curves to determine the distortion exponent. Our formulation does not have that restriction on $R(D)$ and enables the calculation of the distortion exponents for systems, where the diversity-multiplexing trade-off approach is not applicable such as the cases of channel coding diversity and source coding diversity. Also, we consider the proof for a general value of the parameter p, which is different from [22], [23] where only the case of $p = 1$ was considered.

Using the bounds in (17), the minimum expected distortion can be upper- and lower-bounded as

$$\begin{aligned} &\min_D\left\{c_1\left(\frac{\exp(R(D))}{SNR}\right)^p + \left[1 - c_2\left(\frac{\exp(R(D))}{SNR}\right)^p\right]D\right\} \\ &\lessapprox E[D] \lessapprox \min_D\left\{c_2\left(\frac{\exp(R(D))}{SNR}\right)^p\right. \\ &\quad\left. + \left[1 - c_1\left(\frac{\exp(R(D))}{SNR}\right)^p\right]D\right\}. \end{aligned} \tag{19}$$

For sufficiently large SNRs, we have

$$\begin{aligned} &\min_D\left\{c_1\left(\frac{\exp(R(D))}{SNR}\right)^p + D\right\} \lessapprox E[D] \\ &\lessapprox \min_D\left\{c_2\left(\frac{\exp(R(D))}{SNR}\right)^p + D\right\}. \end{aligned} \tag{20}$$

From (3), $\exp(R(D)) = D^{\frac{-1}{2\beta_m}}$, where $\beta_m = N_m/K$ as illustrated in Fig. 1, which leads to

$$\min_D c_1\frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D \lessapprox E[D] \lessapprox \min_D c_2\frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D. \tag{21}$$

By differentiating the lower bound and setting equal to zero, we get the optimizing distortion

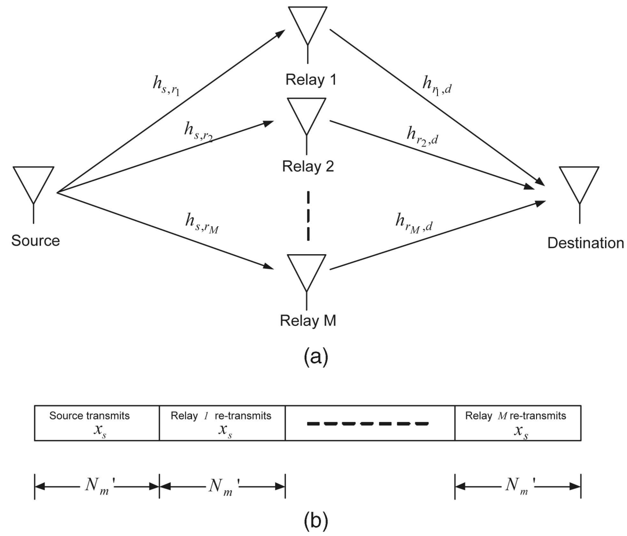

Fig. 2. Two-hop M relays system: (a) system model, (b) time frame structure.

$$D^* = \left(\frac{2\beta_m}{c_1 p}\right)^{\frac{-2\beta_m}{2\beta_m+p}} SNR^{\frac{-2\beta_m p}{2\beta_m+p}}. \tag{22}$$

Substituting (22) into (21), we get

$$C_{LB}\, SNR^{\frac{-2\beta_m p}{2\beta_m+p}} \lessapprox E[D] \lessapprox C_{UB} SNR^{\frac{-2\beta_m p}{2\beta_m+p}}, \tag{23}$$

where C_{LB} and C_{UB} are terms that are independent of the SNR.

From the last equation, the distortion exponent of the two-hop single-relay amplify-and-forward protocol is

$$\Delta_{2H-1R-AF} = \frac{2p\beta_m}{p + 2\beta_m}, \tag{24}$$

where $\beta_m = N_m/K$ and N_m is the number of the source channel uses.

In the sequel, we will use

$$F_Z(t) \approx \acute{c}\left(\frac{t}{SNR}\right)^p, \tag{25}$$

where Z is the scaled harmonic mean of the source-relay and relay-destination signal-to-noise ratios and $\acute{c}$ is a constant. Although the last relation does not follow directly from (17), we use it for simplicity of presentation. The analysis is not affected by this substitution as we can always apply the analysis presented here by forming upper- and lower bounds on the expected distortion and this will yield the same distortion exponent.

We consider now a system consisting of a source, M relay nodes, and a destination as shown in Fig. 2. The M relay nodes amplify the received signals from the source and then retransmit to the destination. The destination selects the signal of the highest quality (highest SNR) to recover the source signal.[3] The distortion exponent of this system is given by the following theorem:

3. The system where the destination selects the signal with the highest quality will have the same distortion exponent as the system where the destination applies maximum ratio combiner (MR) on the received signals from the relay nodes.

Theorem 1. *The distortion exponent of the two-hop M relays selection channel coding diversity amplify-and-forward protocol is*

$$\Delta_{2H-MR-AF} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m}. \quad (26)$$

Proof. Let y_{d_i} be the signal received at the destination due to the ith relay transmission. At sufficiently high SNR, the mutual information between x_s and y_{d_i} is given by

$$I(x_s, y_{d_i}) \approx \log\left(\frac{|h_{s,r_i}|^2 SNR |h_{r_i,d}|^2 SNR}{|h_{s,r_i}|^2 SNR + |h_{r_i,d}|^2 SNR}\right), \ i = 1, 2, \ldots, M.$$

Define the random variables

$$W_i = \frac{|h_{s,r_i}|^2 SNR |h_{r_i,d}|^2 SNR}{|h_{s,r_i}|^2 SNR + |h_{r_i,d}|^2 SNR}, \quad i = 1, 2, \ldots, M.$$

The CDF of W_i can be approximated at high SNR as

$$F_{W_i}(t) \approx \acute{c}\left(\frac{t}{SNR}\right)^p. \quad (27)$$

The minimum end-to-end expected distortion can be computed as

$$\begin{aligned} E[D] &= \min_D \{\Pr[\max(I(x_s, y_{d_1}), I(x_s, y_{d_2}), \ldots, I(x_s, y_{d_M})) \\ &\quad < R(D)] + \Pr[\max(I(x_s, y_{d_1}), \\ &\quad I(x_s, y_{d_2}), \ldots, I(x_s, y_{d_M})) \geq R(D)] \cdot D\} \\ &= \min_D \left\{ \prod_{i=1}^{M} F_{W_i}(\exp(R(D))) \right. \\ &\quad \left. + \left[1 - \prod_{i=1}^{M} F_{W_i}(\exp(R(D)))\right] \cdot D \right\} \\ &\approx \min_D \left\{ \acute{c}^M \frac{D^{\frac{-Mp}{2\beta'_m}}}{SNR^{Mp}} + \left[1 - \acute{c}^M \frac{D^{\frac{-Mp}{2\beta'_m}}}{SNR^{Mp}}\right] \cdot D \right\} \\ &\approx \min_D \left\{ \acute{c}^M \frac{D^{\frac{-Mp}{2\beta'_m}}}{SNR^{Mp}} + D \right\}, \end{aligned} \quad (28)$$

where D is the source encoding distortion and $\beta'_m = N'_m/K$, where N'_m is the number of the source channel uses (refer to Fig. 2). By differentiating and setting equal to zero, we get the optimizing distortion

$$D^* = \left(\acute{c}^M \frac{Mp}{2\beta'_m}\right)^{\frac{2\beta'_m}{Mp+2\beta'_m}} SNR^{\frac{-2M\beta'_m p}{2\beta'_m + Mp}}. \quad (29)$$

By substituting, we get

$$E[D] \approx C_{MR}\, SNR^{\frac{-2M\beta'_m p}{2\beta'_m + Mp}}, \quad (30)$$

where C_{MR} is a term that does not depend on the SNR. Hence, the distortion exponent is given as

$$\Delta_{2H-MR-AF} = \frac{2M\beta'_m p}{2\beta'_m + Mp}. \quad (31)$$

For fair comparison with the single-relay case, we should compare the different systems under the same number of channel uses. So that we have $2N_m = (M+1)N'_m$ (refer to Figs. 1 and 2) from which we have $\beta'_m = \frac{2}{M+1}\beta_m$. By substituting into (31), we get

$$\Delta_{2H-MR-AF} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m}. \quad (32)$$

□

Note that (24) is a special case of Theorem 1 with $M = 1$.

The distortion exponent shows a trade-off between the diversity and the source encoder performance. Increasing the number of relay nodes increases the diversity of the system at the expense of using lower rate source encoder (higher distortion under no outage). To get the optimal number of relays M_{opt}, note that the distortion exponent in (26) can be easily shown to be concave in the number of relays (if we think of M as a continuous variable). By differentiating and setting equal to zero, we get

$$\frac{\partial}{\partial M}\Delta_{2H-MR-AF} = 0 \longrightarrow M_{opt} = 2\sqrt{\frac{\beta_m}{p}}. \quad (33)$$

If M_{opt} in (33) is an integer number, then it is the optimal number of relays. If M_{opt} in (33) is not an integer, substitute into (26) with the largest integer that is less than M_{opt} and the smallest integer that is greater than M_{opt} and choose the one that yields the higher distortion exponent as the optimum number of relay nodes.

