# Near-Optimal Waveform Design for Sum Rate Optimization in Time-Reversal Multiuser Downlink Systems

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Abstract—Utilizing channel reciprocity, the traditional timereversal technique boosts the signal-to-noise ratio at the receiver with very low transmitter complexity. However, the large delay spread gives rise to severe inter-symbol interference (ISI) when the data rate is high, and the achievable transmission rate is further degraded in the multiuser downlink due to the interuser interference (IUI). In this work, we study the weighted sum rate optimization problem by means of waveform design in the time-reversal multiuser downlink where the receiver processing is based on a single sample. Power allocation has a significant impact on the waveform design problem. We propose a new power allocation algorithm named Iterative SINR Waterfilling, which is able to achieve comparable sum rate performance to that of globally optimal power allocation. We further propose another approach called Iterative Power Waterfilling for multiple data streams. Iterative SINR Waterfilling provides better performance than Iterative Power Waterfilling in the scenario of high interference, while Iterative Power Waterfilling can work under multiple data streams. Simulation results show the superior performance of the proposed algorithms in comparison with other waveform designs such as zero-forcing and conventional time-reversal waveform.

Index Terms—Time reversal, waveform design, multiuser downlink.

## I. INTRODUCTION

T HE traditional time-reversal (TR) waveform [1] is able to boost the signal-to-noise ratio at the receiver with very low transmitter complexity in a severe multipath channel. Such a waveform is simply the time-reverse of the channel impulse response which is transmitted by propagating back through each multipath with channel reciprocity. In essence, the environment is performing deconvolution on the fly for the system. It can collect most energy of the multipaths to a single tap. The receiver complexity is hence very low due to the onetap detection, that is, the receiver detects the received signal using only one sample instead of more complicated receive equalization.

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In broadband communication systems, the traditional timereversal technique can be viewed as a simple matched-filter of the multipath channel which maximizes the signal-to-noise ratio (SNR) at the receiver when using single-tap detection. Such a waveform is optimal if only one symbol is transmitted. However, when the symbol rate is high, large delay spreads of the traditional TR waveform result in severe inter-symbol interference (ISI) [2], [3]. Several approaches have been proposed to suppress ISI. In [2], a zero-forcing (ZF) waveform can be adopted to minimize the ISI, but ZF does not take the noise into account. In [3], Emami *et. al.* improved the traditional time-reversal waveform with the minimum mean squared error (MMSE) waveform which suppresses both the ISI and noise.

Although the ZF and MMSE waveforms can successfully suppress the ISI and hence improve the performance of TR systems, they only consider the single-user scenario. In multiuser downlink communications, one transmitter broadcasts different data streams to many receivers at the same time. Since each receiver is only interested in its own data stream, the unintended data streams introduce inter-user interference (IUI) to each receiver. In multiuser communications, due to the low complexity compared to nonlinear methods, linear transmit waveform design can be adopted to enhance the intended signal and suppress the IUI to maximize the transmission rate. Weighted sum rate is an important design criterion since weighting coefficients provide prioritization among different users in various applications. For example, the weights can be chosen as queue lengths to minimize the risk of buffer overflows [4], and the equal weights can be used to maximize the achievable sum rate corresponding to the system capacity.

In the literature, there are some prior works on sum rate optimization for MIMO broadcast channels with linear preprocessing. Some of these works [5]–[7] directly optimize the sum rate in the downlink, and some works [6], [8], [9] exploit the uplink-downlink duality [10]–[13] to iteratively optimize the sum rate. Such an iterative solution based on virtual uplink first appeared in [14], [15]. In [11], the joint beamforming and power control solutions to the max-min SINR problem are developed. Cai *et. al.* further consider the max-min SINR problem subject to a weighted-sum power constraint in multicell downlink networks [13]. The approaches in [6] optimize the weighted sum rate under linear zero-forcing constraints and greedy algorithms are proposed to allocate data streams

