OFDM Channel Estimation Based on Time-Frequency Polynomial Model of Fading Multipath Channel

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Abstract— **Orthogonal frequency division multiplexing (OFDM) scheme gains growing interests in broadband data communications for wireless communications because of its great immunity to fast fading and intersymbol interference. The channel estimation is a crucial aspect in the design of OFDM systems. In this work, we propose a channel estimation algorithm based on polynomial approximation of the channel parameters in both time and frequency domains. The method exploits both the time and frequency correlation of the channel parameters. The estimator is robust and needs a little prior knowledge about the delay and fading properties of the channel. It can even adjust itself to follow the variation of the channel statistics. Our simulation shows it has more than 5dB improvement over the existing method in [5][6][7] under the practical channel conditions.**

I. Introduction

OFDM is now considered an effective technique for broadband wireless communications [1]. It partitions the entire bandwidth into parallel subchannels by dividing the transmit data into parallel low bit rate data streams to modulate the subcarriers corresponding to those subchannels. Thus, OFDM has a relative longer symbol duration which provides great immunity to fast fading and impulse noise. The independence among subchanenls further simplifies the design of the equalizer and provides an easy way for transmitter optimization. Because of all these advantages, OFDM becomes a promising technique in digital video/audio broadcasting and wireless communications [1][2].

Channel estimation and equalization problem is an essential problem in OFDM system design. Without channel information, non-coherent detection has to be used, which incurs performance loss compared to coherent detection. It is observed that the channel parameters for the subchannels are actually correlated, though the subchannels are treated independently when doing the signal detection. The channel estimation algorithms should exploit such correlation to improve the estimation. It is well known that if the correlation function of channel response is known, we can get the MMSE estimation by using the singular value decomposition of the correlation matrix [5]. However, in practice the correlation function is usually not known and the channel statistics may vary by time. Our goal is to design an estimation scheme under the condition that the channel statistics are not known or not completely known. One such scheme proposed in [5][6][7] assumes that the correlation matrix can be diagonalized by Fourier transform. The assumption is true when we consider infinite samples of the channel response. In practice, we can only have finite observations which may cause severe leakage when Fourier transform is performed.

In this work, we consider the problem from the other point of view. It is observed that the fading multipath channel is such a smoothly varying function in time or frequency domains that can be approximated by time-frequency polynomial expansions [12]. The estimation noise therefore can be greatly suppressed by estimating a small number of coefficients of the basis functions over a large number of observations. The approach in [5][6][7] can be considered as the same type of approach as the polynomial model based method by replacing polynomial basis with Fourier basis. Comparing these two types of basis, the polynomial model does not has the leakage problem and is more robust to the channel statistics and system parameters.

A key issue in using the polynomial model to estimate the channel parameters is to decide the model order and time-frequency window dimensions of observations. The approximation error in polynomial model decreases when increasing model order or decreasing the window dimensions, while the residual noise increases. It is important to reach a tradeoff between the model error and noise reduction. We propose an adaptive algorithm that adjusts the window dimensions to balance the tradeoff. The adaptive algorithm can track the tradeoff point without the knowledge of the specific channel correlation function or the fading and delay characteristic.

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II. Time-Frequency Polynomial Channel Model for MCM Systems

The OFDM system divides the whole bandwidth B_d into m subchannels and modulates a block of data onto a set of subcarriers of corresponding subchannels. In most of MCM systems, the subchannels are divided evenly, the bandwidth of the subchannels is $\Delta f = B_d/m$. Input data are first buffered to blocks and then divided into m bit streams. These bit streams are mapped to some complex constellation points $X_{i,k}$, $i = 0, \cdots, m-1$ at kth block. The modulation is implemented by m-point inverse discrete Fourier transform (IDFT). Then the modulated data are passed through P/S converter to form serial data $x_{i,k}$. A cyclic prefix which is the copy of the last v samples of $x_{i,k}$'s is inserted before sending $x_{i,k}$'s to the channel. Now it follows that the symbol duration is $\frac{m}{B_d}$, however, the actual block duration is $T_f = \frac{m+v}{B_d}$ with sampling rate B_d. For a system with $B_d = 800kHz$, $m = 512$ and $v = 64$, the block duration is $T_f = 720\mu s$. Such system will be used in the rest of this paper.