From the result in (33), it is clear that the number of relays decreases, for a fixed β_m, as p increases. For higher channel quality (higher p), the system performance is limited by the distortion introduced by the source encoder in the absence of outage. Then, as p increases, the optimum number of relays decreases to allow for the use of a better source encoder with lower source encoding distortion. In this scenario, the system is said to be a quality-limited system because the dominant phenomena in the end-to-end distortion are source encoding distortion and not outage. Similarly, as β_m increases (higher bandwidth), for a fixed p, the performance will be limited by the outage event rather than the source encoding distortion. As β_m increases, the optimum number of relays increases to achieve better outage performance. In this case, the system is said to be an outage-limited system.

3.1.2 Channel Coding Diversity with Two Relays

We consider now a system, as shown in Fig. 3, comprising a source, two relays, and a destination. After channel encoding, the resulting block is split into two blocks: x_{s_1} and x_{s_2}, which are transmitted to the relay nodes. The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 3. From (11), it can be shown that the maximum mutual information is given by

$$\begin{aligned} I &\approx \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) \\ &\quad + \log\left(1 + \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}\right), \end{aligned} \quad (34)$$

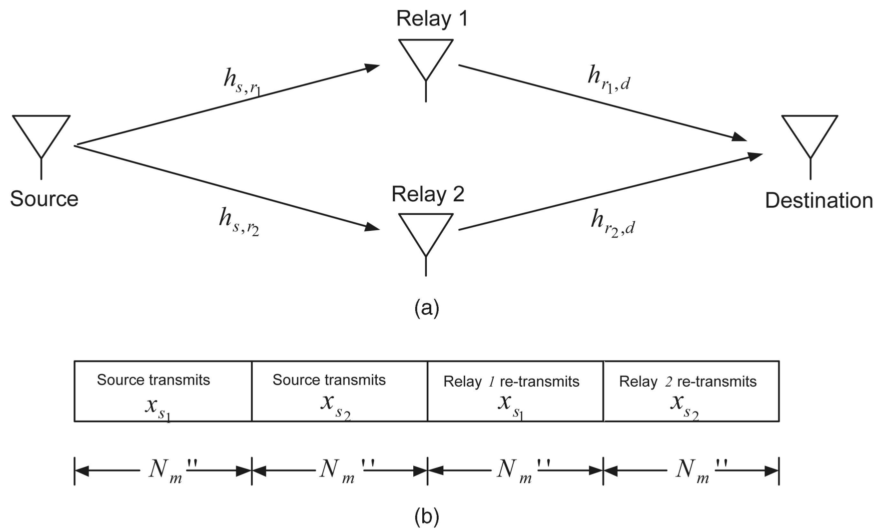

Fig. 3. Two-hop two relays channel coding diversity (source coding diversity) system: (a) system model, (b) time frame structure.

where x_{s_1} and x_{s_2} are independent zero-mean circularly symmetric complex Gaussian random variables with variance $1/2$ per dimension. We can show that the distortion exponent of this system is given by the following theorem:

Theorem 2. *The distortion exponent of the two-hop two-relay channel coding diversity amplify-and-forward system is*

$$\Delta_{2H-2R-OptCC-AF} = \frac{2p\beta_m}{p+\beta_m}. \tag{35}$$

Proof. From [6], the distortion exponent for the channel coding diversity over two parallel channels can be written as

$$\Delta_{2H-2R-OptCC-AF} = \frac{4p\beta''_m}{p+2\beta''_m}. \tag{36}$$

Using (25) and (34) and considering $\beta''_m = N''_m/K$ where N''_m is the number of source channel uses for the x_{s_1} (x_{s_2}) block (refer to Fig. 3), we get for our system the same distortion exponent as (36). For fair comparison with the previous schemes, we should have $2N_m = 4N''_m$, which means that $\beta''_m = \frac{1}{2}\beta_m$. Finally, substituting this relation into (36) yields (35). □

In the context of parallel channels, the notion of multiplexed channel coding diversity was presented in [6]. The gain in the distortion exponent for the multiplexed channel coding diversity scheme (compared to the direct transmission) is a result of the increase of the bandwidth due to the simultaneous use of parallel channels. In the multiplexed channel coding diversity scheme discussed in [6], the two blocks x_{s_1} and x_{s_2} represent a split of a channel-coded message from an SD source encoder over two parallel channels, which will be the two source-relay-destination links in our system. In our system, there is no gain in using multiplexed channel coding diversity because, for fair comparison, using either one relay or two relays does not increase the bandwidth of the channel. This is because only one node, either the source or a relay, is transmitting at a given time slot. The multiplexed channel coding diversity in this case is equivalent to allowing one relay helping the source to forward an SD source-coded message during one block and using the other relay for the next block. Hence, in our system, the multiplexed channel coding diversity is equivalent to the two-hop single-relay system with the same distortion exponent.

3.1.3 Source Coding Diversity with Two Relays

We consider again a system with one source, two relays, and one destination nodes as shown in Fig. 3. The source transmits two blocks x_{s_1} and x_{s_2} to the relay nodes. Each block represents one of the two descriptions generated by the dual descriptions source encoder. In this case, the two blocks are broken up before the channel encoder, that is, each description is fed to a different channel encoder. The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 3. The distortion exponent of this system is given by the following theorem:

Theorem 3. *The distortion exponent of the two-hop two relays source coding diversity amplify-and-forward protocol is*

$$\Delta_{2H-2R-SC-AF} = \max\left[\frac{4p\beta_m}{3p+2\beta_m}, \frac{2p\beta_m}{p+2\beta_m}\right]. \tag{37}$$

Proof. From [6], the distortion exponent for the source coding diversity over two parallel channels can be written as

$$\Delta_{2H-2R-SC-AF} = \max\left[\frac{8p\beta''_m}{3p+4\beta''_m}, \frac{4p\beta''_m}{p+4\beta''_m}\right], \tag{38}$$

Using (25) and (34) and considering $\beta''_m = N''_m/K$ (refer to Fig. 3), we get for our system the same distortion exponent as (38). For fair comparison with the previous schemes, $2N_m = 4N''_m$, which leads to $\beta''_m = \frac{1}{2}\beta_m$. Substituting this equality into (38) completes the proof. □

3.2 Two-Hop Decode-and-Forward Protocol

In this section, we will analyze schemes using two-hop decode-and-forward user cooperation under different channel and source coding diversity schemes. In these cases, the relay nodes decode the received source symbols. Only those relay nodes that had correctly decoded the source symbols will proceed to retransmit them to the destination node. When a relay fails in decoding the source symbols, we say that an outage has occurred. Furthermore, an outage occurs when either the source-relay or the relay-destination channel is in outage, as discussed in Section 2. That is, the quality of the source-relay-destination link is limited by the minimum of the source-relay and relay-destination channels. For the single-relay case, we can formulate the outage as

$$P_{outage} = \Pr[\min(I(x_s, y_{r_1}), I(x_{r_1}, y_d)) < R(D)], \tag{39}$$

where x_{r_1} is the transmitted signal from the relay node. Note that in those schemes using decode-and-forward, the quality (mutual information) of any source-relay-destination link is limited by the minimum of the source-relay and relay-destination links SNRs. On the other hand, for the two-hop amplify-and-forward schemes, the performance is limited by a scaled harmonic mean of the source-relay and the relay-destination links SNRs, which is strictly less than the minimum of the two links SNRs. Hence, the two-hop amplify-and-forward protocol has a higher outage probability (lower quality) than the two-hop decode-and-forward protocol. That is, in terms of outage probability, the two-hop decode-and-forward protocol outperforms the two-hop amplify-and-forward protocol. The above argument is also applicable under different performance measures (for example, if the performance measure was symbol error rate). From our presentation so far, it is clear that the distortion exponents for two-hop decode-and-forward schemes are the same as their corresponding two-hop amplify-and-forward schemes for the repetition channel coding diversity and source coding diversity cases. For example, for the two-hop single-relay decode-and-forward scheme, the minimum expected distortion is given by the lower bound in (23), which has the same distortion exponent as the two-hop single-relay amplify-and-forward scheme. We collect these results in the following theorem:

Theorem 4. *The distortion exponents of the multihop decode-and-forward schemes are:*

- *For the two-hop single relay,*

$$\Delta_{2H-1R-DF} = \frac{2p\beta_m}{p + 2\beta_m}. \tag{40}$$

- *For the two-hop M relays selection channel coding diversity,*

$$\Delta_{2H-MR-DF} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m}. \tag{41}$$

