MIMO-OFDM Channel Estimation via Probabilistic Data Association Based TOAs

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Abstract—The multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) is one of the most promising techniques to achieve broadband wireless communications. Channel estimation is critical in the design of MIMO OFDM systems, since channel parameters are required for coherent detection and decoding. In this work, we propose a channel estimation scheme based on the time of arrivals (TOAs) estimation. The TOAs are first determined using a variant of probabilistic data association (PDA) and employing the minimum description length principle, then the PDA is augmented by group decision feedback to refine the TOA estimates. The proposed algorithm is compared with the Fourier transform based method and the alternating projection (AP) algorithm. Simulation results show that the proposed scheme provides the performance comparable to that of AP and much better than that of the Fourier based method, meanwhile it requires much lower computational cost than that of AP.

I. INTRODUCTION

The future wireless communications are characterized by broadband, high data rate capabilities, integration of services, and flexibility. The multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) is a promising technique to achieve these requirements, since OFDM is greatly resistant to the multipath introduced intersymbol interference (ISI) and transmitter/receiver diversity is effective for combating fading. However, the enhancements in capacity, diversity, and coding gains depend on the availability of the underlying channel information at the receiver, since channel parameters are required for coherent detection and decoding. Without channel information, differential space-time coding (STC) or non-coherent detection has to be used at the expense of performance loss or extra power. Therefore, channel estimation is essential in OFDM system design.

In this paper, we consider channel estimation for transmit diversity OFDM systems using STC. Now the received signal is a superposition of different signals transmitted from different transmit antennas simultaneously. With the assumption of tolerable leakage, a Fourier transform model based algorithm was proposed in [2] and the optimal training sequence was studied [3]. Later, a reduced complexity detection scheme was studied by exploiting the correlation of the adjacent subchannel responses [4]. A polynomial model based approach was developed in [15]. These typical schemes [2], [12], [13] assume a maximum delay profile and do not adapt to sparse channel conditions or higher delay profiles, therefore, the estimation accuracy will be degraded under these conditions. Note that the channel impulse response is characterized by

the delays of the paths, therefore, estimating time of arrivals (TOAs) is one way to improve channel estimation. A multidimensional search for maximum-likelihood (ML) solution is a prohibitively complex task, and a number of alternatives have been proposed to reduce the load. The alternating projection (AP) method [6] is mostly interested due to its good performance. An channel estimation scheme via near-ML TOA estimation, with its root in AP, was presented in [14] with good performance. However, the iterative AP algorithm still could suffer from high computational cost, especially when the number of paths is unknown.

Our goal here is to develop a scheme for TOA estimation with lower computational cost but comparable performance as that of AP. We propose a probabilistic data association (PDA) based TOA estimation. Based on the estimated TOAs, we then estimate the corresponding gain for each path, and compute the channel response accordingly. The key ideas include: (1) Our proposed scheme works on matched filter outputs and apply PDA concept [10], [11] to locate the isolate delay paths due to its simple implementation, high performance and moderate computational cost. The basic idea of PDA is to approximate the interferences from other paths as Gaussian-distributed and iteratively update the covariance matrix. Furthermore, since the number of paths is generally unknown and its value affects the estimation performance, we estimate the number of paths via the minimum description length (MDL) principle; (2) Enhanced resolution is achieved by using the decision feedbacks. Since only finite discrete possible values of delays are examined in PDA, we can further improve the precision of delay estimates by iteratively performing a local maximization with respect to a single delay time while all other delay times are held fixed.

This paper is organized as follows: we describe the system model and problem formulations in Section II. The proposed algorithm is presented in Section III. Simulation results that demonstrate the performance of our algorithm, are studied in comparison with other algorithms in Section IV. Finally, conclusions are given in Section V.

