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Full-Diversity Space-Frequency Codes for MIMO-OFDM Systems

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Abstract — **This paper addresses the problem of space-frequency code design for broadband multi-antenna OFDM systems. We show that space-time codes achieving full diversity in quasi-static flat fading environment can be used to construct space-frequency codes that achieve the maximum diversity available in frequency selective MIMO fading channels. Thus, the abundant classes of existing space-time block and trellis codes can be used for full diversity transmission in MIMO-OFDM systems.**

In order to take advantages of both the MIMO systems and the OFDM, *space-frequency* (SF) coded MIMO-OFDM systems were introduced in [1], and later works [2], [3] also described such systems. The performance criteria for SF coded MIMO-OFDM systems were derived in [4] and [6]. In [4], the authors showed that, in general, existing ST codes cannot exploit the diversity available in frequency selective MIMO channels, and it was suggested that a completely new code design procedure will have to be developed for MIMO-OFDM systems. Later in [5], they proposed a class of SF codes achieving full diversity with the assumption that all of the path delays are located exactly at the sampling instances.

We consider a SF coded MIMO-OFDM system with M_t transmit antennas, M_r receive antennas and N subcarriers. Suppose that the frequency-selective fading channels have L independent delay paths and the same power delay profile. The MIMO channel is assumed to be spatially uncorrelated and constant over each OFDM block period. The channel impulse response from transmit antenna i to receive antenna j can be modeled as $h_{i,j}(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}(l)\delta(\tau - \tau_l)$, where τ_l is the delay and $\alpha_{i,j}(l)$ is the complex amplitude of the l-th path between transmit antenna i and receive antenna j. The $\alpha_{i,j}(l)$'s are modeled as zero-mean complex Gaussian random variables with variances $E|\alpha_{i,j}(l)|^2 = \delta_l^2$, and $\sum_{l=0}^{L-1} \delta_l^2 = 1$. Thus, we consider arbitrary power delay profiles.

Each SF codeword is an $N \times M_t$ matrix $C = \{c_i(n)\}_{0\le n\le N-1, 1\le i\le M_t}$, where $c_i(n)$ denotes the channel symbol transmitted over the n-th subcarrier by transmit antenna i. The OFDM transmitter applies an N-point IFFT to each column of the matrix C. After appending the cyclic prefix, the OFDM symbol corresponding to the i-th ($i = 1, 2, \cdots, M_t$) column of C is transmitted by transmit antenna i. At the receiver, after applying FFT, the received signal at the n-th subcarrier at receive antenna j is given by

$$y_j(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i(n) H_{i,j}(n) + z_j(n), \qquad (1)$$

where $H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-j2\pi n \Delta f \tau_l}$ is the channel frequency response at the n-th subcarrier between transmit antenna i and receive antenna j, $z_j(n)$ is the noise component, and ρ is the average SNR at the receiver.

For SF-coded MIMO-OFDM systems, the maximum achievable diversity is at most $\min\{LM_tM_r, NM_r\}$ ([4], [6], [2], and [7]). In this work, the SF encoder will consist of a ST encoder and a mapping $\mathcal{M}_l$. For each $1 \times M_t$ output vector $(g_1\ g_2 \cdots g_{M_t})$ from the ST encoder and a fixed number $l\,(1 \le l \le L)$, the mapping $\mathcal{M}_l$ is defined as

$$\mathcal{M}_l : (g_1\ g_2 \cdots g_{M_t}) \to \mathbf{1}_{l\times 1}(g_1\ g_2 \cdots g_{M_t}), \qquad (2)$$

where $\mathbf{1}_{l\times 1}$ is an all one matrix of size $l \times 1$. Denoting the output code matrix of the ST encoder by G. For space-time block encoder, G is a concatenation of some block codewords. For space-time trellis encoder, G corresponds to a path of length kM_t starting and ending at the zero state. Then, the code C of size $N \times M_t$ is constructed as $C = \begin{bmatrix} \mathcal{M}_l(G) \\ \mathbf{0}_{(N-klM_t)\times M_t} \end{bmatrix}$, where $\mathcal{M}_l(G) = [I_{kM_t} \otimes \mathbf{1}_{l\times 1}]\,G$. In fact, the SF code C is obtained by repeating each row of G l times and adding some zeros. We have the following result (see [7] for the proof).

Theorem 1: *Suppose that an MIMO-OFDM system equipped with M_t transmit antennas and M_r receive antennas has N subcarriers, and the frequency selective channel has L independent paths. If a space-time (block or trellis) code designed for M_t transmit antennas achieves full diversity for quasi-static flat fading channels, then the space-frequency code obtained from this space-time code via the mapping $\mathcal{M}_l\,(1 \le l \le L)$ will achieve a diversity order of at least $min\{lM_tM_r, NM_r\}$.*

From Theorem 1, we can see that the SF code obtained from a full diversity ST code via the mapping $\mathcal{M}_L$ achieves the maximum achievable diversity $min\{LM_tM_r, NM_r\}$. The symbol rate here is much better than that in [5]. For more details, see [7], in which we also investigate the effects of the delay and power distributions of the channel impulse responses on the performance of the resulting SF codes.

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