- *For the two-hop two relays source coding diversity,*

$$\Delta_{2H-2R-SC-DF} = \max\left[\frac{4p\beta_m}{3p + 2\beta_m}, \frac{2p\beta_m}{p + 2\beta_m}\right]. \tag{42}$$

3.2.1 Channel Coding Diversity with Two Relays

We consider now the use of channel coding with two-relay decode-and-forward protocols. In this case, the relay will perform joint decoding of the two blocks x_{s_1} and x_{s_2} as illustrated in Fig. 4, which means that when any relay decodes correctly, it could forward both x_{s_1} and x_{s_2}. Allowing the first relay to forward only x_{s_1} if it has decoded correctly will cause a degradation in the performance if the second relay decoded erroneously. Hence, if the first relay decoded correctly and the second did not, it is better (in terms of outage probability) for the first relay to forward both x_{s_1} and x_{s_2}. Clearly, a similar argument could be applied to the operation of the second relay. Also, when both relays decode correctly, allowing the second relay to transmit also x_{s_1} and x_{s_2} will cause a loss of diversity. To gain both advantages (lower outage probability when only one relay decodes correctly and diversity when both correctly decode), we propose to use a space-time transmission scheme. In our case, we choose the Alamouti scheme [24], with the time frame structure as shown in Fig. 4. Then, the distortion exponent of this system is given by the following theorem:

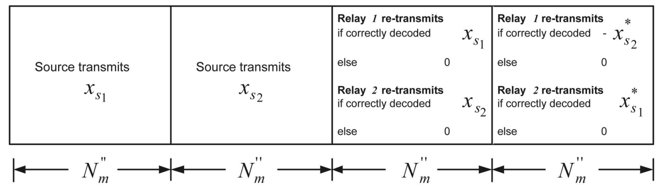

Fig. 4. Two-hop two relays decode-and-forward channel coding diversity system's time frame structure.

Theorem 5. *The distortion exponent of the two-hop two relays channel coding diversity decode-and-forward protocol is*

$$\Delta_{2H-2R-OptCC-DF} = \frac{2p\beta_m}{p + \beta_m}. \tag{43}$$

Proof. The outage probability is given by (proof in Appendix A)

$$P_{outage} = c_o\left(\frac{\exp(pR(D))}{SNR^{2p}}\right). \tag{44}$$

The minimum expected distortion can now be computed as

$$\begin{aligned} E[D] &= \min_D\{P_{outage} + D(1 - P_{outage})\} \\ &\approx \min_D\left\{c_o\left(\frac{\exp(pR(D))}{SNR^{2p}}\right) + D\left[1 - c_o\left(\frac{\exp(pR(D))}{SNR^{2p}}\right)\right]\right\} \\ &\approx \min_D\left\{c_o\left(\frac{\exp(pR(D))}{SNR^{2p}}\right) + D\right\}, \end{aligned} \tag{45}$$

$$\approx \min_D\left\{c_o\frac{D^{-\frac{p}{2\beta''_m}}}{SNR^{2p}} + D\right\}, \tag{46}$$

where D is the source encoder distortion, (45) follows from high SNR approximation, and (46) follows from (3). By differentiating and setting equal to zero, we get the optimizing distortion

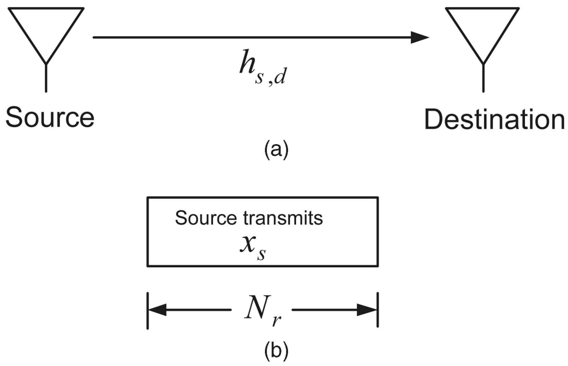

Fig. 5. No diversity (direct transmission) system: (a) system model, (b) time frame structure.

$$D^* = \left(\frac{2\beta''_m}{c_o p}\right)^{\frac{-2\beta''_m}{2\beta''_m+p}} SNR^{\frac{-4\beta''_m p}{2\beta''_m+p}}. \quad (47)$$

Hence, the distortion exponent is given as

$$\Delta_{2H-2R-OptCC-DF} = \frac{4\beta''_m p}{2\beta''_m + p}. \quad (48)$$

For fair comparison, the total number of channel uses should be kept fixed for all schemes. Thus, we have $N_m = 2N''_m$ from which we have $\beta''_m = \frac{1}{2}\beta_m$. Substituting into (48), we get

$$\Delta_{2H-2R-OptCC-DF} = \frac{2p\beta_m}{p + \beta_m}. \quad (49)$$

□

4 Relay Channels

In this section, we present the analysis to derive expressions for the distortion exponents for the different schemes considered before for multihop channels. For comparison purpose, we consider the case when the source transmits a single description source-coded message over the source-destination channel without the help of any relay node. The system is shown in Fig. 5. In this case, the distortion exponent is given by [6]

$$\Delta_{NO-DIV} = \frac{2p\beta_r}{p + 2\beta_r}, \quad (50)$$

where $\beta_r = N_r/K$ and N_r is the number of channel uses for the source block (refer to Fig. 5).

4.1 Amplify-and-Forward Protocol

In this section, we derive the distortion exponent expression for the case of M relays with repetition channel coding diversity by first considering the single-relay channel with an SD source encoder. As for the case of multihop channels, the results show a trade-off between the number of relays (user cooperation diversity) and the quality of the source encoder. We also derive the distortion exponent when using the multiple description coding.

4.1.1 Single Relay

Consider a system consisting of a source, a relay, and a destination as shown in Fig. 6. Transmission of a message is

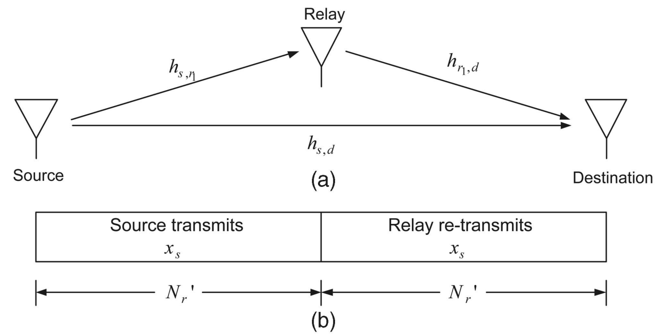

Fig. 6. Single relay system: (a) system model, (b) time frame structure.

done in two phases. In phase 1, the source sends its information to the relay node and the destination. The received signals at the relay and destination nodes are given, respectively, by

$$\begin{aligned} y_{s,r_1} &= h_{s,r_1}\sqrt{P}x_s + n_{s,r_1}, \\ y_{s,d} &= h_{s,d}\sqrt{P}x_s + n_{s,d}, \end{aligned} \quad (51)$$

where the channel coefficients and the noise terms are modeled as in (51).

In phase 2, the relay scales the received signal and retransmits to the destination. The received signal at the destination in phase 2 is as given in (9). We also assume that the destination applies a Maximum Ratio (MR) Combiner to detect the transmitted signal from those received in each phase.

The mutual information of this system is given by Laneman et al. [4]:

$$\begin{aligned} &I(x_s, \mathbf{y}_d) \\ &= \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}\right), \end{aligned} \quad (52)$$

where $SNR = P/N_0$ and $\mathbf{y}_d = [y_{s,d}, y_{r_1,d}]$ is the received data at the destination node during phases 1 and 2. At high SNR, we have

$$\begin{aligned} I(x_s, y_d) &\approx \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) \\ &\approx \log\left(|h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right). \end{aligned} \quad (53)$$

The distortion exponent of this system is given by the following theorem:

Theorem 6. *The distortion exponent of the single-relay amplify-and-forward scheme is*

$$\Delta_{R-1R-AF} = \frac{2p\beta_r}{2p + \beta_r}. \quad (54)$$

Proof. Let

$$W_1 = |h_{s,d}|^2 SNR \text{ and } W_2 = \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}.$$

The outage probability can be calculated as

$$P_{outage} = \Pr[\log(1 + W_1 + W_2) < R(D)] \approx \Pr[W_1 + W_2 < \exp(R(D))]. \quad (55)$$

W_2 is a scaled harmonic mean of the source-relay and relay-destination channels signal-to-noise ratios. Using the following bounds:

$$\Pr(W_1 < w/2)\Pr(W_2 < w/2) < \Pr(W_1 + W_2 < w) < \Pr(W_1 < w)\Pr(W_2 < w),$$

the CDF of $W = W_1 + W_2$ can be upper and lower-bounded as

$$c_3\left(\frac{w}{SNR}\right)^{2p} \lessapprox F_W(w) \lessapprox c_4\left(\frac{w}{SNR}\right)^{2p}, \quad (56)$$

where c_3 and c_4 are constants.