At the receiver, the prefix part is discarded. The demodulation is performed by the discrete Fourier transform (DFT) operation. The demodulated data is $Y_{i,k}$'s. If the cyclic prefix is long enough, the interference between two OFDM blocks is eliminated and the subchannels can be viewed as independent of each other, i.e.,

$$Y_{i,k} = H_{i,k}X_{i,k} + N_{i,k}, \tag{1}$$

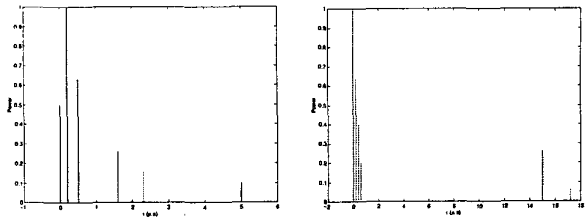
where $H_{i,k}$ is the channel frequency response at $i\Delta f$ of kth block and $N_{i,k}$ is the corresponding channel noise. $N_{i,k}$ is assumed to be white Gaussian process with zero mean and variance σ^2.

Because of this simple relationship, only a one-tap equalizer is needed for each subchannel at the receiver, i.e., $\hat{X}_{i,k} = Y_{i,k}W_{i,k}$, where the equalizer coefficient $W_{i,k}$ is some function of $H_{i,k}$. For example, the zero-forcing equalizer is constructed as $W_{i,k} = \frac{1}{H_{i,k}}$. Then the decision is made upon $\hat{X}_{i,k}$. The problem for us is to estimate $H_{i,k}$'s.

In wireless broadband communications, the channel impulse response can be modeled as [11]:

$$h(t,\tau) = \sum_i \gamma_i(t)\delta(\tau - \tau_i), \tag{2}$$

where $\gamma_i(t)$'s are independent Gaussian process with zero mean and variance p_i. p_i's and τ_i's are delay profiles describing the channel dispersion which is also often characterized by the maximum delay $T_d \triangleq \max_i \tau_i$. Three types of delay profiles are used in this paper, TU,



(a) (b)

Fig. 1. Two typical delay profiles (a) TU (b) HT

HT and 2-ray. The TU and HT delay profiles both have 6 paths [11], while the 2-ray delay profile has two equal power paths. We also assume that the channel is normalized in our simulation, i.e., $\sum_i p_i = 1$.

The channel parameters $H_{i,k}$'s are the samples of $H(t,f) = \int h(t,\tau)e^{-j2\pi\tau f}d\tau$, which is $H_{i,k} = H(kT_f, i\Delta f)$. It is obvious that the Fourier transform of $H(t,f)$ is band limited by f_D and T_d. Therefore, by discarding the high frequency components out of the band, we can reduce the noise and improve the estimation. This is the idea used in [5][6][7]. However, the problem is that we only have finite sample of $H_{i,k}$'s in a practical OFDM system. The Fourier transform over these finite samples may suffer severe leakage, which degrades the performance greatly.

Now let's look at this problem from the other point of view. The channel variation in physical world is smooth in both time and frequency domains. We know from the approximation theory [12] that such a smoothly varying function can be approximated by projecting to a finite set of basis functions. Moreover, since the OFDM channel parameters are located in a time-frequency plane, it is natural to project the parameters over a time-frequency window $(2I+1)\Delta f \times (2K+1)T_f$ to a small set of polynomial basis functions around a center point (i_0, k_0), i.e.,

$$H_{i,k} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} H_{i_0,k_0}(nm)(k-k_0)^m(i-i_0)^n + R_{MN} \tag{3}$$

for $k_0 - K \le k \le k_0 + K$ and$i_0 - I \le i \le i_0 + I$, where $H_{i_0,k_0}(nm) = \frac{T_f^m \Delta f^n}{m!n!} \frac{\partial^m \partial^n H(t,f)}{\partial t^m \partial f^n}\Big|_{t=k_0T_f, f=i_0\Delta f}$ and $R_{MN} = \frac{((k-k_0)T_f)^M}{M!} \frac{\partial^M H(t,f)}{\partial t^M}\Big|_{t=t'} + \frac{((i-i_0)\Delta f)^N}{N!} \frac{\partial^N H(t,f)}{\partial f^N}\Big|_{f=f'} - \frac{((k-k_0)T_f)^M((i-i_0)\Delta f)^N}{M!N!} \frac{\partial^M \partial^N H(t,f)}{\partial t^M \partial f^N}\Big|_{t=t', f=f'}$.

