Outage Probability Optimization with Equal and Unequal Transmission Rates under Energy Harvesting Constraints

Sha Wei*†, Wei Guan*, and K. J. Ray Liu*
*Department of Electrical and Computer Engineering, University of Maryland, College Park, MD, 20742
Email: {swei1, wguan, kjrliu}@umd.edu
†Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai, China, 200240

***Abstract*—In this paper, we study the power scheduling problem to optimize the outage probability with both equal and unequal transmission rates under energy harvesting constraints. In particular, we first model the average outage probability minimization problem over finite horizon, and then reformulate the problem by minimizing its upper bound, which converts the non-convex problem to a more efficient convex optimization problem. Next, we show the optimal power scheduling solution is related to the global minimum of an energy-and-time function which is defined as arithmetic average of harvested energy from current time slot to the arbitrary time slot in the future before the end of transmission. Based on such properties, we propose a *Min-Average* strategy which presents the optimal performance in upper bound of outage probability for equal and non-equal transmission rates. Simulation results show that, our power scheduling scheme achieves better outage performance compared to other alternative strategies, e.g., best-effort, fixed-saving and random saving strategies.**

I. Introduction

In conventional fixed energy supply wireless sensor networks, sensors are equipped with power-limited batteries. When the sensors exhaust with the initial energy, it is not convenient to replace them if they are scattered throughout a broad space; and it even becomes impossible if they are embedded inside a human body. Alternatively, energy harvesting (EH) technology, which obtain energy from surrounding environment, becomes an effective method to provide almost unlimited energy supply and thus extend the life of wireless sensor networks. However, due to the nature of the energy harvesting sources, the sensor may encounter the cases that there is no energy generated, e.g., there is no solar energy at night. Besides, although we expect unlimited amount of energy be harvested in the future, the sensors can only use the currently available energy, which is referred to as the *EH causal constraints* [1]. Therefore, designing efficient transmission algorithms that can optimally manage the randomly arriving energy and adapt to the variation of communication network, e.g., channel and rate variation, becomes indispensable and significant.

Currently, there are many works discussing the optimization problems with the energy harvesting constraints in the wireless communication networks [2], [3], [4]. Regarding to throughput maximization, the authors present the optimal transmission policy that maximizes both the short and long term throughput for battery limited energy harvesting nodes in [5]. In [1], the authors present the throughput maximization problem over a finite number of transmission blocks for a deterministic EH model. The authors in [6] obtain the optimal power scheduling policy to guarantee that the data queue stays stable for the largest possible data rate and the other optimal policy to minimize the mean delay in the data queue. And regarding the outage probability minimization, in [7], the authors propose a forward search algorithm and a threshold-based sub-optimal power scheduling algorithm to minimize the finite-horizon outage probability with fixed transmission rates. The complexity of the forward search algorithm is $\mathcal{O}(N^2)$, where N is EH periods. In [8], the authors consider the energy harvesting circuit model with a main and a secondary energy storage devices, based on which they study the circuit and channel outage performance when the power supply is random.

In this paper, we study the power scheduling problem to optimize the outage probability with both equal and unequal transmission rates under energy harvesting constraints. To find a closed-form power scheduling solution, we first approximate the original objective function, which is in the form of sum of exponentials, as the reciprocal of transmission power's harmonic mean by adopting the arithmetic mean and geometric mean (am-gm) inequality. Next, for equal transmission rate, we discuss the properties of optimal power scheduling solutions to the new optimization problem by analyzing the global minimum of an energy-and-time function, which is defined as arithmetic average of harvested energy from current time slot to the arbitrary time slot in the future before the end of transmission. Based on the properties of the arithmetic mean of harvested energy, we propose a *Min-Average* strategy which presents the optimal performance in upper bound of outage probability for equal transmission rate. The proposed strategy is more efficient on the aspect of complexity since we only need to compute fixed number of arithmetic mean in each time slot. Furthermore, we extend the above solution to unequal transmission rates. Simulation results show that, due to the adaptivity to system parameters, e.g., channel gain, transmission rates and average harvested energy, our power scheduling solution has better outage performance compared to other alternative strategies, e.g., best-effort, fixed and random saving strategies.

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II. System Model and Problem Formulation

A. System Model

In this paper, we investigate a point-to-point wireless communication system with an energy harvesting sensor node and a receiver. We consider a transmission process over T energy harvesting periods, and the harvested energy level for the corresponding period is denoted as E_i for $i = 1, \cdots, T$. In addition, we assume that the battery capacity for storing the harvested energy is infinite, and other consumed energy rather than the transmission energy is relatively small and thus negligible.