II. SYSTEM AND PROBLEM DESCRIPTION

In OFDM, the entire channel is partitioned into parallel subchannels and a block of data are modulated to a set of subcarriers, thereby increasing the symbol duration and reducing the ISI [1]. Transmit diversity has gained great research interests recently, as it has been proved that the

channel capacity is proportional to the number of transmitter or receiver antennas. Therefore, we consider an OFDM system with m_T transmit and m_R receive antenna. At time n, an input data block is mapped into m_T complex constellation sequences $\{X_i[n,k]: k=0,1,...,K-1\}$ for $i=1,...,m_T$, where K is the number of subchannels. The received signal after FFT process at receive antenna i is expressed as:

$$r_j[n,k] = \sum_{i=1}^{m_T} H_{ij}[n,k]X_i[n,k] + w_j[n,k]$$
 (1)

where H_{ij} indicates the channel frequency response from transmitter i to receiver j at the k^{th} tone of the OFDM block at time n, and the noise $w_j[n,k]$ is both spatially and temporally white, with zero mean and variance σ_n^2 . Since channel estimation at each receiver is implemented independently, the index j will be omitted in the following paper. The channel impulse response can be generally modelled as:

$$h(t,\tau) = \sum_{k=1}^{p} \gamma_k \delta(\tau - \tau_k)$$
 (2)

where p is the number of paths, γ_k 's are independent Gaussian processes with zero mean, and τ_k is the corresponding delay of the k^{th} path. Therefore, the vector $\mathbf{H}_i[n]$ can be expressed as the weighted sum of complex sinusoids:

$$\mathbf{H}_i[n] = [\mathbf{s}(\tau_1), \mathbf{s}(\tau_2), ..., \mathbf{s}(\tau_p)] \mathbf{a}_i = \mathbf{S}(\tau) \mathbf{a}_i + \mathbf{w}$$
 (3)

where the vector $\mathbf{a}_i = [\gamma_1,...,\gamma_p]^T$, the complex sinusoid $\mathbf{s}(\tau_k) = [1,e^{-j2\pi\tau_k/T_s},...,e^{-j2\pi\tau_k(K-1)/T_s}]^T$ with T_s being the symbol duration, and $\tau = \{\tau_1,...,\tau_p\}$.

Here we assume that the channels from different transmitters to the same receiver have the same delay and fading property (i.e., same number of paths, and τ_k 's). It is reasonable because the transmitters are very close to each other in practice. We consider the channel estimation based on the training pilots $\{X_i[k]\}$'s. We also apply the simple training strategy proposed in [12] to effectively shift multiple antenna's channels into non-overlapped regions by allowing

$$X_i[k] = X_1[k]e^{-j2\pi k(i-1)T_0/T_s}$$
(4)

where the shift T_0 is larger than the maximum delay, which is defined as $T_d = \max_k \tau_k$, to ensure that TOAs from different channels are non-overlapped. Now, we have the received vector \mathbf{r} in (1) as:

$$\mathbf{r} = \mathbf{X}_1 \odot (\mathbf{S}(\theta)\mathbf{a}) + \mathbf{w} = \mathbf{S}_1(\theta)\mathbf{a} + \mathbf{w}$$
 (5)

where the combined vector $\mathbf{a} = [\mathbf{a}_1^T,...,\mathbf{a}_{m_T}^T]^T$, the combined delays after shifting $\theta = \{[\tau_k,\tau_k+T_0,...,\tau_k+(m_T-1)T_0], k=1,...,p\}$, the symbol \odot represents the Hamard product by multiplying component-wise, and each column of \mathbf{S}_1 is the Hamard product of \mathbf{X}_1 and the corresponding column of \mathbf{S} . Our goal is to estimate the delays τ and the gains \mathbf{a} , based on the observations \mathbf{r} . However, it is worth mentioning that the above assumptions can be relaxed in the algorithms we will develop later.

III. THE PROPOSED SCHEME

Considering the above model (5), we certainly note that our channel estimation problem is mathematically modelled as the estimation of superimposed signals $s(\tau_k) \odot X_1$. Therefore, our channel estimation problem equals to the problem of estimating TOAs, a problem faced in many application areas such as radar, sonar, and geophysics. Many approaches were proposed to estimate TOAs, and one of most promising is the ML estimator computed by the AP algorithm, which is similar to the coordinate descent (CD) algorithm. AP provides high estimation accuracy. However, AP could suffer the high computational cost, especially for problem with high dimension. The situation could be even worse when the number of delay paths is unknown. On the other hand, the Fourier transform based channel estimation in [2] is simple and requires light computational load but its accuracy may not be satisfactory when leakage occurs, under sparse channel conditions, or under longer delay profiles. Therefore, it is desirable to develop algorithms having high accuracy and low computational cost.