Using the bounds in (56), the minimum expected distortion can be asymptotically upper and lower-bounded as

$$\min_D\left\{c_3\left(\frac{D^{\frac{-p}{\beta_r'}}}{SNR^{2p}}\right) + D\right\} \lessapprox E[D] \lessapprox \min_D\left\{c_4\left(\frac{D^{\frac{-p}{\beta_r'}}}{SNR^{2p}}\right) + D\right\}, \quad (57)$$

where $\beta_r' = N_r'/K$ and N_r' is the number of source node channel uses (refer to Fig. 6). By differentiating the lower bound and setting equal to zero, we get the optimal distortion

$$D^* = \left(\frac{\beta_r}{c_3 p}\right)^{\frac{-\beta_r'}{\beta_r'+p}} SNR^{\frac{-2\beta_r' p}{\beta_r'+p}}. \quad (58)$$

Substituting into (57), we get

$$C_{LB}SNR^{-\frac{2\beta_r' p}{\beta_r'+p}} \lessapprox E[D] \lessapprox C_{UB}SNR^{-\frac{2\beta_r' p}{\beta_r'+p}}, \quad (59)$$

where C_{LB} and C_{UB} are constant terms that do not depend on SNR. Hence, the distortion exponent is given as

$$\Delta_{R-1R-AF} = \frac{2\beta_r' p}{\beta_r' + p}. \quad (60)$$

For fair comparison, we should have $N_r = 2N_r'$ from which we have $\beta_r' = \frac{1}{2}\beta_r$. Substituting into (60), we get

$$\Delta_{R-1R-AF} = \frac{2\beta_r p}{\beta_r + 2p}. \quad (61)$$

□

Asymptotically comparing the distortion exponents for the cases with no diversity and with a single relay, we have

$$\lim_{\beta_r/p\to\infty}\frac{\Delta_{R-1R-AF}}{\Delta_{NO-DIV}} = 2, \quad \lim_{\beta_r/p\to 0}\frac{\Delta_{R-1R-AF}}{\Delta_{NO-DIV}} = \frac{1}{2}. \quad (62)$$

Note that as β_r/p increases (bandwidth increases), the system becomes outage limited because the performance is limited by the outage event. In this case, the single-relay amplify-and-forward system will achieve a higher distortion exponent since it achieves diversity. Conversely, as β_r/p tends to zero (higher channel quality), the performance is not limited by the outage event, but is limited by the source encoder quality performance. A similar observation was made in [6] when comparing the performance of selection channel diversity and multiplexed channel diversity over parallel channels. In the case of the multiplexed channel diversity from [6], we can think of the two parallel channels as a single channel with no diversity but with twice the bandwidth. When regarding the multiplexed channel diversity as a single channel, the performance of selection and multiplexed channel diversities for parallel channels can be compared in the same way as (62).

We extended the analysis to the case of the amplify-and-forward scheme with M relay nodes using *repetition* coding for which the distortion exponent, following the same procedure as before, can be shown to be given by

$$\Delta_{R-MR-AF} = \frac{2(M+1)p\beta_r}{2\beta_r + (M+1)^2 p}. \quad (63)$$

Again we can think of selecting the optimum number of relays to maximize the distortion exponent. This is again a trade-off between the diversity and the quality of the source encoder.

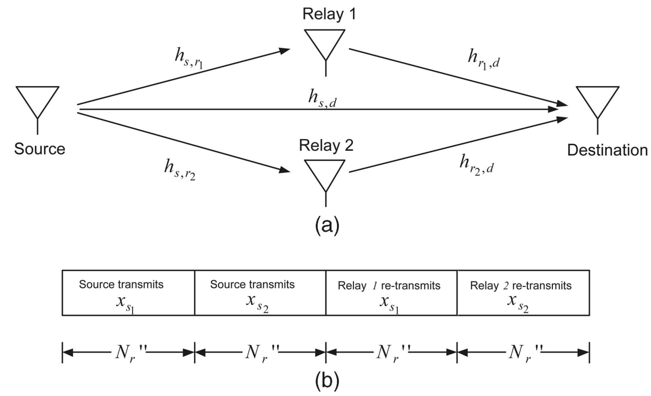

Fig. 7. Two relays system: (a) system model, (b) time frame structure.

4.1.2 Channel Coding Diversity with Two Relays

We consider a system consisting of a source, two relays, and a destination as shown in Fig. 7. The source transmits two channel-coded blocks x_{s_1} and x_{s_2} to the destination and the relay nodes. The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 7. First, we calculate the mutual information for channel coding. In phase 1, the source broadcasts its information to the destination and two relay nodes. The received signals are

$$y_{s,d}(m) = \sqrt{P}h_{s,d}x_{s_m} + n_{s,d}(m), \quad (64)$$

$$y_{s,r_i}(m) = \sqrt{P}h_{s,r_i}x_{s_m} + n_{s,r_i}(m), \quad i = 1,2,\ m = 1,2. \quad (65)$$

Relay 1 will only forward x_{s_1} and relay 2 will only forward x_{s_2}, i.e., relay 1 will amplify-and-forward $y_{s,r_1}(1)$ and relay 2 will amplify-and-forward $y_{s,r_2}(2)$. The received signals at the destination due to relay 1 and relay 2 transmissions are given by

$$y_{r_i,d} = h_{r_i,d}\alpha_i y_{s,r_i}(i) + n_{r_i,d}(i), \quad i = 1, 2, \tag{66}$$

where α_i is the signal amplification performed at the relay, which satisfies the power constraint with equality, that is [4],

$$\alpha_i = \sqrt{\frac{P}{P|h_{s,r_i}|^2 + N_0}}, \tag{67}$$

where all the noise components are modeled as independent zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension.

Define the 4×1 vector $\mathbf{y} = [y_{s,d}(1), y_{s,d}(2), y_{r_1,d}, y_{r_2,d}]^T$. To calculate the mutual information between $\mathbf{x} = [x_{s_1}, x_{s_2}]$ and $\mathbf{y}$, we assume that an MR is applied to $y_{s,d}(1)$ and $y_{r_1,d}$ and another MR is applied to $y_{s,d}(2)$ and $y_{r_2,d}$. The output of the first MR detector is given by

$$r_1 = \zeta_s y_{s,d}(1) + \zeta_1 y_{r_1,d}, \tag{68}$$

where $\zeta_s = \sqrt{P}h^*_{s,d}/N_0$ and $\zeta_1 = \frac{\sqrt{P}\zeta_1 h^*_{r_1,d}h^*_{s,r_1}}{(\zeta_1^2|h_{r_1,d}|^2+1)N_0}$. We can write r_1 in terms of x_{s_1} as

$$r_1 = \left(|h_{s,d}|^2 SNR + \frac{|h_{r_1,d}|^2 SNR |h_{s,r_1}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}\right) x_{s_1} + n_1, \tag{69}$$

where n_1 is a zero-mean complex Gaussian noise of variance

$$|h_{s,d}|^2 SNR + \frac{|h_{r_1,d}|^2 SNR |h_{s,r_1}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}.$$

Similarly, we can have r_2, representing the output of the second MR detector, given by

$$r_2 = \left(|h_{s,d}|^2 SNR + \frac{|h_{r_2,d}|^2 SNR |h_{s,r_2}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR + 1}\right) x_{s_2} + n_2, \tag{70}$$

where n_2 is a zero-mean complex Gaussian noise of variance

$$|h_{s,d}|^2 SNR + \frac{|h_{r_2,d}|^2 SNR |h_{s,r_2}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR + 1}.$$

Next, the pdf of $\mathbf{y}$ given $\mathbf{x}$ and the channel state information (*CSI*) is given by

$$p(\mathbf{y}/\mathbf{x}, CSI) = p(y_{s,d_1}, y_{r_1,d}/x_{s_1}, CSI)p(y_{s,d_2}, y_{r_2,d}/x_{s_2}, CSI). \tag{71}$$

The pdf of $\mathbf{y}$ given $\mathbf{x}$ and the channel state information represents an exponential family of distributions [25]. Therefore, it can be easily shown that given the channel coefficients, r_1 and r_2 are sufficient statistics for $\mathbf{x}$, that is,

$$\begin{aligned} p(\mathbf{y}/\mathbf{x}, r_1, r_2, CSI) &= p(\mathbf{y}/r_1, r_2, CSI) \\ &= p(y_{s,d_1}, y_{r_1,d}/r_1, CSI)p(y_{s,d_2}, y_{r_2,d}/r_2, CSI). \end{aligned} \tag{72}$$

Since r_1 and r_2 are sufficient statistics for $\mathbf{x}$, then the mutual information between $\mathbf{x}$ and $\mathbf{y}$ equals the mutual information between $\mathbf{x}$ and $\mathbf{r} = [r_1, r_2]$ [18], that is,

$$I(\mathbf{x}; \mathbf{r}) = I(\mathbf{x}; \mathbf{y}). \tag{73}$$

For any covariance matrix of $\mathbf{x}$, the mutual information is maximized when $\mathbf{x}$ is zero-mean complex Gaussian random vector [18]. The maximum mutual information can be easily proved to be given by

$$\begin{aligned} I \approx \log&\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) \\ &+ \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}\right), \end{aligned} \tag{74}$$

which is achieved when x_{s_1} and x_{s_2} are independent.

