Waveform Design for Sum Rate Optimization in Time-Reversal Multiuser Downlink Systems

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Abstract— **Utilizing channel reciprocity, the traditional time-reversal technique boosts the signal-to-noise ratio at the receiver with very low transmitter complexity. However, the large delay spread gives rise to severe inter-symbol interference (ISI) when the data rate is high, and the performance is further degraded in the multiuser downlink due to the inter-user interference (IUI). In this work, we study the weighted sum rate optimization problem in the time-reversal multiuser downlink using waveform design. By exploiting the relation between the allocated power and the SINR targets, an iterative algorithm is proposed to optimize the waveform design through a new power allocation algorithm. Simulation results are shown to demonstrate the superior performance of the proposed algorithm in comparison with other traditional methods.**

I. INTRODUCTION

In wideband communication systems, multipath channel with a very large delay spread results in severe inter-symbol interference (ISI) when the data rate is high. In multiuser downlink communication, one transmitter broadcasts different data streams to many receivers at the same time. Since each receiver is only interested in its own data stream, the unintended data streams result in the inter-user interference (IUI).

In the time-reversal communication system [1], the receiver complexity is very low due to the one-tap detection, that is, the receiver detects the received signal using only one sample instead of more complicated receive equalization. With the channel reciprocity, the transmitter can estimate the channel state information (CSI) through the periodical training signals from the receivers. Among the transmit equalizers, the traditional time-reversal technique is a simple matched-filter (MF) with very low complexity and can maximize the signal-to-noise ratio (SNR) at the receiver. However, when the data rate is high, the large delay spread gives rise to severe ISI [2], [3]. Therefore, the performance of MF is limited when the transmitted power is high. Several approaches are proposed to deal with the interference. With multiple-input-single-output (MISO) precoding [2], a zero-forcing (ZF) waveform can be adopted to reduce the ISI. However, ZF does not take the noise into account. In [3], Emami *et. al.* improve the traditional time-reversal waveform with the minimum mean squared error (MMSE) waveform which suppresses both the ISI and noise, but this MMSE waveform is not optimized under different data rates.

Although the ZF and MMSE waveforms can successfully suppress the ISI, they [2], [3] only consider a single-user scenario with a fixed data rate. In multiuser communication, due to the low complexity compared to nonlinear methods, linear precoding is usually adopted to enhance the intended signal and suppress the IUI. However, to the best of our knowledge, previous works in linear precoding for multiuser MIMO broadcast channels assume flat fading and take no consideration into the ISI introduced by multipath. In this paper, we study the sum rate optimization problem in the time-reversal multiuser downlink with waveform design to simultaneously suppress the ISI and IUI.

In the literature, the sum rate optimization methods for MIMO broadcast channels with linear precoding have been proposed in [4]–[9]. Some of these works [4]–[6] directly optimize the sum rate in the downlink, and some works [5], [7]–[9] exploit the uplink-downlink duality [10]–[12] to alternatingly optimize the sum rate. Such an iterative solution based on virtual uplink first appeared in [13], [14].

In this paper, we assume that the transmitter and each receiver are equipped with a single antenna (multiuser SISO downlink). Despite only one antenna, the abundance of multipath can provide numerous degrees of freedom as virtual antennas, and hence the multipath channel is analogous to a MIMO channel except that each user has a self-interfering component (namely, ISI). With the well-known uplink-downlink duality, the waveform design for the downlink can be obtained using virtual uplink, given any power allocation. However, the power allocation for sum rate is non-convex for either uplink or downlink. We propose the waveform design through a new power allocation algorithm to alleviate the ISI and IUI. The proposed power allocation algorithm exploits the relation between the allocated power and the SINR targets. Although the resulting sum rate is local optimum, simulation results show that such local optimum can still achieve a much better weighted sum rate performance than traditional methods.

This paper is organized as follows. In Section II, the system model and problem formulation are described. In Section III, we propose an iterative algorithm which alternatingly optimizes between calculating the waveform and the power allocation vector. Finally, the numerical simulation in Section IV illustrates the performance compared with traditional methods, and conclusion is drawn in Section V.