Consider a Rayleigh fading channel, the received signal of the i-th communication period is given by

$$y_i = h_i \sqrt{P_i} x_i + n_i, \;\; i = 1, \cdots, T, \tag{1}$$

where y_i is the received signal at the destination, x_i is the corresponding channel input from sensor node, h_i is a Rayleigh distributed channel coefficient with zero mean and variance λ, P_i is the transmission power, and n_i is the complex additive white Gaussian noise (AWGN) at the destination with zero mean and unit variance.

For each energy harvesting period, the corresponding instantaneous mutual information of the channel assuming the optimal Gaussian codebook for the transmitted signals is given by

$$I_i(x_i; y_i) = \log_2 \left(1 + |h_i|^2 P_i\right). \tag{2}$$

In this paper, we first discuss the optimal power scheduling scheme if all T transmission periods are transmitted with the same constant rate R, and then extend the results to the unequal rate scenarios. Based on (2), the outage probability at the i-th transmission period is defined as

$$\begin{aligned} \mathcal{O}(P_i, R_i) &= \Pr\left[I_i(h_i, P_i) < R_i\right] \\ &= 1 - \exp\left(-\frac{\lambda(2^{R_i} - 1)}{P_i}\right), \end{aligned} \tag{3}$$

which is a function of the transmission power P_i and rate R_i.

B. Problem Formulation

In this subsection, we formulate the average outage probability minimization problem over finite T horizon with energy harvesting constraints.

We assume that the harvested power levels of all the T EH periods are known prior to the transmissions by estimation. The level of harvested energy source, for example the solar energy, can be easily predicted with high accuracy based on additive decomposition model in [9]. Hence, the transmission power for each communication block is limited by the following EH constraints

$$\sum_{i=1}^{t} P_i \le \sum_{i=1}^{t} E_i, \tag{4}$$

where $t = 1, \cdots, T$. Then, the average outage probability minimization problem over the finite T communication periods is formulated as

$$\begin{aligned} (\text{P1}): \min_{\{P_i\}} \; & 1 - \frac{1}{T} \sum_{i=1}^{T} \exp\left(-\frac{\lambda(2^{R_i} - 1)}{P_i}\right) \\ \text{s.t.} & \sum_{i=1}^{t} P_i \le \sum_{i=1}^{t} E_i, P_i \ge 0, 1 \le i \le T. \end{aligned} \tag{5}$$

Since $\exp\left(-\frac{\lambda(2^{R_i}-1)}{P_i}\right)$ is a non-convex function over P_i and the constraints in (5) are all linear, Problem (P1) is non-convex in general, and thus difficult to be solved by conventional convex optimization techniques. Note that, the objective function is the sum of a series real numbers $\exp\left(-\frac{\lambda(2^{R_i}-1)}{P_i}\right)$ given P_i and R_i. Thus, to find the closed-form optimal power scheduling solution, we approximate the original objective function by using the am-gm inequality as

$$\begin{aligned} & \exp\left(-\frac{\lambda(2^{R_1} - 1)}{P_1}\right) + \cdots + \exp\left(-\frac{\lambda(2^{R_T} - 1)}{P_T}\right) \\ & \ge T\left\{\exp\left[-\left(\frac{\lambda(2^{R_1} - 1)}{P_1} + \cdots + \frac{\lambda(2^{R_T} - 1)}{P_T}\right)\right]\right\}^{\frac{1}{T}}, \end{aligned} \tag{6}$$

where the equality holds if and only if $\exp(-\frac{\lambda(2^{R_1}-1)}{P_1}) = \cdots = \exp(-\frac{\lambda(2^{R_T}-1)}{P_T})$. Consequently, the problem (P1) is reformulated by minimizing its upper bound, which converts the non-convex problem to a more efficient convex optimization problem as

$$\begin{aligned} (\text{P2}): \min_{\{P_i\}} \; & \frac{1}{T} \sum_{i=1}^{T} \frac{\lambda(2^{R_i} - 1)}{P_i} \\ \text{s.t.} & \sum_{i=1}^{t} P_i \le \sum_{i=1}^{t} E_i, P_i \ge 0, 1 \le i \le T. \end{aligned} \tag{7}$$