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to users. In [8], the receiver is assumed to know the transmit power allocation, and thus, the receiver is able to normalize the received signal with the transmit power allocation and the resulting problem is shown to be convex. In [9], the weighted sum rate maximization is modelled into minimizing the product of MSE, and sequential quadratic programming is used to locate a local optimum of the minimization. Most previous works on beamforming for multiuser MIMO downlink channels assume flat fading and do not consider the ISI introduced by multipath. ISI degrades the user's achievable rate as a self-interfering term proportional to its own transmit power. To the best of our knowledge, the systems with single-tap detection considering ISI and IUI have not been considered before. In order to tackle this problem, we propose a near-optimal waveform design to maximize the weighted sum rate by simultaneously suppressing the ISI and IUI. Pre-equalization for ISI and IUI is proposed in [16], where the design criterion is MSE and thus the problems they considered are convex. In this work, the waveform design in the multiuser downlink systems where the receiver processing is based on a single tap is formulated and shown to be similar to the downlink beamforming problem. Beamforming problems with the max-min SINR criteria are convex [11], [13] and thus can be solved optimally, but beamforming for weighted sum rate maximization is known to be a non-convex optimization problem. In tackling the non-convex sum rate maximization problem, d.c. (difference of convex functions) programming has been applied in recent literature (e.g., [17], [18]) by exploiting the fact that the sum rate can be written as difference of convex functions. In [17], Kha et. al. proposed an iterative algorithm in which the solution to a convex optimization problem is calculated at each iteration, which is accomplished by another iterative algorithm such as the interior point method. Thus, the overall complexity of such a method is quite high. Other d.c. programming approaches (e.g., [18]) claimed to be able to obtain the global optimum are mostly based on combinatorial optimization such as branchand-bound global search and usually require demanding computational complexity. A practical approach is provided in [19] to maximize weighted sum rate for MIMO-OFDM systems but each user has only a single data stream. In this work, we further provide an efficient solution to the weighted sum rate maximization problem for multiple data streams. For single data stream, the proposed algorithm is shown to perform better than [19] in the scenario of high interference.

The proposed algorithms are based on the well-known uplink-downlink duality, i.e., the waveform design for the downlink can be obtained using virtual uplink, given any power allocation. However, the power allocation problem for sum rate optimization is non-convex for either uplink or downlink. By exploiting the relation between the allocated power and the SINR targets, we propose a power allocation algorithm called *Iterative SINR Waterfilling* which can achieve comparable performance to the globally-optimal power allocation. The essential idea of the proposed scheme is to first allocate the SINRs to the users to maximize the weighted sum rate, and with the allocated target SINRs, the corresponding power allocation can easily be determined. For multiple data streams, we also propose an iterative power allocation algorithm



Fig. 1. The schematic diagram of the time reversal system.

called *Iterative Power Waterfilling* which is the multiple-datastream extension of the *modified iterative waterfilling* in [20]. Simulation results show that both the proposed approaches significantly outperform traditional waveform designs such as zero-forcing and time-reversal waveforms.

This paper is organized as follows. In Section II, the system model and problem formulation are described. In Section III, we introduce the proposed waveform design which alternately optimizes between calculating the waveform and the power allocation vector. The waveform design for multiple data streams is proposed in Section IV. Finally, the numerical simulation in Section V illustrates the performance compared with traditional methods, and conclusion is drawn in Section VI.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In the time reversal system [1], the receiver first sends an impulse signal, which is then received by the transmitter as a channel impulse response. Utilizing the channel impulse response, the transmitter forms the TR waveform and sends data symbols using the TR waveform. Figure 1 shows the schematic diagram of the time reversal system. In this paper, we consider multiuser downlink multipath channels with one transmitter and K users. The receive signal of the kth user at time m,  $y_k[m]$ , can be written as

$$y_k[m] = \sum_{l} h_k[m-l]s[l] + n_k[m],$$
 (1)

where s[m] is the transmit signal and  $h_k[m]$  denotes the channel impulse response of user k. The channel length of  $h_k[m]$  is denoted by  $L_k$ , i.e.,  $h_k[m] = 0$  for m < 0 and  $m \ge L_k$ . Writing (1) in a matrix form, we have the receive signal vector of the kth user as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k = \mathbf{H}_k \left( \sum_{j=1}^K \mathbf{u}_j \sqrt{p_j} x_j \right) + \mathbf{n}_k,$$
 (2)

where  $\mathbf{y}_k$  is a  $(2L-1) \times 1$  vector with  $L = \max_k L_k$ ,  $\mathbf{u}_j$  is the transmit waveform,  $p_j$  is the transmit power allocated to user j,  $x_j$  is the intended signal for user j, and  $\mathbf{n}_k$  is the additive white Gaussian noise (AWGN) with mean zero and variance  $\sigma^2$ . In (2),  $\mathbf{H}_k$  is a  $(2L-1) \times L$  Toeplitz matrix with each column vector being the shifted version of  $\{h_k[m]\}_{m=1}^L$ ,

In the time-reversal communication system, user k estimates the received signal by only  $y_k[L]$ . Let  $\mathbf{H}_k^{(l)}$  denote the *l*th row of  $\mathbf{H}_k$ , the symbol at time slot *l* for user k as  $x_k(l)$ , and  $[\mathbf{n}_k]_L$  as the *L*th element of  $n_k$ . The complete characterization of the signal with ISI and IUI is given by

$$y_{k}[L] = \mathbf{H}_{k}^{(L)} \mathbf{u}_{k} \sqrt{p}_{k} x_{k}(L) + \mathbf{H}_{k}^{(L)} \left( \sum_{j=1, j \neq k}^{K} \mathbf{u}_{j} \sqrt{p_{j}} x_{j}(L) \right)$$
$$+ \sum_{l=1, l \neq L}^{2L-1} \mathbf{H}_{k}^{(l)} \left( \sum_{j=1}^{K} \mathbf{u}_{j} \sqrt{p_{j}} x_{j}(l) \right) + [\mathbf{n}_{k}]_{L}.$$
(3)