II. SYSTEM MODEL

Consider a multiuser downlink multipath channel with K users. The transmitter is assumed to have perfect CSI. The receive signal of the kth user at time m, $y_k[m]$, can be written as

$$y_k[m] = \sum_l h_k[m-l]s[l] + n_k[m], \qquad (1)$$

where $s[m]$ is the transmit signal and $h_k[m]$ denotes the channel impulse response of user k. The channel length of $h_k[m]$ is denoted by L_k, i.e., $h_k[m] = 0$ for $m < 0$ and $m \geq L_k$. Writing (1) in a matrix form, we have the receive signal vector of the kth user as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k$$
$$= \mathbf{H}_k \left(\sum_{j=1}^{K} \mathbf{u}_j \sqrt{p_j} x_j \right) + \mathbf{n}_k, \qquad (2)$$

where $\mathbf{y}_k$ is a $(2L-1) \times 1$ vector with $L = \max_k L_k$, $\mathbf{u}_j$ is the transmit waveform, p_j is the transmit power allocated to user j, x_j is the intended signal for user j, and $\mathbf{n}_k$ is the additive white Gaussian noise (AWGN) with mean zero and variance σ^2. In (2), $\mathbf{H}_k$ is a $(2L-1) \times L$ Toeplitz matrix with each column vector being the shifted version of $h_k[m]$.

In the time-reversal communication system, the users detect the received signal with only one tap. The Lth element of $\mathbf{y}_k$ can be expressed as

$$y_k[L] = \mathbf{H}_k^{(1)} \left(\mathbf{u}_k \sqrt{p_k} x_k + \sum_{j=1, j \neq k}^{K} \mathbf{u}_j \sqrt{p_j} x_j \right) + [\mathbf{n}_k]_L. \qquad (3)$$

where $\mathbf{H}_k^{(1)}$ denotes the Lth row of $\mathbf{H}_k$ and $\mathbf{H}_k^{(0)}$ consists of all other rows . The summation term indicates the inter-user interference. $[\mathbf{n}_k]_L$ denotes the Lth element in the vector $\mathbf{n}_k$.

In (3), we have explicitly considered the inter-user interference and the noise. However, (3) is only for the scenario with one symbol transmitted. When symbols are transmitted consecutively, then ISI must be considered. The amount of ISI depends on the symbol rate. Therefore, we introduce the decimation ratio D, which represents the ratio of the symbol duration to the signal sampling duration. Each element in $\mathbf{y}_k$ is a signal sample, and the data symbols are transmitted every D signal samples. Clearly, higher D results in less ISI but lower data rate. In other words, one symbol induces ISI to at most $\lfloor 2(L-1)/D \rfloor$ other symbols. Therefore, with decimation ratio D, the channel matrix $\mathbf{H}_k$ can be decimated by keeping only $\lfloor 2(L-1)/D \rfloor + 1$ rows and deleting the other rows for simplicity.

Therefore, the SINR which considers both ISI and IUI for user k is

$$\text{SINR}_k^{\text{DL}} = \frac{\mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k p_k}{\mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k p_k + \sum_{j=1, j \neq k}^{K} \mathbf{u}_j^H \mathbf{R}_k \mathbf{u}_j p_j + \sigma^2}, \qquad (4)$$

where $\mathbf{R}_k^{(1)} = \mathbf{H}_k^{(1)H} \mathbf{H}_k^{(1)}$, $\mathbf{R}_k^{(0)} = \mathbf{H}_k^{(0)H} \mathbf{H}_k^{(0)}$, and $\mathbf{R}_j = \mathbf{H}_j^H \mathbf{H}_j$. The superscript DL denotes the downlink. The first term and the second term in the denominator denotes ISI and IUI, respectively.

In this paper, we jointly design the waveform $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_K]$ and power allocation vector $\mathbf{p} = [p_1, ..., p_K]^T$ to maximize the weighted sum rate subject to a total power constraint P_{max}, i.e.,

$$\mathcal{P}_{\text{Rate}}^{\text{DL}} : \quad \max_{\mathbf{p}, \mathbf{U}} \sum_{k=1}^{K} \alpha_k \log \left(1 + \text{SINR}_k^{\text{DL}} \right) \qquad (5a)$$
$$\text{s.t.} \quad \mathbf{1}^T \mathbf{p} \leq P_{\text{max}}, \qquad (5b)$$

where α_k denotes the rate weighting coefficient for user k.