The distortion exponent of this system is given by the following theorem (proof in Appendix B):

Theorem 7. *The distortion exponent of the two relays channel coding diversity with the amplify-and-forward protocol is given by*

$$\Delta_{R-2R-OptCC-AF} = \frac{3p\beta_r}{3p + \beta_r}. \tag{75}$$

4.1.3 Source Coding Diversity with Two Relays

We continue analyzing a system as in Fig. 7, but now we assume that each of the two blocks sent from the source, x_{s_1} and x_{s_2}, represents one description generated from a dual descriptions source encoder. The first relay will only forward the block x_{s_1} and the second relay will only forward x_{s_2} as shown in Fig. 7. The distortion exponent of this system is given by the following theorem:

Theorem 8. *The distortion exponent of the two relays source coding diversity with the amplify-and-forward protocol is given by*

$$\Delta_{R-2R-SC-AF} = \max\left[\frac{2p\beta_r}{2p + \beta_r}, \frac{3p\beta_r}{4p + \beta_r}\right]. \tag{76}$$

Proof. The receiver applies an MR detector on the received data to detect x_{s_1} and x_{s_2}. Let $W_1 = |h_{s,d}|^2 SNR$,

$$W_2 = \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}, \quad \text{and}$$
$$W_3 = \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}$$

The minimum expected end-to-end distortion is given by

$$\begin{aligned} E[D] \approx \min_{D_0, D_1} &\Pr[\log(1 + W_1 + W_2) < R_{md}(D_0, D_1)/2, \\ &\log(1 + W_1 + W_3) < R_{md}(D_0, D_1)/2] \\ &+ \Big(\Pr[\log(1 + W_1 + W_2) < R_{md}(D_0, D_1)/2, \\ &\log(1 + W_1 + W_3) > R_{md}(D_0, D_1)/2] \\ &+ \Pr[\log(1 + W_1 + W_2) > R_{md}(D_0, D_1)/2, \\ &\log(1 + W_1 + W_3) < R_{md}(D_0, D_1)/2]\Big) D_1 \\ &+ \Pr[\log(1 + W_1 + W_2) > R_{md}(D_0, D_1)/2, \\ &\log(1 + W_1 + W_3) > R_{md}(D_0, D_1)/2] D_0, \end{aligned} \tag{77}$$

where R_{md}, D_0, and D_1 are as introduced in Section 2. To calculate the minimum expected distortion, we need to calculate the following probabilities in (77):

$$\begin{aligned} P_1' &= \Pr[\log(1+W_1+W_2) < R_{md}(D_0,D_1)/2, \\ &\quad \log(1+W_1+W_3) < R_{md}(D_0,D_1)/2] \\ &= \Pr[\log(1+W_1+\max(W_2,W_3)) < R_{md}(D_0,D_1)/2] \\ &\approx c_{s1}\frac{1}{SNR^{3p}}\exp\left(\frac{3p}{2}R_{md}(D_0,D_1)\right), \end{aligned} \tag{78}$$

$$\begin{aligned} P_2' &= \Pr[\log(1+W_1+W_2) > R_{md}(D_0,D_1)/2, \\ &\quad \log(1+W_1+W_3) > R_{md}(D_0,D_1)/2] \\ &= \Pr[\log(1+W_1+\min(W_2,W_3)) > R_{md}(D_0,D_1)/2] \\ &= 1-\Pr[\log(1+W_1+\min(W_2,W_3)) < R_{md}(D_0,D_1)/2] \\ &\approx 1-c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1)), \end{aligned} \tag{79}$$

$$\begin{aligned} P_3' &= \Pr[\log(1+W_1+W_2) < R_{md}(D_0,D_1)/2, \\ &\qquad \log(1+W_1+W_3) > R_{md}(D_0,D_1)/2] \\ &\quad + \Pr[\log(1+W_1+W_2) > R_{md}(D_0,D_1)/2, \\ &\qquad \log(1+W_1+W_3) < R_{md}(D_0,D_1)/2] \\ &= 1-P_1'-P_2' \approx c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1)) \\ &\quad - c_{s1}\frac{1}{SNR^{3p}}\exp\left(\frac{3p}{2}R_{md}(D_0,D_1)\right) \\ &\approx c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1)). \end{aligned} \tag{80}$$

The minimum expected distortion in (77) can now be calculated as

$$\begin{aligned} E[D] &\approx \min_{D_0,D_1}\left\{c_{s1}\frac{1}{SNR^{3p}}\exp\left(\frac{3p}{2}R_{md}(D_0,D_1)\right)\right. \\ &\quad + c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1))D_1 \\ &\quad \left. + \left(1-c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1))\right)D_0\right\} \\ &\approx \min_{D_0,D_1}\left\{c_{s1}\frac{1}{SNR^{3p}}\exp\left(\frac{3p}{2}R_{md}(D_0,D_1)\right)\right. \\ &\quad \left. + c_{s2}\frac{1}{SNR^{2p}}\exp(pR_{md}(D_0,D_1))D_1 + D_0\right\}. \end{aligned} \tag{81}$$

Substituting from (6) yields upper- and lower bounds for the minimum expected end-to-end distortion as

$$\begin{aligned} E[D] &\gtrapprox \min_{D_0,D_1}\frac{c_{s1}}{SNR^{3p}}\left(\frac{1}{4D_0D_1}\right)^{\frac{3p}{4\beta_r''}} + \frac{c_{s2}}{SNR^{2p}}\left(\frac{1}{4D_0D_1}\right)^{\frac{p}{2\beta_r''}}. \\ &\quad D_1 + D_0 \\ E[D] &\lessapprox \min_{D_0,D_1}\frac{c_{s1}}{SNR^{3p}}\left(\frac{1}{2D_0D_1}\right)^{\frac{3p}{4\beta_r''}} + \frac{c_{s2}}{SNR^{2p}}\left(\frac{1}{2D_0D_1}\right)^{\frac{p}{2\beta_r''}}. \\ &\quad D_1 + D_0. \end{aligned} \tag{82}$$

Note that for $p \geq 2\beta_r''$, the minimum expected distortion increases as D_1 decreases. Hence, the optimal choice of D_1 approaches a constant that is bounded away from zero [6].

In the context of parallel channels, the notion of multiplexed channel coding diversity was presented in [6]. Compared to the direct transmission, the gain in the distortion exponent for the multiplexed channel coding diversity scheme in [6] is a result of the increase of the bandwidth due to the simultaneous use of parallel channels. In the multiplexed channel coding diversity scheme discussed in [6], the two blocks x_{s_1} and x_{s_2} represent a split of a channel-coded message from an SD source encoder over two parallel channels, which will be the two source-relay-destination links in our system. In our system, there is no gain in using multiplexed channel coding diversity because, for fair comparison, using either one relay or two relays does not increase the bandwidth of the system. The multiplexed channel coding diversity in this case is equivalent to allowing one relay helping the source to forward an SD source-coded message during one block and using the other relay for the next block. Hence, in our system, the multiplexed channel coding diversity is equivalent to the single relay system with the same distortion exponent.

For $D_1 \geq 1/2$, the source coding rate is given by (7) and not (6). The optimal system in this case degenerates to the channel multiplexed scheme, which is equivalent, in our system, to the single-relay system as described above. Thus, the distortion exponent is given by

$$\Delta_{R-2R-SC-AF} = \frac{2p\beta_r}{2p+\beta_r}, \quad p \geq \frac{1}{2}\beta_r = 2\beta_r''. \tag{83}$$

For $p < 2\beta_r''$, we can find the optimal value of D_1 by differentiating the lower bound in (82) and setting equal to zero. We get

$$D_1^* = \left(\frac{c_{s_1}}{c_{s_2}}\left(\frac{3p}{(\beta_r-2p)}\right)\right)^{\frac{\beta_r}{p+\beta_r}} SNR^{-\frac{p\beta_r}{p+\beta_r}}(4D_0)^{-\frac{p}{p+\beta_r}}, \; p < \frac{1}{2}\beta_r, \tag{84}$$

where, for fair comparison, we fix the total number of channel uses and get $\beta_r'' = \frac{1}{4}\beta_r$. For the case when $p < \frac{1}{2}\beta_r$, substituting (84) in the lower bound into (82), we get

$$E[D] \gtrapprox \min_{D_0} C.D_0^{-\frac{3p}{p+\beta_r}}.SNR^{-\frac{3p\beta_r}{p+\beta_r}} + D_0, \quad p < \frac{1}{2}\beta_r, \tag{85}$$

where C is a constant that does not depend on D_0 and the SNR. By differentiating and setting equal to zero, we can get the expression for the optimizing D_0 as