III. Outage Probability Optimization without Battery Capacity Constraint

In this section, we study the outage probability minimization problem with equal or unequal transmission rates. Specifically, we first discuss two important properties of optimal solution to Problem (P2) given in (7) by analyzing a series of arithmetic means of harvested energy from current time slot t_c to the t_m-th time slot defined as

$$f(t_c, t_m) = \frac{\sum\limits_{k=t_c}^{t_m} E_k}{t_m - t_c + 1}, \quad t_m = t_c, \cdots, t_T, \tag{8}$$

where E_k is the harvested energy at the k-th time slot and $t_T = T$ for expression unification. We then define the moment t_g that achieves the global minimum of (8) for the current time slot t_c as

$$t_g = \arg\min_{t_c \le t_m \le t_T} f(t_c, t_m). \tag{9}$$

In the following, we present the optimal power scheduling scheme with equal transmission rate in Theorem 1, and then extend the conclusion to unequal transmission rates in Theorem 2.

We first investigate the power scheduling scheme in equal rate scenario. Specifically, we first introduce the scheme if the global minimum defined in (9) is achieved at the finalized time slot, i.e., $t_g = t_T$. Afterwards, we show a piecewise optimal power scheduling scheme when $t_g < t_T$. Consequently, we present the optimal power scheduling scheme by taking both cases into account for all the time slots.

As stated previously, the harvested power levels for all time slots are assumed to be known prior to the transmission or can be explicitly estimated. The objective function in problem (P2) in (7) can be simplified as

$$\min_{P_i} \frac{1}{T} \sum_{i=1}^{T} \frac{\alpha}{P_i}, \tag{10}$$

which is the reciprocal of the harmonic mean of transmission power, and $\alpha = \lambda(2^R - 1)$ is defined as the system transmission index.

Lemma 1: (**Energy Equipartition**) If $t_T = \arg\min_{t_c \le t_m \le t_T} f(t_c, t_m)$, the optimal power scheduling solution to Problem (P2) from the current time slot t_c to T-th time slot is given as

$$P_i^* = f(t_c, t_T), \quad \text{for} \quad t_c \le i \le t_T. \tag{11}$$

Proof: We first prove that the probably optimal solution is achieved if all the harvested energies are allocated equally for time slots from t_c to t_T. Given that the harmonic mean of a sequence of positive numbers is less or equal to its arithmetic mean, the objective function in (10) is lower bounded by

$$\frac{1}{T} \sum_{i=t_c}^{T} \frac{\alpha}{P_i} \ge \frac{\alpha T}{\sum\limits_{i=t_c}^{T} P_i} = \frac{\alpha T}{\sum\limits_{i=t_c}^{T} E_i}. \tag{12}$$

Note that the lower bound is achieved if and only if all the allocated power levels are equal to each other, i.e., $P_i = f(t_c, t_T), \forall t_c \le i \le t_T$.

In the following, we prove that the equally partitioned power levels $f(t_c, t_T)$ satisfy the constraints in (7) by adopting mathematical induction, i.e., the optimal solution is achievable.

The initial step: In this step, we verify that the if $P_{t_c} = f(t_c, t_T)$, then P_{t_c+1} can also achieve $f(t_c, t_T)$.

Since $t_T = \arg\min_{t_c \le t_m \le t_T} f(t_c, t_m)$, we have $f(t_c, t_T) < E_{t_c}$ and $f(t_c, t_T)$ can be assigned to P_{t_c}. Moreover, we can also obtain

$$\frac{E_{t_c} + E_{t_c+1}}{2} \ge f(t_c, t_T), \tag{13}$$

Hence, the available energy at the $t_c + 1$ time slot is

$$\tilde{E} = E_{t_c} - P_{t_c} + E_{t_c+1} \ge 2f(t_c, t_T) - f(t_c, t_T) = f(t_c, t_T),$$

which demonstrates that the global minimum $f(t_c, t_T)$ satisfying the energy harvesting constraint at $t_c + 1$, and P_{t_c+1} can also be $f(t_c, t_T)$.

The inductive step: In this step, we will prove that if there is an integer $N \in [t_c, t_T - 1]$ such that $P_n^* = f(t_c, t_T)$ for all $n \in [t_c, N]$, then P_{N+1}^* can also achieve $f(t_c, t_T)$. For $t_T = \arg\min_{t_c \le t_m \le t_T} f(t_c, t_m)$, the arithmetic means of harvested energy from current time slot t_c to the $N+1$-th time slot is larger than that to the T-th time slot, i.e., $f(t_c, N+1) \ge f(t_c, t_m)$.