Here we propose an approach from a different perspective to reduce the complexity in TOA estimation. Specifically, if one discretizes the delays that a path can inhabit, and assigns a binary variable to indicate its existence, then the TOA problem becomes recognizable as CDMA modulation. We can examine the relationship between the approaches for our channel estimation and those for multi-user detection. For instance, the Fourier transform model based algorithm proposed in [2] can be regarded as a decorrelator in CDMA. In this paper, we draw the comparison, and focus on one of the very promising approach: probabilistic data association (PDA). The so-called PDA detector [10] for CDMA has its root in the popular PDA filter of target tracking [11], and provides very good performance and tractable computation cost. Encouraged by this, we apply the PDA concept to our channel estimation problems and carry out a PDA-based algorithm to obtain TOA estimates. Since the number of paths p is unknown, we also need to apply information theoretic criteria for its determination. We apply the MDL principle [8], [9] to estimate the number of paths. To further improve the estimation accuracy, for each single path delay, we apply the decision feedback idea and perform a local estimation with higher precision. Overall, our proposed scheme includes three stages: pre-processing, the PDA-based searching, and the refinement via decision feedback, as presented in more details at the following sub-sections.

A. Pre-processing

The goal of pre-processing is to decrease the problem dimension and thus reduce the computational cost. We choose to work in the matched filter (MF) domain instead of the raw data domain. Suppose we allow only $\{t_k\}$, for k=1,...,N, as possible values of path delay where N is the total number of t_k , then we can consider $\{\mathbf{s}(t_k) \odot \mathbf{X}_1\}$ as all possible signatures, and hence with referring to the observation model

in (5), the model of the MF outputs at the receiver is given by:

$$y = RAb + n = RBa + n \tag{6}$$

where \mathbf{v} is the MF output vector with length N, the vector $\mathbf{b} \in \{0,1\}^N$ indicates the delays of paths with $b_i = 1$ meaning there is a path at delay t_i . Therefore, if there are p paths, the number of 1 elements in b is p and the corresponding $\{t_i\}$'s indicate the real path delays τ . **B** and **A** are diagonal matrices whose diagonal elements are $\{b_i\}$'s and $\{a_i\}$'s respectively, \mathbf{R} is the N-by-N signature covariance matrix, and \mathbf{n} is a colored noise vector following a $CN(0, \sigma_n^2 \mathbf{R})$ distribution, with $CN(0, \sigma_n^2 \mathbf{R})$ representing the complex Gaussian vector distribution with zero mean and covariance matrix $\sigma_n^2 \mathbf{R}$. In our problem, we assume the amplitudes $\{a_i\}$ are unknown, but follow an i.i.d. $CN(0, \sigma_s^2)$ where σ_s^2 is known ¹. It is clear that the number of paths can vary between 0 and N. It is worth mentioning that the possible path delays t_i 's can either be chosen uniformly, or chosen via an amplitudeweighted way. We give a simple example of the later case as the following: suppose at first $\{t_i\}$'s are uniformly spaced, then each new t_i is adjusted based on the MF amplitudes observed at the adjacent old t_i and t_{i+1} , therefore $t_i^{new} =$ $(|y(i)|t_i+|y(i+1)|t_{i+1})/(|y(i)|+|y(i+1)|)$. This amplitudeweighted approach is commonly employed in target location estimation for radar applications. To further efficiently reduce the dimension of N, we could compare the magnitude of y to a threshold and consider only values that are above the threshold, which gives the advantage of only working within regions that have good signal-to-noise ratio (SNR). Because the DOAs from different transmitters are assumed the same and due to the training strategy given in (4), we further reduce the dimension from N to $N_1 = N/m_T$ by assuming $t_{(j-1)N_1+i} = t_i + (j-1)T_0$ and accordingly b(i) = $b((j-1)N_1+i)$, for $j=1,...,m_T$ and $i=1,...,N_1$. Here without loss of generality, we assume N is dividable by m_T . We focus on this specific model in our following derivations.