Without loss of generality, assuming $M = N$ and using the Rayleigh fading model, the mean squared model error is bounded by

$$\begin{aligned} \mathrm{E}[\|R_{MM}\|^2] &\leq \frac{2M!(2\pi KT_f f_D)^{2M}}{2^{2M}(M!)^4} + \frac{(2\pi I\Delta f T_d)^{2M}}{(M!)^2} \\ &+ \frac{2M!(4\pi^2 KIT_f \Delta f f_D T_d)^{2M}}{2^{2M}(M!)^6}. \end{aligned} \quad (4)$$

When we ignore the prefix part since usually $v \ll m$, the first term in (4) is determined by $f_D T_f = \frac{m f_D}{B_d}$, while the second term is determined by $\Delta f T_d = \frac{B_d T_d}{m}$. The third term is actually determined by $f_D T_d$ and is much smaller than the first two terms if they are both smaller than one. To make the model error small, we can choose larger model order M if $f_D T_f < 1$ and $\Delta f T_d < 1$. However, the goal for using modeling is to express the channel responses by a small number of model coefficients, which means we want M to be small. The other way to reduce the above bound is to adjust the window dimensions K and I. When m is large, the first term is dominating, then we should choose smaller K to make the model error small. On the other hand, when m is small and the second term is dominating, then we should choose smaller I. By carefully choosing the window dimensions, the time-frequency model error can be limited to certain level once the Doppler frequency f_D, maximum delay T_d and the bandwidth B_d are fixed. It should be pointed out that unlike using only time or frequency domain model [8][9][10], the model error of time-frequency model does not depend on the subchannel number m.

III. Channel Estimation with Polynomial Model

Suppose we have chosen the appropriate model order and window dimensions such that the following approximation is valid.

$$\mathbf{H}_{i_0,k_0} \simeq \mathbf{Q}_{M,N}(I,K)\mathbf{b}_{i_0,k_0}, \quad (5)$$

where $\mathbf{H}_{i_0,k_0} = [H_{-I+i_0,-K+k_0} \cdots H_{-I+i_0,K+k_0} \cdots H_{I+i_0,-K+k_0} \cdots H_{I+i_0,K+k_0}]^T$, $\mathbf{b}_{i_0,k_0} = [H_{i_0,k_0}(0,0) \cdots H_{i_0,k_0}(0,N-1) \cdots H_{i_0,k_0}(M-1,0) \cdots H_{i_0,k_0}(M-1,N-1)]^T$ and $\mathbf{Q}_{M,N}(I,K) =$

$$\begin{bmatrix} q_{0,0}^{-I,-K} & \cdots & q_{0,N}^{-I,-K} & q_{1,0}^{-I,-K} & \cdots & q_{M,N}^{-I,-K} \\ \vdots & & \vdots & \vdots & & \vdots \\ q_{0,0}^{I,-K} & \cdots & q_{0,N}^{I,-K} & q_{1,0}^{I,-K} & \cdots & q_{M,N}^{I,-K} \\ \vdots & & \vdots & \vdots & & \vdots \\ q_{00}^{I,K} & \cdots & q_{0,N}^{I,K} & q_{1,0}^{I,K} & \cdots & q_{M,N}^{I,K} \end{bmatrix}$$

with $q_{m,n}^{i,k} = i^n k^m$, for $i = -I, \cdots, 0, \cdots, I$, $k = -K, \cdots, 0, \cdots, K, m = 0, \cdots, M$ and $n = 0, \cdots, N$.