The available energy at the $N+1$ time slot is

$$\begin{aligned} B_{N+1} &= B_{t_c} \sum_{k=t_c+1}^{N+1} E_k - (N - t_c + 1) f(t_c, t_T) \\ &\ge (N - t_c + 2) f(t_c, t_T) - (N - t_c + 1) f(t_c, t_T) \\ &= f(t_c, t_T). \end{aligned} \tag{14}$$

which infers that the global minimum $f(t_c, t_T)$ satisfying the energy harvesting constraint at $N+1$-th time slot. This completes the inductive step.

So far, we have prove that the optimal power scheduling solution to Problem (P2) is to allocate the harvested energy equally for each time slot if $t_T = \arg\min_{t_m} f(t_c, t_m)$. This finishes the proof of Lemma 1. ■

In Lemma 1, we have explore the optimal power scheduling solution when the global minimum of function $f(t_c, t_m)$ occurs at $t_g = t_T$. When $t_c = 1$, if the global minimum of function $f(t_c, t_m)$ occurs before the last time slot t_T, i.e., $t_g < t_T$, we can segment the sequence of harvested energy $[E_0, \cdots, E_{t_T-1}]$ into two pieces at $t = t_g$, and denote the cutting point as t_{g_1}. Obviously, from Lemma 1, the optimal power scheduling scheme for the first segment (from $t = 1$ to $t = t_{g_1}$) is $P_t^* = f(1, t_{g_1}), \forall t = 1, \cdots, t_{g_1}$. Similarly, we can determine the global minimum at $t_c = t_{g_1+1}$ according to (9), and divide the remaining sequence into two segments at this moment, denoted as $t = t_{g_2}$. Therefore, a series of global minimums can be obtained iteratively until we reach the end of the sequence, which can be denoted as $\{t_{g_i}\}_{i=1}^{N_g}$ with N_g being the total number of the global minimums.

Lemma 2: (**Piecewise Optimal**) Given $t_{g_{i+1}} = \arg\min_{t_{g_i+1} \le t_m \le t_T} f(t_{g_i+1}, t_m)$, the sequence of harvested energy $[E_0, \cdots, E_{t_T-1}]$ is segmented into pieces $[t_{g_i} + 1, t_{g_{i+1}}]$ with $i = 1, \cdots, N_g$. If the energy is exhausted at the moment t_{g_i}, the piecewise optimal power scheduling of each piece is given by

$$P_m^* = f(t_{g_i} + 1, t_{g_{i+1}}), \ \forall \ t_{g_i} + 1 < m \le t_{g_{i+1}}. \tag{15}$$

Proof: At the first time slot, the cutting point t_{g_1} is obtained by (9) as $t_{g_1} = \arg\min_{1 \le t_m \le t_T} f(1, t_m)$, $\forall m = 1, 2, \cdots, T$. After cutting the sequence at $t = t_{g_1}$, the optimal power scheduling solution of first piece $[1, t_{g_1}]$ can be conducted according to Lemma 1 as

$$P_t^* = f(1, t_{g_1}), \quad \text{for} \quad t \in [1, t_{g_1}]. \tag{16}$$

Note that the energy is exhausted at the moment t_{g_1} under this power scheduling scheme. Iteratively, we repeat the operation like first time slot, and find the i-th cutting point t_{g_i}. Thus, for the i-th piece, the optimal allocated power is

$$P_m^* = f(t_{g_{i-1}} + 1, t_{g_i}), \quad \text{for} \quad t_{g_{i-1}} < m \le t_{g_i}. \tag{17}$$

Eventually, we obtain N_g global minimums iteratively until the end of the sequence is reached. Note that the N_g global minimums has the following relationship

$$f(1, t_{g_1}) \leq f(t_{g_1}+1, t_{g_2}) \leq \cdots \leq f(t_{g_{N_g-1}}+1, t_T). \quad (18)$$

■

With the properties of optimal solution to Problem (P2) shown in Lemma 1 and Lemma 2, we give the following closed-form power scheduling solution for arbitrary T as:

Theorem 1: (**Min-Average Strategy for Equal Rate**) Given *a priori* known harvested energy $[E_1, E_2, \cdots, E_T]$ and a fixed transmission rate R, the optimal power scheduling solutions $[P_1^*, P_2^*, \cdots, P_T^*]$ for arbitrary T time slots to the Problem (P2) is

$$\begin{aligned} t_{k+1}^* &= \underset{t_k+1 \leq t_m \leq t_T}{\arg\min}\ f(t_k+1, t_m),\ \ \forall k \\ P_k^* &= f\left(t_k+1, t_{k+1}\right),\ \ \forall k. \end{aligned} \quad (19)$$

Proof: In Lemma 2, we have obtained the optimal power scheduling solution to each segmented pieces. Now, we want to prove the piecewise optimal power scheduling result is also the global optimal result to Problem (P2) by showing that the energy should be used up in each cutting point t_{g_i} with mathematical induction.