III. ITERATIVE ALGORITHM FOR THE WEIGHTED SUM RATE OPTIMIZATION

In this section, we develop an iterative algorithm for the weighted sum rate optimization in multiuser downlink multipath channels. The algorithm first solves for the waveforms and power allocation in the virtual uplink system, and then transforms the solution into the original downlink problem.

With the uplink-downlink duality [10], [11], the downlink optimal waveform can be found in the virtual uplink, given any power allocation. However, the power allocation problem for sum rate maximization is non-convex. Hence, we propose a new waterfilling power allocation algorithm for the non-convex objective function.

A. Uplink-Downlink Duality

The virtual uplink problem is constructed as follows.

$$\mathcal{P}_{\text{Rate}}^{\text{UL}} : \quad \max_{\mathbf{q}, \mathbf{U}} \sum_{k=1}^{K} \alpha_k \log \left(1 + \text{SINR}_k^{\text{UL}} \right) \qquad (6a)$$
$$\text{s.t.} \quad \mathbf{1}^T \mathbf{q} \leq P_{\text{max}} \qquad (6b)$$

where $\mathbf{q} = [q_1, ..., q_K]^T$ is the power allocation in the virtual uplink, the downlink transmit waveform $\mathbf{U}$ becomes the uplink receive waveform, and the uplink SINR for user k is

$$\text{SINR}_k^{\text{UL}} = \frac{\mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k q_k}{\mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k q_k + \sum_{j=1, j \neq k}^{K} \mathbf{u}_k^H \mathbf{R}_j \mathbf{u}_k q_j + \sigma^2}, \qquad (7)$$

where q_k is the transmit power of user k in the virtual uplink, and the superscript UL denotes the virtual uplink.

By exploiting the fact that the SINR achievable regions are the same [10] for the two dual problems, we develop an iterative algorithm to solve $\mathcal{P}_{\text{Rate}}^{\text{DL}}$ by first solving $\mathcal{P}_{\text{Rate}}^{\text{UL}}$.

It is now well-known [11] that for given SINR targets $\{\gamma_k\}_{k=1}^{K}$, the duality shows that the minimum required total power for the downlink and its virtual uplink is the same. On the other hand, given a sum power constraint P_{max}, the same SINR region is achievable for both the downlink and its virtual uplink. Therefore, we propose an iterative algorithm to first solve $\mathcal{P}_{\text{Rate}}^{\text{UL}}$ and obtain the solution for Problem $\mathcal{P}_{\text{Rate}}^{\text{DL}}$. The algorithm iterates between computing the waveform $\mathbf{U}$ and solving for the uplink power vector $\mathbf{q}$. After the iteration for virtual uplink is completed, the downlink power vector $\mathbf{p}$ is then calculated with the virtual uplink power vector $\mathbf{q}$.

It is noteworthy that although the uplink-downlink duality and the waveform design of $\mathbf{U}$ given a fixed power allocation vector is not new, the proposed power allocation is the key element to the iterative sum rate optimization algorithm. Without the power allocation, one cannot iteratively update the waveform $\mathbf{U}$ accordingly.

In the following two subsections, we describe the waveform design and the power allocation algorithm in detail.

B. Waveform Design

The $\mathrm{SINR}_k^{\mathrm{UL}}$ in (7) can also be written as

$$\mathrm{SINR}_k^{\mathrm{UL}} = \frac{q_k \mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k}{\mathbf{u}_k^H \left(q_k \mathbf{R}_k^{(0)} + \sum_{j=1, j \neq k}^K q_j \mathbf{R}_j + \sigma^2 \mathbf{I} \right) \mathbf{u}_k}. \quad (8)$$

Given a fixed $\mathbf{q}$, we can choose $\mathbf{u}_k$ to be the maximum SINR waveform, which turns out to be the MMSE waveform

$$\mathbf{u}_k^{\mathrm{MMSE}} = c_k^{\mathrm{MMSE}} \left(\sum_{j=1}^K q_j \mathbf{R}_j + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}_k^{(1)H}. \quad (9)$$

Here, c_k^{MMSE} is a constant such that the norm of $\mathbf{u}_k^{\mathrm{MMSE}}$ is normalized to unit.