Then construct $\tilde{\mathbf{H}}_{i_0,k_0} = [\tilde{H}_{-I+i_0,-K+k_0} \cdots \tilde{H}_{-I+i_0,K+k_0} \cdots \tilde{H}_{I+i_0,-K+k_0} \cdots \tilde{H}_{I+i_0,K+k_0}]^T$ with $\tilde{H}_{i,k} = \frac{Y_{i,k}}{X_{i,k}} = H_{i,k} + \frac{N_{i,k}}{X_{i,k}}$ as the temporary estimation, where $N_{i,k}$ is the channel noise and is assumed to be white Gaussian with zero mean and variance σ^2. $X_{i,k}$'s can be obtained either from training or from the detected signal. Then

$$\tilde{\mathbf{H}}_{i_0,k_0} = \mathbf{H}_{i_0,k_0} + \mathbf{N}_{i_0,k_0} \simeq \mathbf{Q}_{M,N}(I,K)\mathbf{b}_{i_0,k_0} + \mathbf{N}_{i_0,k_0},$$

where $\mathbf{N}_{i_0,k_0} = \left[\frac{N_{-I+i_0,-K+k_0}}{X_{-I+i_0,-K+k_0}} \cdots \frac{N_{-I+i_0,K+k_0}}{X_{-I+i_0,K+k_0}} \cdots \frac{N_{I+i_0,-K+k_0}}{X_{I+i_0,-K+k_0}} \cdots \frac{N_{I+i_0,K+k_0}}{X_{I+i_0,K+k_0}}\right]^T$.

Using LS methods, we can get the estimation of the coefficients of the polynomial basis from the temporary estimation

$$\hat{\mathbf{b}}_{i_0,k_0} = \mathbf{Q}_{M,N}^{\dagger}(I,K)\tilde{\mathbf{H}}_{i_0,k_0}, \quad (6)$$

where $\mathbf{Q}_{M,N}^{\dagger}(I,K)$ is the pseudo inverse of $\mathbf{Q}_{M,N}(I,K)$.

The channel estimation then can be constructed as

$$\begin{aligned} \hat{H}_{i,k} &= \mathbf{q}_{M,N}(i-i_0,k-k_0)^T\hat{\mathbf{b}}_{i_0,k_0} \\ &= \mathbf{q}_{M,N}(i-i_0,k-k_0)^T\mathbf{Q}^{\dagger}(I,K)\tilde{\mathbf{H}}_{i_0,k_0}, \end{aligned} \quad (7)$$

where $\mathbf{q}_{M,N}(i-i_0,k-k_0) = [q_{0,0}^{i-i_0,k-k_0} \cdots q_{0,N-1}^{i-i_0,k-k_0} \cdot q_{M-1,0}^{i-i_0,k-k_0} \cdots q_{M-1,N-1}^{i-i_0,k-k_0}]^T$. Usually we fix the value of $i-i_0$ and $k-k_0$, i.e., fix the point of estimation inside the window and slide the window to get all the estimations. Then the estimator can be viewed as a two-dimensional filtering process. Moreover, the polynomial basis has the symmetric property and a recursive algorithm can be derived to implement the filtering which reduces the computation complexity.

With estimation point chosen at the center of the frequency domain window and end point at the time domain window, the estimation error from (7) becomes

$$\epsilon_{I,K} = \mathrm{E}[\|H_{i_0,k_0} - \hat{H}_{i_0,k_0}\|^2] = \epsilon_h + \epsilon_n, \quad (8)$$

where

$$\begin{aligned} \epsilon_h &= r_H(0,0) - \mathrm{E}[H_{i_0,k_0}\mathbf{H}_{i_0,k_0}^T]\mathbf{Q}_{M,N}^{\dagger T}(I,K)\mathbf{q}_{M,N}(0,K) \\ &- \mathbf{q}_{M,N}(0,K)^T\mathbf{Q}_{M,N}^{\dagger}(I,K)\mathrm{E}[\mathbf{H}_{i_0,k_0}H_{i_0,k_0}^*] \\ &+ \mathbf{q}_{M,N}(0,K)^T\mathbf{Q}_{M,N}^{\dagger}(I,K)\mathrm{E}[\mathbf{H}_{i_0,k_0}\mathbf{H}_{i_0,k_0}^T] \\ &\quad \mathbf{Q}_{M,N}^{\dagger T}(I,K)\mathbf{q}_{M,N}(0,K) \end{aligned}$$

and

$$\epsilon_n = \sigma^2\mathbf{q}_{M,N}(0,K)^T\mathbf{Q}_{M,N}^{\dagger}(I,K)\mathbf{Q}_{M,N}^{\dagger T}(I,K)\mathbf{q}_{M,N}(0,K).$$