The initial step: In this step, we verify that the energy $\sum_{k=0}^{t_{g_1}-1} E_k$ should be used up at t_{g_1}. Thus, the power scheduling solution in (15) can achieve the optimal performance to Problem (P2) when the sequence of harvested energy is segmented into two pieces, namely $[1, t_{g_1}]$ and $[t_{g_1}+1, t_{g_2} = t_T]$.

Suppose from first to t_{g_1}-th time slot, we totally consume $\sum_{k=0}^{t_{g_1}-1} E_k - E_s > 0$ amount of energy, and E_s is saved to second segment. Although saving energy during the period between first and t_{g_1}-th time slot may cause the change of global minimum point t_{g_1}, according to (12), the lower bound of (10) is achieved by equally allocate $\frac{\sum_{k=0}^{t_{g_1}-1} E_k - E_s}{t_{g_1}}$ to each time slot. In this case, the objective function in (10) from first time slot to t_{g_1}-th time slot can be lower bounded by

$$\frac{1}{t_{g_1}} \sum_{k=1}^{t_{g_1}} \frac{\alpha}{P_k} \geq \frac{\alpha(t_{g_1})}{\sum\limits_{k=0}^{t_{g_1}-1} E_k - E_s} \triangleq y_{11}. \quad (20)$$

Moreover, in order to achieve the optimal performance for the first two segments, the energy $\sum_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s$ should be used up at t_{g_2}-th slot. Because the objective function in (10) increases as the P_k decreases, i.e., for $0 \leq \Delta_k < P_k$, we always have

$$\begin{aligned} \frac{1}{t_{g_2}-t_{g_1}} \sum_{k=t_{g_1}+1}^{t_{g_2}} \frac{\alpha}{P_k - \Delta_k} &\geq \frac{\alpha(t_{g_2}-t_{g_1})}{\sum\limits_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s - \sum\limits_{k=t_{g_1}+1}^{t_{g_2}} \Delta_k} \\ &\geq \frac{\alpha(t_{g_2}-t_{g_1})}{\sum\limits_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s}. \end{aligned} \quad (21)$$

In this case, the objective function in (10) from t_{g_1+1}-th time slot to t_{g_2}-th time slot can be lower bounded by

$$\frac{1}{t_{g_2}-t_{g_1}} \sum_{k=t_{g_1}+1}^{t_{g_2}} \frac{\alpha}{P_k} \geq \frac{\alpha(t_{g_2}-t_{g_1})}{\sum\limits_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s} \triangleq y_{12} \quad (22)$$

After combining (20) and (22), we have the objective function in (10) from first time slot to t_{g_2}-th time slot as

$$\begin{aligned} y_1 &\triangleq \tfrac{1}{t_{g_2}} \left[t_{g_1} y_{11} + (t_{g_2}-t_{g_1}) y_{12}\right] \\ &= \tfrac{1}{t_{g_2}} \left[\frac{\alpha(t_{g_1})^2}{\sum\limits_{k=0}^{t_{g_1}-1} E_k - E_s} + \frac{\alpha(t_{g_2}-t_{g_1})^2}{\sum\limits_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s} \right]. \end{aligned} \quad (23)$$

And the objective function with transmission power in (15) is given by

$$y_2 \triangleq \frac{1}{t_{g_2}} \left[\frac{\alpha(t_{g_1})^2}{\sum_{k=0}^{t_{g_1}-1} E_k} + \frac{\alpha(t_{g_2}-t_{g_1})^2}{\sum_{k=t_{g_1}}^{t_{g_2}-1} E_k} \right]. \quad (24)$$

After some manipulations, the difference between y_1 and y_2 is shown in (25).

Since $f(1, t_{g_1}) \leq f(t_{g_1}+1, t_{g_2})$, we have

$$(t_{g_1}) \left(\sum_{k=t_{g_1}}^{t_{g_2}-1} E_k \right) - (t_{g_2}-t_{g_1}) \left(\sum_{k=0}^{t_{g_1}-1} E_k \right) \geq 0, \quad (26)$$

and the difference in (25) is always greater than 0. Hence, the energy $\sum_{k=1}^{t_{g_1}} E_k$ should be used up at t_{g_1}.