TABLE 1
Distortion Exponents for the Amplify-and-Forward (Decode-and-Forward) Multihop and Relay Channels

	Multi-Hop Channels	Relay Channel
Single relay	$\frac{2p\beta_m}{p+2\beta_m}$	$\frac{2p\beta_r}{2p+\beta_r}$
Selective channel coding diversity with M relays	$\frac{4Mp\beta_m}{M(M+1)p+4\beta_m}$	$\frac{2(M+1)p\beta_r}{2\beta_r+(M+1)^2p}$
Channel coding diversity with 2 relays	$\frac{2p\beta_m}{p+\beta_m}$	$\frac{3p\beta_r}{3p+\beta_r}$
Source coding diversity with 2 relays	$\max\left[\frac{4p\beta_m}{3p+2\beta_m}, \frac{2p\beta_m}{p+2\beta_m}\right]$	$\max\left[\frac{2p\beta_r}{2p+\beta_r}, \frac{3p\beta_r}{4p+\beta_r}\right]$

$$D_0^* = C'.SNR^{-\frac{3p\beta_r}{4p+\beta_r}}, \quad p < \frac{1}{2}\beta_r. \tag{86}$$

Hence, from (86), we have

$$C'_{LB}SNR^{-\frac{3p\beta_r}{4p+\beta_r}} \lessapprox E[D] \lessapprox C'_{UB}SNR^{-\frac{3p\beta_r}{4p+\beta_r}}, \quad p < \frac{1}{2}\beta_r. \tag{87}$$

From (83) and (87), we conclude that the distortion exponent for the source diversity system is given by

$$\Delta_{R-2R-SC-AF} = \max\left[\frac{2p\beta_r}{2p+\beta_r}, \frac{3p\beta_r}{4p+\beta_r}\right], \tag{88}$$

where the second term in (88) is the maximum for the case $p < \frac{1}{2}\beta_r$. □

4.2 Decode-and-Forward Relay Channel

We now analyze the decode-and-forward relay channel. In the case of channel coding diversity with two-relay decode-and-forward protocol, the relay will perform joint decoding of the two blocks x_{s_1} and x_{s_2} as illustrated in Fig. 4, which means that when any relay decodes correctly, it could forward both x_{s_1} and x_{s_2}. Allowing the first relay to forward only x_{s_1} if it has decoded correctly will cause a degradation in the performance if the second relay decoded erroneously. Hence, if the first relay decoded correctly and the second did not, it is better (in terms of outage probability) for the first relay to forward both x_{s_1} and x_{s_2}. Clearly, a similar argument applies to the operation of the second relay. Also, when both relays decode correctly, allowing the second relay to transmit also x_{s_1} and x_{s_2} will cause a loss in diversity. To gain both advantages (lower outage probability when only one relay decodes correctly and diversity when both relays correctly decode), we propose to use a space-time transmission scheme in the relay transmission phase. In our case, we choose the Alamouti scheme [24], with the time frame structure as shown in Fig. 4.

The distortion exponents for the different schemes can be derived following the same procedure as in the previous sections. Due to the space limitations, we collect the corresponding results in the following theorem:

Theorem 9. *The distortion exponents of the decode-and-forward relay channel are:*

- *For the single-relay channel,*

$$\Delta_{R-1R-DF} = \frac{2p\beta_r}{2p+\beta_r}. \tag{89}$$

- *For the M relays repetition channel coding diversity,*

$$\Delta_{R-MR-DF} = \frac{2(M+1)p\beta_r}{2\beta_r + (M+1)^2 p}. \tag{90}$$

- *For the channel coding with two relays, with the same time frame structure as shown in Fig. 4,*

$$\Delta_{R-2R-OptCC-DF} = \frac{3p\beta_r}{3p+\beta_r}. \tag{91}$$

- *For the source coding diversity with two relays,*

$$\Delta_{R-2R-SC-DF} = \max\left[\frac{2p\beta_r}{2p+\beta_r}, \frac{3p\beta_r}{4p+\beta_r}\right]. \tag{92}$$

In summary, the distortion exponents for the decode-and-forward relay channel are the same as the amplify-and-forward relay channel.

5 Discussion

The distortion exponents for the various schemes analyzed in this paper are given in Table 1. From the results in Table 1, we can see that the channel coding diversity scheme always results in a higher distortion exponent than the source coding diversity scheme at any bandwidth expansion factor (the result is valid over both the multihop and relay channels). This suggests that, between source and channel coding, it is better to exploit diversity at the channel encoder level. Comparing the expressions for the distortion exponents for the single relay and M relay nodes, we can see that increasing the number of relays does not always result in an increase in the distortion exponent, showing that there is a trade-off between the quality (resolution) of the source encoder and the amount of cooperation (number of relays).

Fig. 8 compares the distortion exponent for the various systems as a function of β_m for the two-hop channel. The results in Fig. 8 confirm that the channel coding diversity gives better distortion exponent than the source coding diversity. A similar observation was made in [6] for the case of parallel channels. Note that as β_m increases, the factor that limits the distortion exponent performance is the diversity (number of relays nodes). In this case (high β_m), the system is said to be an outage-limited system as the outage probability, rather than the quality of the source encoder, is the main limiting factor in the end-to-end distortion. Fig. 8 shows that in this scenario, the distortion exponent

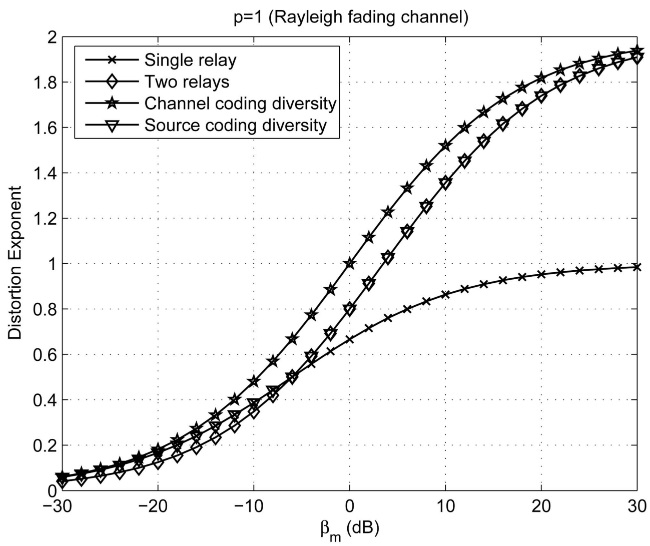

Fig. 8. Distortion exponents for two-hop amplify-and-forward (decode-and-forward) protocol.

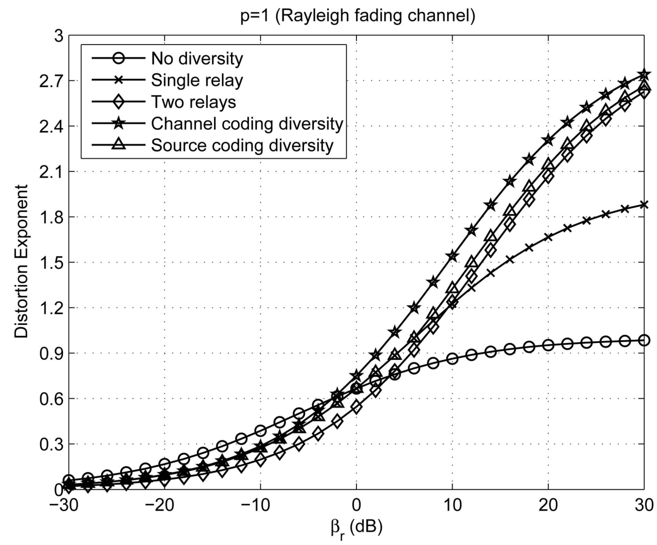

Fig. 9. Distortion exponents for amplify-and-forward (decode-and-forward) relay channel.

performance is improved by increasing the number of relays so as to increase diversity. At low β_m, the system is said to be quality limited as the quality of the source encoder (distortion under no outage), rather than the outage probability, is the main limiting factor in the end-to-end distortion. In this case, the gain from using a better source encoder, which has a higher resolution, is more significant than the gain from increasing the number of relay nodes. Fig. 8 shows that in this scenario, the distortion exponent performance is improved by using only a single-relay node allowing for the use of a higher resolution source encoder.

Fig. 9 shows the distortion exponent versus β_r for the various relay channel schemes. Fig. 9 confirms that the scheme with channel coding diversity yields better distortion exponent than the one with source coding diversity. As was the case for two-hop schemes, as β_m increases, diversity becomes the limiting factor for the distortion exponent, in which case, Fig. 9 shows that increasing the number of relays improves the distortion exponent results. Again, at low β_m, direct transmission (no-diversity) results in a lower end-to-end distortion, which can be interpreted in the same way as for the multihop channel.