TABLE I

WINDOW DIMENSION ADAPTIVE ALGORITHM

1. *Initialization*: use $I_0 \times K_0$ calculate estimation and $\hat{\epsilon}_0 = \hat{\epsilon}_{I_0,K_0}$.
2. Use window dimensions $I \times K$ to estimate the kth block and compute the estimated estimation error $\hat{\epsilon}_{I,K}$, $\hat{\epsilon}_{I+1,K}$ and $\hat{\epsilon}_{I,K+1}$.
3. If $\hat{\epsilon}_{I,K} < \hat{\epsilon}_0$, then $I_0 = I$, $K_0 = K$, $\hat{\epsilon}_0 = \hat{\epsilon}_{I,K}$,
   a) If $|\hat{\epsilon}_{I,K} - \hat{\epsilon}_{I+1,K}| < \epsilon^f_{th}$, then I unchanged. Otherwise, if $\hat{\epsilon}_{I,K} > \hat{\epsilon}_{I+1,K}$, then $I = I + 1$, or if $\hat{\epsilon}_{I,K} < \hat{\epsilon}_{I+1,K}$, then $I = I - 1$.
   b) If $|\hat{\epsilon}_{I,K} - \hat{\epsilon}_{I,K+1}| < \epsilon^t_{th}$, then K unchanged. Otherwise, if $\hat{\epsilon}_{I,K} > \hat{\epsilon}_{I,K+1}$, then $K = K + 1$, or if $\hat{\epsilon}_{I,K} < \hat{\epsilon}_{I,K+1}$, then $K = K - 1$.
   Otherwise, $I = I_0$ and $K = K_0$.
4. Go to step 2 for block $k + 1$.

The channel estimation error consists of two parts, one is the model error ϵ_h related to model approximation, and the other is the residual noise ϵ_n. The model error $\epsilon_h = 0$, if $\mathbf{Q}^{\dagger}_{M,N}(I,K)\mathrm{E}[\mathbf{H}_{i_0,k_0}\mathbf{H}^T_{i_0,k_0}]\mathbf{Q}^{\dagger T}_{M,N}$ is a diagonal matrix. This can be realized when the eigen-basis of the channel correlation function is used. However, the statistics of the channel must be known which is difficult in practice and also difficult to implement. Then for a model basis like polynomial model, the model error increases when the model order $M \times N$ becomes small or the window dimension $I \times K$ becomes large. However, the noise is reduced more in this case. With fixed polynomial model order M and N, the optimal window dimension is obtained by

$$\min_{I,K} \epsilon_{I,K} = \epsilon_h + \epsilon_n. \tag{9}$$

Usually, there are several local minima in this optimization problem. We prefer the one with the small window dimension which has less computation complexity and often has the least estimation error since the model error is small.

Suppose the noise statistics is known, we can calculate the estimated estimation error from the estimates and the received signal by

$$\begin{aligned}\hat{\epsilon}_{I,K} &= \sum_i \sum_k \|\tilde{H}_{i_0,k_0} - \hat{H}_{i_0,k}\|^2 - \sigma^2 \\ &+ \mathrm{E}[N_{i_0,k_0}\mathbf{N}^H_{i_0,k_0}]\mathbf{Q}^{\dagger}_{M,N}(I,K)\mathbf{q}_{M,N}(0,K) \\ &+ \mathbf{q}_{M,N}(0,K)^T\mathbf{Q}^{\dagger}_{M,N}(I,K)\mathrm{E}[\mathbf{N}_{i_0,k_0}N^*_{i_0,k_0}]\end{aligned}$$

Use this approximation, the window dimension adaptive algorithm is given in Table I.