The inductive step: In this step, we assume that in order to achieve optimal performance for the first N segments, the energy should be used up at $t_{g_{N-1}}$-th time slot. And we want to prove that in order to achieve optimal performance for the first $N+1$ segments, the energy should be used up at t_{g_N}-th time slot, where $N+1 \leq N_g$.

If the energy is not used up at t_{g_N}-th time slot, we suppose that the sensor totally consume $\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k - E_s' > 0$ amount of energy from $(t_{g_{N-1}}+1)$-th to t_{g_N}-th time slot, and E_s' is saved to the $(N+1)$-th segment. Note that, if the energy is saved from any time previous to $(t_{g_{N-1}}+1)$-th time slot, we can not guarantee the optimal performance of the first N segments, which is contradicted to the assumption. Thus, the energy can only be saved during the period between $(t_{g_{N-1}}+1)$-th and t_{g_N}-th time slot.

Although saving energy during the period between $(t_{g_{N-1}}+1)$-th and t_{g_N}-th time slot may cause the change of global minimum point t_{g_N}, according to (12), the lower bound of (10) is achieved by equally allocate $\frac{\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k - E_s'}{t_{g_N}-t_{g_{N-1}}}$ to each time slot. In this case, from $(t_{g_{N-1}}+1)$-th to t_{g_N}-th time slot, the objective function in (10) with energy saving is lower bounded by

$$\frac{1}{t_{g_N}-t_{g_{N-1}}} \sum_{k=t_{g_{N-1}+1}}^{t_{g_N}} \frac{\alpha}{P_k} \geq \frac{\alpha(t_{g_N}-t_{g_{N-1}})}{\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k - E_s'} \triangleq y_{11}'. \quad (28)$$

$$y_1 - y_2 = \frac{\alpha E_s^2 \left[(t_{g_1}) \sum_{k=t_{g_1}}^{t_{g_2}-1} E_k + (t_{g_2} - t_{g_1}) \sum_{k=0}^{t_{g_1}-1} E_k \right] + \alpha E_s \left[(t_{g_1}) \left(\sum_{k=t_{g_1}}^{t_{g_2}-1} E_k \right) - (t_{g_2} - t_{g_1}) \left(\sum_{k=0}^{t_{g_1}-1} E_k \right) \right]}{t_{g_2} \sum_{k=0}^{t_{g_1}-1} E_k \sum_{k=t_{g_1}}^{t_{g_2}-1} E_k \left(\sum_{k=0}^{t_{g_1}-1} E_k - E_s \right) \left(\sum_{k=t_{g_1}}^{t_{g_2}-1} E_k + E_s \right)} \tag{25}$$

$$y_1{}' - y_2{}' = \frac{\left(\begin{array}{l} (E_{s'}) \left[\left((t_{g_N} - t_{g_{N-1}}) \sum_{k=t_{g_N}}^{t_{g_{N+1}-1}} E_k \right) + \left((t_{g_{N+1}} - t_{g_N}) \sum_{k=t_{g_{N-1}+1}}^{t_{g_N}} E_k \right) \right] \\ +\alpha E_{s'} \left[(t_{g_N} - t_{g_{N-1}}) \left(\sum_{k=t_{g_N+1}}^{t_{g_N+1}} E_k \right) - (t_{g_{N+1}} - t_{g_N}) \left(\sum_{k=t_{g_{N-1}+1}}^{t_{g_N}} E_k \right) \right] \end{array} \right)}{(t_{g_{N+1}} - t_{g_{N-1}}) \sum_{k=t_{g_{N-1}+1}}^{t_{g_N}} E_k \sum_{k=t_{g_N+1}}^{t_{g_N+1}} E_k \left(\sum_{k=t_{g_{N-1}+1}}^{t_{g_N}} E_k - E_{s'} \right) \left(\sum_{k=t_{g_N+1}}^{t_{g_N+1}} E_k + E_{s'} \right)} \tag{27}$$

Moreover, in order to achieve the optimal performance for the first $N+1$ segments, the energy $\sum_{k=t_{g_N}}^{t_{g_{N+1}}-1} E_k + E'_s$ should be used up at $t_{g_{N+1}}$-th slot, because the objective function in (10) increases as the P_k decreases. In this case, from $(t_{g_N}+1)$-th to $t_{g_{N+1}}$-th time slot, the lower bound of the objective function in (10) with additional energy is given by

$$y'_{12} \triangleq \frac{\alpha(t_{g_{N+1}} - t_{g_N})}{\sum_{k=t_{g_N}}^{t_{g_{N+1}}-1} E_k + E'_s}. \tag{29}$$