B. PDA-based searching

The goal of PDA-based searching is to locate TOAs with affordable computational cost. The basic idea of PDA is to iteratively approximate the interference from other paths as Gaussian noise. Based on model (6), recall that the elements of a follow the i.i.d. $CN(0, \sigma_s^2)$ distribution, according to the properties of the vector-valued Gaussian distribution, we have

$$\mathbf{y}|\mathbf{b} \sim CN(0, \mathbf{R}\mathbf{B}\boldsymbol{\Sigma}_{s}\mathbf{B}^{H}\mathbf{R}^{H} + \sigma_{n}^{2}\mathbf{R})$$
 (7)

where $\Sigma_s = \sigma_s^2 \mathbf{I}_N$, with \mathbf{I}_N being the identity matrix of dimension N-by-N. We apply the PDA concept to estimate \mathbf{b} given the observation \mathbf{y} . Define $P_b(i)$ as the probability that a path arrives at the delay t_i . The key in PDA is to iteratively update $P_b(i)$ based on the Gaussian approximation

until convergence. By using the binary nature of b_i 's and applying the Gaussian approximation based on moment-match, for $i = 1, ..., N_1$, we can show that

$$f\left[\mathbf{y}|b(i) = 0, \{P_b(j)\}_{j \neq i, j \in [1, N_1]}\right] = CN(0, \Sigma_{0i}), \quad (8)$$

$$f\left[\mathbf{y}|b(i) = 1, \{P_b(j)\}_{j \neq i, j \in [1, N_1]}\right] = CN(0, \Sigma_{1i}), \text{ with}$$

$$\Sigma_{0i} = \sigma_s^2 \sum_{j \neq i} P_b(j) \left(\sum_{m=1}^{m_T} \mathbf{R}_{(m-1)N_1 + j} \mathbf{R}_{(m-1)N_1 + j}^H \right) + \sigma_n^2 \mathbf{R},$$

$$\mathbf{\Sigma}_{1i} = \mathbf{\Sigma}_{0i} + \sigma_s^2 \left(\sum_{m=1}^{m_T} \mathbf{R}_{(m-1)N_1+i} \mathbf{R}_{(m-1)N_1+i}^H
ight)$$

where \mathbf{R}_j is the j^{th} column of the matrix \mathbf{R} . Because of the special form of the covariance matrices Σ_{0i} and Σ_{0i} , great computational cost can be saved by using the rank one matrix inverse formula. Now based on the above Gaussian approximation of the distribution, at each iteration q, $P_b(i)$ is updated according to the Bayesian formula as:

$$P_b(i)^{\{q\}} = \frac{f(\mathbf{y}|b(i) = 1, \{P_b(j)^{\{q-1\}}\}_{j \neq i})Pr(b(i) = 1)}{\sum_{b(i)} f(\mathbf{y}|b(i), \{P_b(j)^{\{q-1\}}\}_{j \neq i})Pr(b(i))}$$
(9)

for $i = 1, ..., N_1$, where Pr(b(i) = 1) is the prior probability and we choose 0.5 in our problem. Normally, $\{P_b(i)\}$ converges within 2-4 iterations as shown in [10].

With the preparation of the above statistic analysis, now we apply the PDA idea and the MDL principle to estimate b:

- 1. Ordering of MF bins and updating $\{P_b(i)\}$: We first initialize the probabilities $P_b(i) = 0.5$, $\forall i$. At the first iteration, the indices i's are sorted in descending order of magnitude of \mathbf{y} , and each $P_b(i)$ in the ordered index is updated according to (9). In succeeding iterations they are sorted in descending order of $\{P_b(i)\}$, and $P_b(i)$'s are sequentially updated based on the current value of $\{P_b(j), j \neq i\}$. Repeat the process until each $P_b(i)$ is converged.
- **2. Determining the number of paths:** Based on the probabilities $\{P_b(i)\}$'s obtained above, we further proceed to decide b. Since the number of active paths p is unknown, any attempt to estimate it by maximizing $f(\mathbf{y}|\mathbf{b})$ directly will yield as many paths as possible. Therefore, a suitable order selection algorithm should be included. A popular such principle is MDL [8], [9]. Starting from picking only one path whose delay yields the largest probability $P_b(i)$, we sequentially add one more path by applying the MDL principle to decide the number of paths p. Note that the penalty term in MDL has the basic form $n_p \ln(N)/2$ where n_p is the total number of parameters estimated. Since in our problem the total number of estimated parameters is p (e.g. p contains p non-zero elements), now we have

$$\begin{split} MDL(\mathbf{y},p) &= -\ln(f(\mathbf{y}|\hat{\mathbf{b}})) + p\ln(N)/2, \text{ where} \quad \ (10) \\ \hat{b}(i) &= \begin{cases} 1, & \text{if } P_b(i) \in \{ \text{ the } p \text{ largest } P_b(i) \}; \\ 0, & \text{elsewhere,} \end{cases} \end{split}$$