C. Power Allocation Algorithm: Iterative SINR Waterfilling

Given fixed $\mathbf{U}$, the problem $\mathcal{P}_{\mathrm{Rate}}^{\mathrm{UL}}$ becomes solving the power allocation vector $\mathbf{q}$ given a sum power constraint $P_{\max}$. It can be verified that this problem is non-convex so that the global optimal solution is difficult to search. Therefore, our objective of the power allocation algorithm is to efficiently obtain a near-optimal solution.

We propose a novel power allocation algorithm called iterative SINR waterfilling. Firstly, the optimization variables are changed from $\{q_k\}_{k=1}^K$ to $\{\gamma_k\}_{k=1}^K$ through the following mapping.

$$\gamma_k = \mathrm{SINR}_k^{\mathrm{UL}} = \frac{\mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k q_k}{\mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k q_k + \sum_{j=1, j \neq k}^K \mathbf{u}_k^H \mathbf{R}_j \mathbf{u}_k q_j + \sigma^2} \quad (10)$$

Let $\mathbf{D}$ be a diagonal matrix with $[\mathbf{D}]_{kk} = \gamma_k / \mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k$, and

$$[\mathbf{\Phi}]_{kj} = \begin{cases} \mathbf{u}_j^H \mathbf{R}_k \mathbf{u}_j, & k \neq j \\ \mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k, & k = j \end{cases}. \quad (11)$$

Rewriting (10), we can represent the power allocation vector $\mathbf{q}$ in terms of $\{\gamma_k\}_{k=1}^K$ by

$$\mathbf{q} = \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma}. \quad (12)$$

Now, instead of allocating power $\{q_k\}_{k=1}^K$, we allocate the SINR targets $\{\gamma_k\}_{k=1}^K$.

Thus, the weighted sum rate optimization problem in terms of $\{\gamma_k\}_{k=1}^K$ becomes

$$\max_{\gamma_1, \ldots, \gamma_K} \quad \sum_{k=1}^K \alpha_k \log \left(1 + \gamma_k \right), \quad (13)$$

$$\mathrm{s.t.} \quad \mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma} \leq P_{\max}, \quad (14)$$

$$\rho \left(\mathbf{D}\mathbf{\Phi}^T \right) < 1, \quad (15)$$

where $\rho(\cdot)$ denotes the spectral radius.

According to the Karush-Kuhn-Tucker (KKT) conditions, the optimum γ_k must satisfy

$$\gamma_k = \left(\frac{\alpha_k}{\lambda t_k} - 1 \right)^+, \quad (16)$$

$$\mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma} = P_{\max}, \quad (17)$$

$$\rho \left(\mathbf{D}\mathbf{\Phi}^T \right) < 1, \quad (18)$$

where

$$t_k = \frac{\mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k}{\gamma_k^2} \mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\mathbf{e}_k \mathbf{e}_k^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma}, \quad (19)$$

and $\mathbf{e}_k$ is the kth column of a $K \times K$ identity matrix.

Next, in order to solve for λ, we show the monotonicity of λ in the left-hand-side expression of (17) and (18).

Lemma 1: Let $\mathbf{\Lambda}$ be a square diagonal matrix with positive diagonal elements, and $\mathbf{S}$ be a square matrix with positive elements. Then $\rho(\mathbf{\Lambda}\mathbf{S}) \leq \rho(\mathbf{\Lambda})\rho(\mathbf{S})$.

Proposition 1: $\rho(\mathbf{D}\mathbf{\Phi}^T)$ *is monotonically decreasing with* λ. $\mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma}$ *is also monotonically decreasing with* λ *if* $\rho(\mathbf{D}\mathbf{\Phi}^T) < 1$.