After combining (28) and (29), we have the objective function in (10) from $(t_{g_{N-1}}+1)$-th time slot to $t_{g_{N+1}}$-th time slot as

$$\begin{aligned} y'_1 &\triangleq \frac{1}{t_{g_{N+1}} - t_{g_{N-1}}} \left[(t_{g_N} - t_{g_{N-1}}) y'_{11} + (t_{g_{N+1}} - t_{g_N}) y'_{12} \right] \\ &= \frac{1}{t_{g_{N+1}} - t_{g_{N-1}}} \left[\frac{\alpha(t_{g_N} - t_{g_{N-1}})^2}{\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k - E'_s} + \frac{\alpha(t_{g_{N+1}} - t_{g_N})^2}{\sum_{k=t_{g_N}}^{t_{g_{N+1}}-1} E_k + E'_s} \right]. \end{aligned} \tag{30}$$

And the objective function with transmission power in (15) is given by

$$y'_2 \triangleq \frac{1}{t_{g_{N+1}} - t_{g_{N-1}}} \left[\frac{\alpha(t_{g_N} - t_{g_{N-1}})^2}{\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k} + \frac{\alpha(t_{g_{N+1}} - t_{g_N})^2}{\sum_{k=t_{g_N}}^{t_{g_{N+1}}-1} E_k} \right]. \tag{31}$$

After some manipulations, the difference between y'_1 and y'_2 is shown in (27).

Since $f(t_{g_{N-1}}+1, t_{g_N}) \leq f(t_{g_N}+1, t_{g_{N+1}})$, we have

$$(t_{g_N} - t_{g_{N-1}}) \left(\sum_{k=t_{g_N}}^{t_{g_{N+1}}-1} E_k \right) \geq (t_{g_{N+1}} - t_{g_N}) \left(\sum_{k=t_{g_{N-1}}}^{t_{g_N}-1} E_k \right) \tag{32}$$

Thus, we can conclude that the lower bound of (10) with energy saving from the N-th segment to $N+1$-th segment has poorer performance comparing to the solution in (15), which infers that in order to achieve optimal performance for the first $N+1$ segments, the energy should be used up at t_{g_N}-th time slot.

So far we have shown that the energy should be used up in each cutting point t_{g_i}, and the piecewise optimal power scheduling result in (15) is globally optimal, which is equivalent to the power scheduling solution in (19). ■

Now, we discuss the optimal power scheduling solution to Problem (P2) given unequal rates R_i. Denote $\alpha_i = \lambda(2^{R_i} - 1)$ for $i = 1, \cdots, T$. And redefine the $f(\cdot)$-function as

$$F(t_c, t_m) = \frac{1}{\sum_{i=t_c}^{t_m} \sqrt{\alpha_i}} \left(\sum_{i=t_c-1}^{t_m-1} E_i \right). \tag{33}$$

We can obtain the optimal power scheduling solution for unequal rates case with the new defined $F(t_c, t_m)$ function.

Theorem 2: (**Min-Average Strategy for Unequal Rates**) Given *a priori* known harvested energy E_i for $i = 0, 1, \cdots, T-1$, transmission rates R_k for $k = 1, 2, \cdots, T$ and an infinite battery capacity, the optimal power scheduling solutions $[P_1^*, P_2^*, \cdots, P_T^*]$ for arbitrary T time slots to the Problem (P2) is

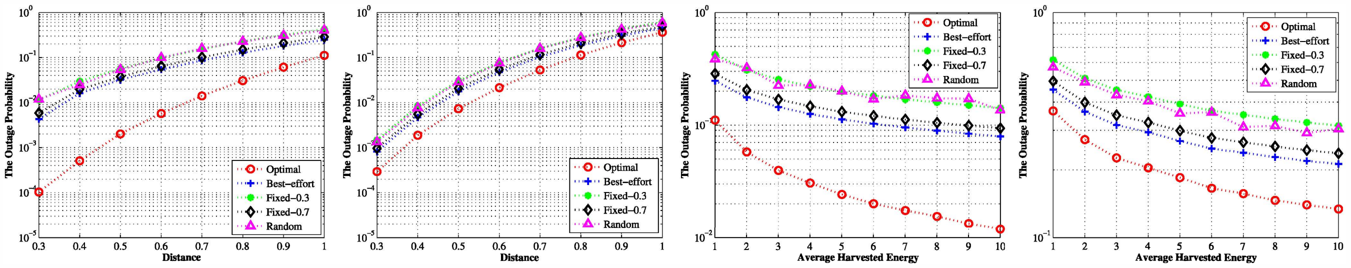
$$\begin{aligned} t^*_{k+1} &= \arg\min_{t_k+1 \leq t_m \leq t_T} F(t_k+1, t_m), \ \forall k \\ P^*_k &= \sqrt{\alpha_k} F(t_k+1, t_{k+1}), \ \forall k. \end{aligned} \tag{34}$$