6 Conclusion

In this paper, we have studied the performance limit of systems that may present diversity in the form of source coding, channel coding, and user cooperation diversity, and their possible combinations. In the case of source coding, diversity is introduced through the use of dual-description source encoders. Channel coding diversity is obtained from joint decoding of channel-coded blocks sent through different channels. User cooperation diversity is achieved via schemes that operate over multihop or relay channels. We have considered user cooperation using either the amplify-and-forward or the decode-and-forward techniques.

The presented study focused on analyzing the achievable performance limit, which was measured in terms of the distortion exponent. The distortion exponent measures the rate of decay of the end-to-end distortion at high SNRs. Our results show that for both relay and multihop channels, channel coding diversity provides better performance, followed by the source coding diversity. In the case of relay channels, we note that at low bandwidth, it is not the channel outage event, but the distortion introduced at the source coding stage is the dominant factor limiting the distortion exponent performance. Therefore, in these cases, it is better not to cooperate and use a lower distortion source encoder. Similarly, we showed that as the bandwidth expansion factor increases, the distortion exponent is improved by increasing the number of relays because user cooperation diversity is the main limiting factor. In these cases, the system is said to be an outage-limited system. Therefore, it is better to cooperate in these cases, which results in minimizing the outage probability, and consequently, minimizing the end-to-end distortion.

Appendix A

Outage Analysis for Optimal Channel Coding Diversity with Two Relays over Two-Hop Channels Using the Decode-and-Forward Scheme

Let $S\longrightarrow R_i$ and $R_i\longrightarrow D$ denote the channel between the source and the ith relay and the channel between the ith relay and the destination, respectively. Let $R_1, R_2\longrightarrow D$ denote the channel between the two relays and the destination when both relays decode correctly.

The event of outage can be split into four disjoint events from which the outage probability can be calculated as $P_{outage} = P_{o_1} + P_{o_2} + P_{o_3} + P_{o_4}$, where

$$\begin{aligned} P_{o_1} &= \Pr[S\longrightarrow R_1 \text{ in outage},\ S\longrightarrow R_2 \text{ in outage}] \\ &= \Pr[S\longrightarrow R_1 \text{ in outage}].\Pr[S\longrightarrow R_2 \text{ in outage}] \\ &= \Pr[2\log(1+|h_{s,r_1}|^2 SNR) < R(D)]. \\ &\quad \Pr[2\log(1+|h_{s,r_2}|^2 SNR) < R(D)] \\ &\approx c_{o_1}\left(\frac{\exp(pR(D))}{SNR^{2p}}\right), \end{aligned} \tag{93}$$

$$\begin{aligned} P_{o_2} &= \Pr[S \longrightarrow R_1 \text{ in outage},\ S \longrightarrow R_2 \text{ not in outage}, \\ &\quad R_2 \longrightarrow D \text{ in outage}] \\ &= \Pr[S \longrightarrow R_1 \text{ in outage}].\ \Pr[S \longrightarrow R_2 \text{ not in outage}]\ . \\ &\quad \Pr[R_2 \longrightarrow D \text{ in outage}] \\ &\approx c_{o_2}\left(\frac{\exp(pR(D))}{SNR^{2p}}\right), \end{aligned} \tag{94}$$

$$\begin{aligned} P_{o_3} &= \Pr[S \longrightarrow R_2 \text{ in outage},\ S \longrightarrow R_1 \text{ not in outage}, \\ &\quad R_1 \longrightarrow D \text{ in outage}] \\ &\approx c_{o_3}\left(\frac{\exp(pR(D))}{SNR^{2p}}\right), \end{aligned} \tag{95}$$

$$\begin{aligned} P_{o_4} &= \Pr[S \longrightarrow R_1 \text{ not in outage},\ S \longrightarrow R_2 \text{ not in outage}, \\ &\quad R_1, R_2 \longrightarrow D \text{ in outage}] \\ &= \Pr[S \longrightarrow R_1 \text{ not in outage}]\ .\Pr[S \longrightarrow R_2 \text{ not in outage}]. \\ &\quad \Pr[R_1, R_2 \longrightarrow D \text{ in outage}] \\ &\approx \Pr\left[2\log\left(1 + \frac{1}{2}(|h_{r_1,d}|^2 SNR + |h_{r_2,d}|^2 SNR)\right) < R(D)\right], \end{aligned} \tag{96}$$

where the factor $1/2$ in (96) is due to the loss in SNR because of the use of transmit diversity [24]. To calculate P_{o_4} in (96), we need to calculate the cumulative distribution function (CDF) of the random variable $|h_{r_1,d}|^2 SNR + |h_{r_2,d}|^2 SNR$. Let $W_1 = |h_{r_1,d}|^2 SNR$ and $W_2 = |h_{r_2,d}|^2 SNR$. The pdf of $W_1 + W_2$ can be computed as

$$\begin{aligned} f_{W_1+W_2}(w) &= \int_0^w f_{W_1}(\tau) f_{W_2}(w-\tau)d\tau \\ &\approx \frac{c_{11}c_{22}p^2}{SNR^{2p}} \int_0^w \tau^{p-1}(w-\tau)^{p-1}d\tau \\ &= c_{11}c_{22}p^2 \frac{w^{2p-1}}{SNR^{2p}} B(p,p), \end{aligned} \tag{97}$$

where $B(.,.)$ is the Beta function [26]. The CDF of $W_1 + W_2$ can be computed as

$$F_{W_1+W_2}(w) = \int_0^w f_{W_1+W_2}(\tau)d\tau = c_{33}\left(\frac{w}{SNR}\right)^{2p}, \tag{98}$$

from which we have

$$P_{o_4} \approx c_{o_4}\left(\frac{\exp(pR(D))}{SNR^{2p}}\right). \tag{99}$$

Then, the outage probability is

$$P_{outage} = P_{o_1} + P_{o_2} + P_{o_3} + P_{o_4} \approx c_o\left(\frac{\exp(pR(D))}{SNR^{2p}}\right). \tag{100}$$

In the proof, we have assumed that x_{s_1} and x_{s_2} are independent zero-mean complex Gaussian with variance $1/2$ per dimension. We can easily show that this choice of x_{s_1} and x_{s_2} is the optimal choice for maximizing the mutual information (minimizing the outage probability) by inspection of the individual outage events in (100).

Appendix B

The Distortion Exponent for the Channel Coding Diversity with Two Relays

In this appendix, we will derive expressions for a lower bound as well as an upper bound on the required distortion exponent. The upper bound and the lower bound turn to be the same, which gives us an expression for the desired distortion exponent.

We start with the analysis of a suboptimal system at the destination node. This suboptimal system will give a lower bound on the distortion exponent. In the suboptimal system, the detector (suboptimal detector) selects the paths with the highest SNR and does not apply an MR detector (the optimal detector is the one that applies MR on the received signals). For example, for x_{s_1}, it either selects the source-destination link or the source-relay-destination link based on which one has higher SNR. The mutual information for the suboptimal system can be easily proved to be

$$\begin{aligned} I_{sub} &\approx \log\left(1 + \max\left(|h_{s,d}|^2 SNR, \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right)\right) \\ &\quad + \log\left(1 + \max\left(|h_{s,d}|^2 SNR, \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}\right)\right). \end{aligned} \tag{101}$$

Let

$$W_1 = |h_{s,d}|^2 SNR, W_2 = \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}, \quad \text{and}$$

$$W_3 = \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}.$$

The outage probability of the suboptimal system is given by

$$\begin{aligned} P_{outage} &= \Pr[I_{sub} < R] = \Pr[\log(1 + \max(W_1, W_2)) \\ &\quad + \log(1 + \max(W_1, W_3)) < R] \\ &= \Pr\Big[\{2\log(1 + W_1) < R, W_1 > W_2, W_1 > W_3\} \\ &\quad \bigcup \{\log(1 + W_1) + \log(1 + W_3) < R, W_1 \\ &\quad > W_2, W_3 > W_1\} \bigcup \{\log(1 + W_2) + \log(1 + W_1) \\ &\quad < R, W_2 > W_1, W_1 > W_3\} \\ &\quad \bigcup \{\log(1 + W_2) + \log(1 + W_3) \\ &\quad < R, W_2 > W_1, W_3 > W_1\}\Big] \\ &= \Pr[2\log(1 + W_1) < R, W_1 > W_2, W_1 > W_3] \\ &\quad + \Pr[\log(1 + W_1) + \log(1 + W_3) < R, W_1 \\ &\quad > W_2, W_3 > W_1] + \Pr[\log(1 + W_2) + \log(1 + W_1) \\ &\quad < R, W_2 > W_1, W_1 > W_3] + \Pr[\log(1 + W_2) \\ &\quad + \log(1 + W_3) < R, W_2 > W_1, W_3 > W_1], \end{aligned} \tag{102}$$

where the last equality follows from the events being disjoint. In the last equation, we used R instead of $R(D)$ for simplicity of presentation. The joint pdf of W_1, W_2, and W_3, which are independent random variables, is given by

$$f(w_1, w_2, w_3) \approx c_j p^3 \left(\frac{w_1^{p-1} w_2^{p-1} w_3^{p-1}}{SNR^{3p}} \right), \tag{103}$$