Proof: Assume $\hat{\lambda} > \lambda$. From (16), we have $\hat{\gamma}_k \leq \gamma_k$. $\rho(\hat{\mathbf{D}}\mathbf{D}^{-1}) \leq 1$. With *Lemma 1*,

$$\begin{aligned} \rho(\hat{\mathbf{D}}\mathbf{\Phi}^T) &= \rho(\hat{\mathbf{D}}\mathbf{D}^{-1}\mathbf{D}\mathbf{\Phi}^T) \\ &\leq \rho(\hat{\mathbf{D}}\mathbf{D}^{-1})\rho(\mathbf{D}\mathbf{\Phi}^T) \\ &\leq \rho(\mathbf{D}\mathbf{\Phi}^T). \end{aligned}$$

Thus, $\rho(\mathbf{D}\mathbf{\Phi}^T)$ is monotonically decreasing with λ.

If $\rho(\mathbf{D}\mathbf{\Phi}^T) < 1$, then $\left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} = \sum_{r=0}^\infty \left(\mathbf{D}\mathbf{\Phi}^T \right)^r$.

$$\begin{aligned} \mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma} &= \mathbf{1}^T \sum_{r=0}^\infty \left(\mathbf{D}\mathbf{\Phi}^T \right)^r \mathbf{D}\boldsymbol{\sigma} \\ &\geq \mathbf{1}^T \sum_{r=0}^\infty \left(\hat{\mathbf{D}}\mathbf{\Phi}^T \right)^r \hat{\mathbf{D}}\boldsymbol{\sigma}. \end{aligned}$$

Thus, $\mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma}$ is also monotonically decreasing with λ if $\rho(\mathbf{D}\mathbf{\Phi}^T) < 1$. $\qquad\square$

Since the γ_k in (16), $\rho(\mathbf{D}\mathbf{\Phi}^T)$, and $\mathbf{1}^T \left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T \right)^{-1} \mathbf{D}\boldsymbol{\sigma}$ are all monotonic with λ, the bisection search can be applied to efficiently compute the λ such that the power constraint is satisfied. In the one dimensional bisection search, the initial upper bound of λ can be set as $\max_k \alpha_k / t_k$ since the SINR targets $\{\gamma_k\}_{k=1}^K$ are all zero for λ higher than this value. The lower bound can be set as a small positive number, which corresponds to very large values of $\{\gamma_k\}_{k=1}^K$.

(16) is a waterfilling-like solution with a feasibility constraint (18) and a nonlinear power constraint (17). The t_k can be considered as a modification term to the water level due to the effect of the interference [15]. In solving the optimum γ_k, we can first fix t_k, and then SINR target γ_k is found by using bisection search for λ and substituting λ into (16). The new γ_k is then used to update t_k as in (19). The procedure is

TABLE I
ITERATIVE SINR WATERFILLING

(i) Given $\mathbf{q}$, initialize γ_k with (10).
(ii) **Loop**:
    1. Calculate t_k using (19).
    2. Bisection search λ with (16), i.e.,
        (a) Set bisection upper bound $\lambda^{\max} = \max_k \alpha_k/t_k$,
            and lower bound $\lambda^{\min} = \delta > 0$.
        (b) **Loop**:
            Set $\lambda = \frac{1}{2}(\lambda^{\max} + \lambda^{\min})$.
            Compute $\gamma_k = \left(\frac{\alpha_k}{\lambda t_k} - 1\right)^+$.
            If $\rho\left(\mathbf{D}\mathbf{\Phi}^T\right) < 1$ **then**
                If $\mathbf{1}^T\left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T\right)^{-1}\mathbf{D}\boldsymbol{\sigma} < P_{\max}$ **then**
                    $\lambda^{\max} = \lambda$.
                else
                    $\lambda^{\min} = \lambda$.
            else
                $\lambda^{\min} = \lambda$.
            Until $|\mathbf{1}^T\left(\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T\right)^{-1}\mathbf{D}\boldsymbol{\sigma} - P_{\max}| < \epsilon$.
    3. With γ_k obtained in last step, compute $\mathbf{q}$ by (12).
    Until $\mathbf{q}$ converges or the max. number of iterations is reached.