IV. Simulation Results

In this section, we evaluate the performance of the optimal power scheduling algorithms by simulations. For the harvested energy model, we implement the same EH sources assumptions as [7]. In particular, we assume a periodic power profile model from a predictable EH source, and the harvested power profile is given by

$$E_i = \tilde{E} \left[\sin\left(\frac{i-1}{T} 2\pi - \theta \right) + 1 \right], \tag{35}$$

for $i = 1, 2, \cdots, T$, where $\tilde{E}$ is the average transmission energy, and θ is the phase shift. And we choose the length of the communication as $T = 10$. For power scheduling with equal rate, we set $R = 1$ bit/sec/Hz. And for power scheduling with unequal rate, R is a uniformly distributed random variable between in the interval $[1, 2]$ bit/sec/Hz. Throughout our



(a) (b) (c) (d)

Fig. 1. Simulation Results. (a) (Equal rate) The outage probability VS. distance; (b) (Unequal rates) The outage probability VS. distance; (c) (Equal rate) The outage probability VS. averaged energy; and (d) (Unequal rates) The outage probability VS. averaged energy.

simulations, we use the path loss model $\lambda = d^{-3}$, where d is the distance between sensor node and destination.

In order to investigate the performance of our optimal power scheduling algorithm comprehensively, we compare our optimal power scheduling algorithm with other strategies: (1) the best-effort power scheduling strategy, which allocates all available power to the current communication block. (2) The fixed-β power scheduling strategy, which saves the fixed β amount of power in the first $T-1$ periods and used up the saving energy at T-th period. And β is chosen to be 0.3 and 0.7 to demonstrate different levels of energy saving. (3) The random saving power scheduling strategy, which saves η amount of power in the first $T-1$ communication block and uses up the saved energy at T-th communication block. And η is a uniformly distributed random variable in the interval $[0, 1]$.

Firstly, we present the outage performance with respect to channel gain λ indicated by different distances, which will generate different system transmission index α. Outage performance for equal and unequal transmission rates are shown in Fig. 1 (a) and Fig. 1 (b), respectively. In this experiment, we assume $\theta = \pi/3$, with which the harvested energy in (35) reflects the solar radiance of a day between eight am to eighteen pm, averaged harvested energy $\tilde{E} = 1$ and the average SNR is equal to 10dB. It is shown that our optimal power scheduling strategy in (19) is the best among all other strategies in all distance realizations given equal transmission rate. And our optimal scheduling strategy in (34) performs best among all other strategies in all distance realizations given unequal transmission rates. As the distance is getting further, the gap between the *optimal* strategy and other strategies getting closer. And it infers that fixed saving, random saving, and best-effort strategies, which do not adapt to the channel gain, are not efficient.

Secondly, we show the outage performance with respect to the average harvested energy $\tilde{E}$, where the equal and unequal transmission rates cases are shown in Fig. 1 (c) and Fig. 1 (d), respectively. In this experiment, we assume $\theta = \pi/3$, distance is normalized to unity, and the average SNR is equal to 10dB. It is shown that our optimal power scheduling strategy in (19) is the best among all other strategies in all $\tilde{E}$ realizations given equal transmission rate. And our optimal scheduling strategy in (34) performances best among all other strategies in all $\tilde{E}$ realizations given unequal transmission rates. As the $\tilde{E}$ getting larger, the gap between the *optimal* strategy and other strategies getting larger.

V. Conclusion

In this paper, we analyze the optimal power scheduling method to minimize the outage probability with both equal and unequal transmission rates. We first convert the non-convex average outage probability minimization problem to a convex problem by adopting the am-gm inequality approximation. Then, based on the properties of the global minimum for the energy-and-time function $f(t_c, t_m)$, we give the general optimal power scheduling solutions to the new optimization problem for both equal and unequal transmission rate cases, respectively. Simulation results show that, due to the better adaptivity to the system parameters, our optimal solution has better outage performance compared to alternative strategies.

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