where c_j is a constant. To find the outage probability, we calculate the probability of the individual outage events in (102),

$$\begin{aligned} P_1 &= \Pr[2\log(1+W_1) < R, W_1 > W_2, W_1 > W_3] \\ &= \int_{w_1=0}^{\exp(R/2)} \int_{w_3=0}^{w_1} \int_{w_2=0}^{w_1} f(w_1, w_2, w_3) dw_2 dw_3 dw_1 \\ &\approx \frac{c_j \exp\left(\frac{3pR}{2}\right)}{3SNR^{3p}}, \end{aligned} \tag{104}$$

$$\begin{aligned} P_2 &= \Pr[\log(1+W_1) + \log(1+W_3) < R, W_1 > W_2, W_3 > W_1] \\ &\approx \Pr[\log(W_1) + \log(W_3) < R, W_1 > W_2, W_3 > W_1] \\ &= \int_{w_1=0}^{\exp(R/2)} \int_{w_3=w_1}^{\frac{\exp(R)}{w_1}} \int_{w_2=0}^{w_1} f(w_1, w_2, w_3) dw_2 dw_3 dw_1 \\ &\approx \int_{w_1=0}^{\exp(R/2)} \int_{w_3=w_1}^{\frac{\exp(R)}{w_1}} \int_{w_2=0}^{w_1} c_j p^3 \left(\frac{w_1^{p-1} w_2^{p-1} w_3^{p-1}}{SNR^{3p}} \right) dw_2 dw_3 dw_1 \\ &= \frac{c_j p^2}{SNR^{3p}} \int_{w_1=0}^{\exp(R/2)} \int_{w_3=w_1}^{\frac{\exp(R)}{w_1}} w_1^{2p-1} w_3^{p-1} dw_3 dw_1 \\ &= \frac{2c_j \exp\left(\frac{3pR}{2}\right)}{3SNR^{3p}}, \end{aligned} \tag{105}$$

$$\begin{aligned} P_3 &= \Pr[\log(1+W_2) + \log(1+W_1) < R, W_2 > W_1, W_1 > W_3] \\ &\approx \Pr[\log(W_1) + \log(W_3) < R, W_1 > W_2, W_3 > W_1] \\ &\approx \frac{2c_j \exp\left(\frac{3pR}{2}\right)}{3SNR^{3p}}, \end{aligned} \tag{106}$$

$$\begin{aligned} P_4 &= \Pr[\log(1+W_2) + \log(1+W_3) < R, W_2 > W_1, W_3 > W_1] \\ &\approx \Pr[\log(W_2) + \log(W_3) < R, W_2 > W_1, W_3 > W_1] \\ &= \int_{w_1=0}^{\exp(R/2)} \int_{w_2=w_1}^{\frac{\exp(R)}{w_1}} \int_{w_3=w_1}^{\frac{\exp(R)}{w_2}} f(w_1, w_2, w_3) dw_3 dw_2 dw_1 \\ &\approx \int_{w_1=0}^{\exp(R/2)} \int_{w_2=w_1}^{\frac{\exp(R)}{w_1}} \int_{w_3=w_1}^{\frac{\exp(R)}{w_2}} c_j p^3 \left(\frac{w_1^{p-1} w_2^{p-1} w_3^{p-1}}{SNR^{3p}} \right) dw_3 dw_2 dw_1 \\ &= \frac{4c_j \exp\left(\frac{3pR}{2}\right)}{3SNR^{3p}}, \end{aligned} \tag{107}$$

where we have $\lim_{w_1 \to 0^+} w_1^p \log w_1 = 0$ for $p \geq 1$. The outage probability for the suboptimal system is

$$P_{outage} = P_1 + P_2 + P_3 + P_4 \approx \frac{c_m \exp\left(\frac{3pR}{2}\right)}{SNR^{3p}},$$

where c_m is a constant. The minimum expected end-to-end distortion can now be computed as

$$\begin{aligned} E[D] &= \min_D \{P_{outage} + (1 - P_{outage})D\} \\ &\approx \min_D \left\{ \frac{c_m \exp\left(\frac{3pR}{2}\right)}{SNR^{3p}} + \left(1 - \frac{c_m \exp\left(\frac{3pR}{2}\right)}{SNR^{3p}}\right) D \right\} \\ &\approx \min_D \left\{ \frac{c_m D^{\frac{-3p}{4\beta_r''}}}{SNR^{3p}} + \left(1 - \frac{c_m D^{\frac{-3p}{4\beta_r''}}}{SNR^{3p}}\right) D \right\} \\ &\approx \min_D \left\{ \frac{c_m D^{\frac{-3p}{4\beta_r''}}}{SNR^{3p}} + D \right\}, \end{aligned} \tag{108}$$

where $\beta_r'' = N_r''/K$ (refer to Fig. 7), D is the source encoder distortion, and we have used both high SNR approximations and (3). By differentiating and setting equal to zero, we get the optimizing distortion

$$D^* = \left(\frac{4\beta_r''}{3c_m p} \right)^{\frac{-4\beta_r''}{4\beta_r''+3p}} SNR^{\frac{-12\beta_r'' p}{4\beta_r''+3p}}. \tag{109}$$

By substituting, we get the distortion exponent for this suboptimal system as

$$\Delta_{SUBOPTIMAL} = \frac{12\beta_r'' p}{4\beta_r'' + 3p}. \tag{110}$$

For fair comparison, the total number of channel uses is fixed, and thus, $\beta_r'' = \frac{1}{4}\beta_r$.

For the optimal detector (the one using an MR detector), the distortion exponent satisfies

$$\Delta_{R-2R-OptCC-AF} \geq \Delta_{SUBOPTIMAL} = \frac{3\beta_r p}{\beta_r + 3p}. \tag{111}$$

Next, we find an upper bound on the distortion exponent for the optimal system. In this case, the mutual information in (74) can be upper- and lower-bounded as

$$\begin{aligned} &\log(1 + 2W_1 + W_2 + W_3) \leq \log(1 + W_1 + W_2) \\ &\quad + \log(1 + W_1 + W_3) \leq 2\log\left(1 + W_1 + \frac{1}{2}W_2 + \frac{1}{2}W_3\right), \end{aligned}$$

where

$$W_1 = |h_{s,d}|^2 SNR, W_2 = \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}, \text{ and}$$

$$W_3 = \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}$$

are nonnegative numbers. The upper bound follows from the concavity of the $\log$-function. Therefore, the outage probability P_o of the optimal system can be upper- and lower-bounded as

$$\begin{aligned} &\Pr\left[2\log\left(1 + W_1 + \frac{1}{2}W_2 + \frac{1}{2}W_3\right) < R\right] \leq P_o \\ &\quad \leq \Pr[\log(1 + 2W_1 + W_2 + W_3) < R]. \end{aligned} \tag{112}$$

From (112), we can easily show that

$$C_L \frac{\exp\left(\frac{3pR}{2}\right)}{SNR^{3p}} \lessapprox P_o \lessapprox C_U \frac{\exp(3pR)}{SNR^{3p}}, \tag{113}$$

where C_L and C_U are two constants that do not depend on the SNR. Similar to the suboptimal system, and using (113), the minimum expected end-to-end distortion for the optimal system can be lower-bounded as

$$\begin{aligned} E[D] &\gtrapprox \min_D \left\{ C_L \frac{\exp\left(\frac{3pR}{2}\right)}{SNR^{3p}} + \left(1 - C_U \frac{\exp(3pR)}{SNR^{3p}}\right) D \right\} \\ &\approx \min_D \left\{ \frac{C_L D^{\frac{-3p}{4\beta''_r}}}{SNR^{3p}} + \left(1 - \frac{C_U D^{\frac{-3p}{2\beta''_r}}}{SNR^{3p}}\right) D \right\} \\ &\approx \min_D \left\{ \frac{C_L D^{\frac{-3p}{4\beta''_r}}}{SNR^{3p}} + D \right\}. \end{aligned} \quad (114)$$

By differentiating the lower bound and setting equal to zero, we get the optimizing distortion as

$$D^* = \left(\frac{4\beta''_r}{3pC_L}\right)^{\frac{-4\beta''_r}{4\beta''_r+3p}} SNR^{\frac{-12\beta''_r p}{4\beta''_r+3p}}. \quad (115)$$

By substituting, we get

$$E[D] \gtrapprox C_{LO} SNR^{\frac{-12\beta''_r p}{4\beta''_r+3p}} \quad (116)$$

from which we can upper bound the distortion exponent of the optimal system as

$$\Delta_{R-2R-OptCC-AF} \leq \frac{12\beta''_r p}{4\beta''_r + 3p} = \frac{3\beta_r p}{\beta_r + 3p}. \quad (117)$$

Finally, from (111) and (117), we get

$$\Delta_{R-2R-OptCC-AF} = \frac{3\beta_r p}{\beta_r + 3p}. \quad (118)$$

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