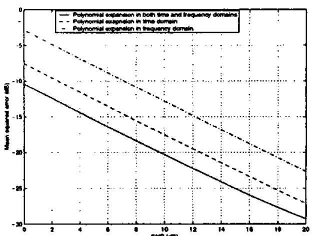


Fig. 2. Estimation error vs. SNR (2-ray, $f_D = 30Hz$ and $T_d = 25\mu s$, $I \times K = 6 \times 6$ and $M \times N = 3 \times 3$)

IV. SIMULATION RESULTS

The MCM system used in the simulations is the system introduced in section II. Fig. 2 shows the comparison of mean-squared estimation errors of the channel estimations based on expansions in both time and frequency domains with those based on expansion either in time or frequency domain. We can see that the estimation error with both time and frequency domain expansions is about 3dB less at SNR of 10dB compared to frequency domain expansion [9] and more than 7dB less compared to time domain expansion [8].

Fig. 3 shows the estimation error under different delay Profiles with Doppler shift of 40Hz. Fig. 3(a) shows the estimation error with TU delay profile and 2-ray delay profile with the same maximal delay as TU and (b) shows the estimation error with HT delay profile and 2-ray delay profile with the same maximal delay as HT. The results using the Fourier transform based method of [7] are also shown for comparison. With limited subchannels, all the delay paths of the channel have to be at the sampling instances of the system to avoid leakage otherwise severe performance loss occurs. For 2-ray channel with $T_d = 5\mu s$, the method in [7] has better performance since the two delay paths at $T_d = 0$ and $T_d = 5\mu s$ are all at the sampling instance of the system. There is no leakage in Fourier transform which is used as frequency domain estimator in [7]. However, the leakage becomes large for TU or HT delay profiles because not all their delay paths are at the sampling instances and the performance of the Fourier transform based method is much degraded. However, the proposed polynomial model method does not such severe leakage problem and has more than 5dB gain over the Fourier transform based method and is more robust to the channel statistics variation. There is only small difference between the TU or HT and its corresponding 2-ray channel with same T_d respectively.

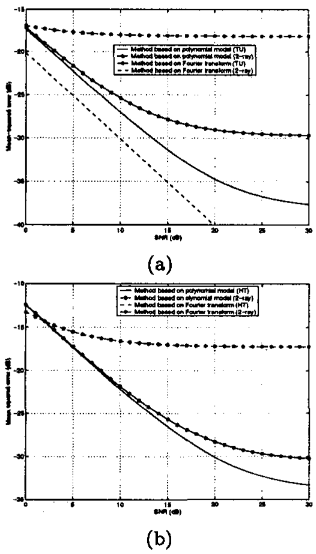


Fig. 3. Estimation error vs. SNR ($M \times N = 3 \times 3$) (a) $I \times K = 40 \times 4$ (b) $I \times K = 11 \times 5$

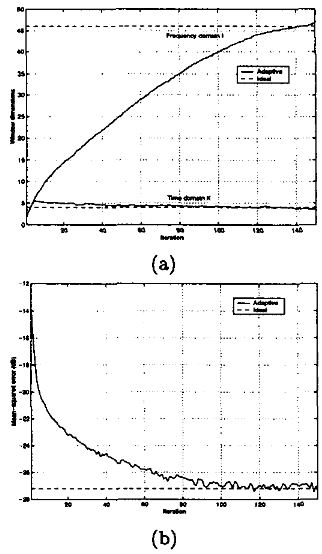


Fig. 4. Window dimensions adaptation (TU, SNR=10dB, $f_D = 40Hz$) (a) Window dimensions (b) Estimation error

Fig. 4 shows the window dimension adaptation. Both the window dimension variation and the estimation error variation with iteration are shown in Fig 4 (a) and (b), respectively. At the beginning the window dimension is 2×2. Then, after 100 iteration, the algorithm converges to a window dimension of 46×4 and an estimation error under 27dB. With this adaptation algorithm, the polynomial model based method is not only robust to the specific correlation of the channel variation and dispersion, but also robust to T_d and f_D and can follow the variation of the channel statistics.

V. Conclusion

In this paper, we studied the channel estimation problem for the OFDM system when the statistics of the multipath fading channel is not known or partialy known. A channel estimation approach based on polynomial approximation of the channel parameters is proposed. The method exploits the channel correlation in both time and frequency domain. It is shown in simulation that the method is robust to different channel statistics. Moreover, a window dimension adaptation algorithm is proposed to adapt the channel estimator to the channel statistics which further improves the robustness of the system.

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