TABLE II
ITERATIVE WEIGHTED SUM RATE OPTIMIZATION ALGORITHM

(i) Initialize $q_k = P_{\max}/K$.
(ii) **Loop** (uplink optimization):
    1. Calculate $\mathbf{U}$ by (9).
    2. Calculate $\mathbf{q}$ using *Iterative SINR Waterfilling*.
    Until $\mathbf{q}$ and $\mathbf{U}$ converges or the max. number of iterations is reached.
(iii) Compute γ_k by (10).
(iv) Obtain downlink power vector $\mathbf{p}$ by (21).

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repeated until convergence. The power allocation algorithm is summarized in Table I.

As in [15], we can incorporate a memory term for γ_k to slow down the update and the convergence can be improved. In the nth iteration, the $\gamma_k(n)$ can be calculated by

$$\gamma_k(n) = \beta\gamma_k^{\text{new}}(n) + (1-\beta)\gamma_k(n-1), \qquad (20)$$

where $\gamma_k^{\text{new}}(n)$ is the one obtained after the bisection search and β is the forgetting factor with $0 < \beta < 1$.

D. Iterative Sum Rate Optimization Algorithm

The iterative sum rate optimization algorithm iterates between calculating the waveform $\mathbf{U}$ using (9) and the power allocation $\mathbf{q}$ using Table I in the virtual uplink. After convergence or maximum number of iterations is reached, we can compute the corresponding achievable SINR targets $\{\gamma_k\}_{k=1}^K$ and the downlink power allocation $\mathbf{p}$ can then be obtained similar to (12), i.e.,

$$\mathbf{p} = (\mathbf{I} - \mathbf{D}\mathbf{\Phi})^{-1}\mathbf{D}\boldsymbol{\sigma}. \qquad (21)$$

The proposed iterative algorithm for the weighted sum rate optimization algorithm is summarized in Table II.

IV. NUMERICAL SIMULATION

In this section, we use numerical simulation to demonstrate the performance of the proposed iterative sum rate optimization algorithm. In the simulation, each path is assumed to be an i.i.d. complex Gaussian random variable with zero mean and variance of $\frac{1}{2L}$ per dimension. Figure 1 shows the sum rate performance of a 2-user system with $L = 8$, $D = 2$, $\alpha_1 = \alpha_2 = 1$. The weighted sum rate performance of a 4-user system is shown in Figure 2, where $L = 10$, $D = 4$, and $\alpha_1 = \alpha_2 = 2, \alpha_3 = \alpha_4 = 1$. Each rate is averaged over 1000 channel realizations.

In both figures, the forgetting factor β is set as $1/K$. MF denotes the matched-filter; ZF denotes the zero-forcing waveform. The proposed power allocation is the iterative SINR waterfilling algorithm described in Section III-C. The equal power allocation is to split the total power equally to each user, i.e., $p_k = P_{\max}/K$. The optimal power allocation is simulated by exhaustive search.

From the figures, it is clear that the proposed power allocation can improve the performance of equal power allocation for all waveform designs, since the iterative SINR waterfilling is able to find a local optimum by taking the channel gains into consideration. The improvement for the MMSE waveform is especially significant at high power region. In Figure 1, the MMSE waveform with the proposed power allocation performs almost the same as the optimal power allocation. We can see that even with the MMSE waveform, which is optimal given any power allocation, the equal power allocation still saturates at high power region.

Note that since the sub-optimal waveforms MF and ZF are not related to the power allocation, these methods cannot iteratively optimize between the waveform design and power allocation. For the MMSE with equal power allocation, since the power allocation remains the same, the MMSE waveform cannot update accordingly. Therefore, these methods are not iterative and thus require lower computational complexity compared to the proposed algorithm.

It is well-known [16] that since MF only maximizes the received signal power without considering the interference, it saturates at a lower rate, as shown in both figures. ZF cancels the interference but sacrifices the received signal power resulting in worse performance at low power region. The proposed method strikes a balance between the two by reducing the interference including ISI and IUI, while keeping a high received signal power.

Figure 3 shows typical convergence behaviors of the inner loop (Table I) and outer loop (Table II) of the proposed algorithm. The maximum iteration number of inner loop is set to be 20. Since the power allocation problem is non-convex, the KKT condition is only necessary, but not sufficient for a global optimum. Thus, the iterative SINR waterfilling only converges to a local optimum, if it converges. However, very fast convergence of the outer loop in is always observed (typically 2 to 8 iterations).