for $p = 1, 2, ..., N_1$. Then p is estimated as

$$\hat{p} = \arg \min_{p} \{MDL(\mathbf{y}, p)\}.$$

 $^{^{1}}$ In practice, $\mathbf{a_{i}}$'s are independent Gaussian processes with different but unknown variances. Therefore, we choose a prior that all variances are equally chosen based on the powers of observations. The results here can easily be generalized to non-identical models. However, our experience suggests that the performance is not sensitive to the values of variances.

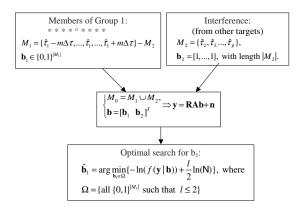


Fig. 1. Procedure of the refinement via DF idea within one iteration: the searching procedure in group 1.

Based on the values of possible delays $\{t_i\}$, this "rough" procedure usually yields a reasonably good resolution. To further improve the estimation accuracy, the following stage is indicated.

C. Refinement via Decision Feedback (DF)

Since we only have finite samples of ${\bf r}$ in a practical OFDM, the path arrival times τ are roughly estimated in the above procedure, and we record the TOAs results as $\hat{\tau}=\{\hat{\tau}_1,\dots,\hat{\tau}_{\hat{p}}\}$. We improve the precision of estimates by performing a local maximization with respect to a single arriving time τ_i while all other arriving times are held fixed; the process is performed iteratively until it converges. The idea behind is very similar to the decision feedback algorithm in CDMA. We begin with the previously-determined TOAs $\hat{\tau}$, and we re-define ${\bf M}=\hat{\tau}$, and we then:

- 1. Sort the paths in descending-power order. Assuming that paths arrive at delays M, the corresponding complex amplitudes of paths can be obtained by the ML estimate with referring to model (6).
- **2.** For each path i, for $i=1,\ldots,\hat{p}$, with \hat{p} meaning the size of \mathbf{M} , we optimally search the nearby region (a group of delays) of $\hat{\tau}_i$ and record the updated estimates as $\{\hat{\tau}_i\}$ sequentially. We then update $\mathbf{M}=\{\{\hat{\tau}_1\},\{\hat{\tau}_2\},\ldots,\{\hat{\tau}_{\hat{p}}\}\}$. Here as an example, we illustrate the optimal search in group 1 in Fig. 1, where group 1 is characterized by $\hat{\tau}_1$ and paths arriving at other delays $\{\hat{\tau}_2,\ldots,\hat{\tau}_{\hat{p}}\}$ are regarded as interference. Based on $\hat{\tau}_1$, the members of group 1 with size (2m+1) are selected as the nearby delays around $\hat{\tau}_1$, which is in turn reasonably defined by the constant delay difference between adjacent members $\Delta \tau$. This $\Delta \tau$ is referred as precision factor. We use \mathbf{M}_1 to indicate the members in group 1 but not part of the interference, and \mathbf{M}_2 to indicate the delay locations of interfering paths. We consider model (6), and apply the MDL principle to detect the number of paths in group 1

$$\hat{\mathbf{b}}_1 = \arg\min_{\mathbf{b}_1 \in \Omega} \left\{ -\ln(f(\mathbf{y}|\mathbf{b})) + (l/2)\ln(N) \right\}$$
 (11)

where $\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2]^T$, $\mathbf{b}_2 = [1, \dots, 1]$ with length $|\mathbf{M}_2|$ indicating the interfering targets, and $\mathbf{b}_1 \in \{0, 1\}^{|\mathbf{M}_1|}$ is used to indicate the locations and number of paths in \mathbf{M}_1 .

Since the purpose of this decision feedback process is to improve estimation of τ_1 through $\hat{\mathbf{b}}_1$, we only consider three possibilities: there are 0, 1, or 2 paths in \mathbf{M}_1 , indicating by Ω .

$$\Omega = \{ \text{all } \mathbf{b}_1 \text{ such that } l \leq 2 \}$$

where l means the number of non-zero elements (paths) in \mathbf{b}_1 . Due to the small size of Ω , problem (11) can be solved by exhaustive search. Based on $\hat{\mathbf{b}}_1$, we update $\{\hat{\tau}_i\}$. We similarly update $\{\hat{\tau}_i\}$, for $i=2,\ldots,|\mathbf{M}|$. Note that the possibility that two targets are found at this stage enables the iterative procedure to detect a large number of closely-spaced paths.