Finally, we note that we have no proof whether our iterative algorithms converge to the global optimum or merely local
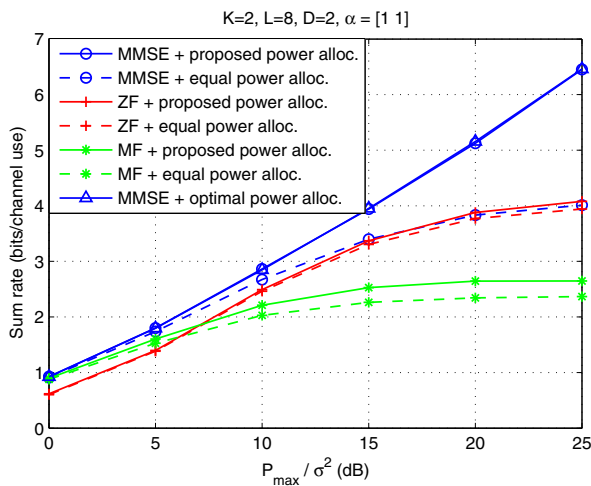
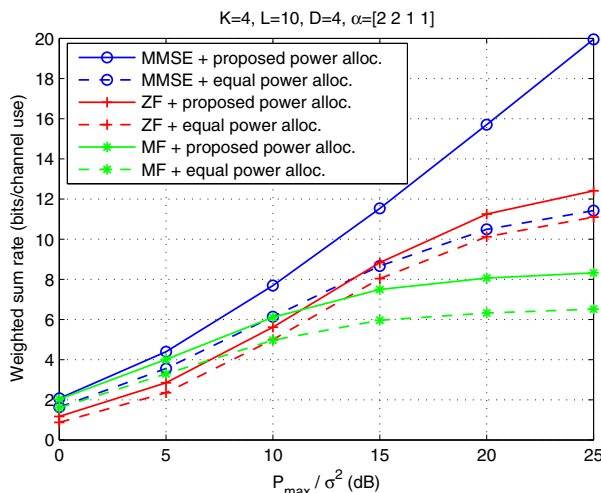Fig. 1.   Sum rate performance for different methods with 2 users.

Fig. 2.   Weighted sum rate performance for different methods with 4 users. The weighting coefficients are $\alpha_1 = \alpha_2 = 2$, and $\alpha_3 = \alpha_4 = 1$.

Fig. 3.   Convergence behaviors of the inner loop (power allocation) and the outer loop (sum rate optimization).

optima. However, as shown by the simulation results, the local optima still result in much better performance than other traditional methods.

V. CONCLUSION

In this paper, we explored the weighted sum rate optimization problem of the waveform design for the time-reversal multiuser downlink communication system. We proposed an iterative algorithm which alternatingly optimizes the waveform and the power allocation. Utilizing the duality for the SINR constrained problems, the proposed iterative SINR waterfilling provides an efficient numerical method for the non-convex power allocation problem.

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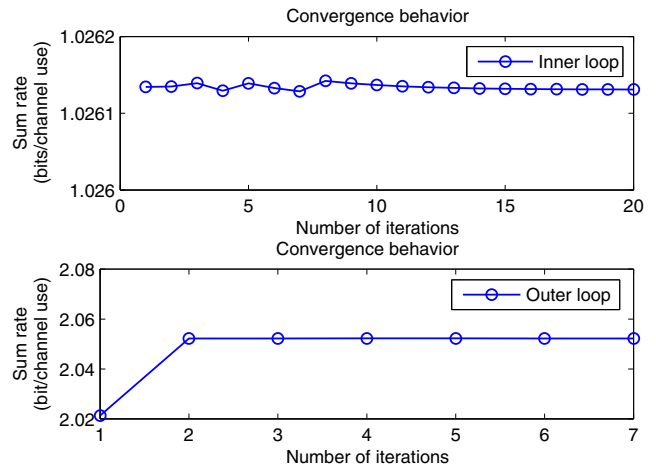
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