We continue until the process converges. The final estimates of path delays are in the subset M.

IV. SIMULATION RESULTS

To demonstrate the performance of the proposed channel estimation approach, simulations have been conducted for the space-time code based OFDM. The system bandwidth is 1600kHz and divided into 128 subchannels. The length of the cyclic prefix is 32. This results in a symbol duration $80\mu s$. A Doppler frequency of 40 Hz is used to represent the mobile environment. The delay profiles used are the well known typical urban (TU) and the hilly terrain (HT) delay profiles [15]. Two transmit and two receive antennas are used for diversity. The training sequence is designed as in (4). After the training block, the channel estimates are used to decode the next data block [3]. Recently, space-time coding at the transmitter equipped with efficient decoder at the receiver has been shown to offer high code efficiency and good performance [5]. Here the space time block code using 8PSK proposed in [5] is employed and the ML detection scheme is adopted for decoding.

The system performance is measured by bit error rate (BER) and the mean-squared estimation error (MSE). Fig. 2 and Fig. 3 show the performance comparison of the proposed scheme with the DFT based scheme in [2] and the AP method [6] for the TU and HT profile channels respectively, where BER vs. SNR and MSE vs. SNR are plotted. In AP implementation, the knowledge of the number of paths in each channel link is assumed known, while the number of paths is estimated via MDL in our implementation, therefore, AP serves as an upper bound of the performance. It is clear that estimating TOAs via our scheme yields a much better estimation performance than the Fourier transform based estimation, with the gap increases for larger SNR. We also note some performance degradations of our scheme compared with AP, however this performance difference is minor when compared with the difference from the DFT based scheme. If the number of paths is also unknown in AP, the performance of AP can be degraded due to the necessariness for estimating the number of paths.

Another concern is the computational cost. The comparison of CPU time requirement for different approaches is shown in Table I, where we report the ratio of the CPU time required by the proposed scheme to that of AP method. It is worth mentioning that the DFT based scheme is much more computationally efficient. However, we prefer the proposed scheme

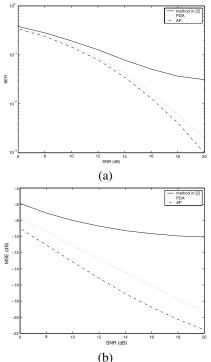


Fig. 2. Performance comparison of different channel estimators for the TU channel with 40Hz Doppler frequency. (a) BER (b) MSE

and AP due to the significant BER gain. From this table, we note that the ratio of the average CPU time requirement of the proposed scheme to that of AP is around 0.2. This ratio is even much smaller if the order selection issue is also addressed in AP implementation. Clearly, the proposed scheme is much faster than AP and has the comparable estimation accuracy. Therefore, the proposed PDA based TOA estimation scheme provides a very promising technique for MIMO-OFDM channel estimation.

CPU TIME REQUIREMENT OF OUR SCHEME OVER THAT OF AP.

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delay profile / SNR (dB)	6	10	16
TU	0.193	0.215	0.235
HT	0.212	0.231	0.250

V. CONCLUSION

In this paper, we presented a PDA-based TOA estimators to address the channel estimation problem for OFDM system. The scheme works on matched filter outputs and incorporates the PDA and decision feedback methods to locate path delays, with the number of paths determined via the MDL principle. It was shown in simulations that the scheme provides comparable performance to that of AP, which has perfect knowledge of the number of paths and thus serves as a performance upper bound. Moreover, our scheme requires much lower computational cost than AP. Therefore, it provides a promising technique for MIMO-OFDM channel estimation.

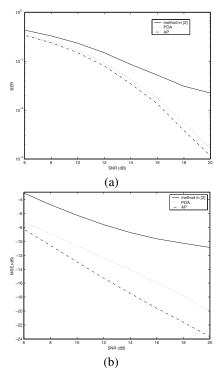


Fig. 3. Performance comparison of different channel estimators for the HT channel with 40Hz Doppler frequency. (a) BER (b) MSE

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