Assume that user k only decodes its own current symbol  $x_k(L)$  and considers the interferences (IUI and ISI) as noise. Then the SINR of user k is given as

$$\operatorname{SINR}_{k}^{\mathrm{DL}} = \frac{\mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(1)} \mathbf{u}_{k} p_{k}}{\mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(0)} \mathbf{u}_{k} p_{k} + \sum_{j=1, j \neq k}^{K} \mathbf{u}_{j}^{H} \mathbf{R}_{k} \mathbf{u}_{j} p_{j} + \sigma^{2}},$$
(4)

where  $\mathbf{R}_{k}^{(1)} = \mathbf{H}_{k}^{(L)H} \mathbf{H}_{k}^{(L)}$ ,  $\mathbf{R}_{j} = \mathbf{H}_{j}^{H} \mathbf{H}_{j}$ , and  $\mathbf{R}_{k}^{(0)} = \mathbf{R}_{k} - \mathbf{R}_{k}^{(1)}$ . The superscript DL denotes the downlink. The first term and the second term in the denominator denote ISI and IUI, respectively.

In this paper, we jointly design the waveform  $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_K]$  and power allocation vector  $\mathbf{p} = [p_1, ..., p_K]^T$  to maximize the weighted sum rate subject to a total power constraint  $P_{\text{max}}$ , i.e.,

$$\mathcal{P}_{\text{Rate}}^{\text{DL}}: \max_{\mathbf{p}, \mathbf{U}} \sum_{k=1}^{K} \alpha_k \log \left(1 + \text{SINR}_k^{\text{DL}}\right)$$
  
s.t.  $\mathbf{1}^T \mathbf{p} \le P_{\max}, \mathbf{u}_i^H \mathbf{u}_i = 1, p_i \ge 0, \forall i,$ (5)

where  $\alpha_k$  denotes the rate weighting coefficient for user k, and 1 is an all-one vector with K elements.

# III. ITERATIVE ALGORITHM FOR THE WEIGHTED SUM RATE OPTIMIZATION

In this section, we develop an iterative algorithm for the weighted sum rate optimization in multiuser downlink time-reversal system. Since the waveform design structure is decoupled in the virtual uplink system and the uplinkdownlink duality [10]–[12] builds a bridge between the two systems, the proposed algorithm first solves the waveform design and power allocation in the virtual uplink system, and then transforms the solution into the original downlink problem.

The optimal power allocation problem for sum rate maximization is non-convex either in downlink or virtual uplink. In general, solving the global optimum for a non-convex problem requires an exhaustive search, which is computationally impractical. Hence, we propose an algorithm to efficiently attain a satisfactory near-optimal solution for the non-convex power allocation problem. We will show in Section V by simulations that the proposed algorithm can reach a solution which is very closed to global optimum.

#### A. Uplink-Downlink Duality

As shown in (4), the SINR of every user depends on the waveforms of all users, so all users' waveforms have to be jointly designed at the same time. Thus, the waveform design is complicated in the downlink system. With the uplinkdownlink duality [10]–[12], the downlink optimal waveform can be individually decided in the virtual uplink with fixed power allocation.

The virtual uplink problem is constructed as follows.

$$\mathcal{P}_{\text{Rate}}^{\text{UL}}: \quad \max_{\mathbf{q},\mathbf{U}} \sum_{k=1}^{K} \alpha_k \log \left(1 + \text{SINR}_k^{\text{UL}}\right)$$
  
s.t.  $\mathbf{1}^T \mathbf{q} \le P_{\max}, \mathbf{u}_i^H \mathbf{u}_i = 1, q_i \ge 0, \forall i,$ (6)

where  $\mathbf{q} = [q_1, ..., q_K]^T$  is the power allocation in the virtual uplink, the downlink transmit waveform  $\{\mathbf{u}_j\}_{j=1}^K$  becomes the uplink receive waveform, and the uplink SINR for user k is

$$\operatorname{SINR}_{k}^{\operatorname{UL}} = \frac{\mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(1)} \mathbf{u}_{k} q_{k}}{\mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(0)} \mathbf{u}_{k} q_{k} + \sum_{j=1, j \neq k}^{K} \mathbf{u}_{k}^{H} \mathbf{R}_{j} \mathbf{u}_{k} q_{j} + \sigma^{2}},$$
(7)

where  $q_k$  is the transmit power of user k in the virtual uplink, and the superscript UL denotes the virtual uplink. Examining the difference between (4) and (7), we can see that  $SINR_k^{UL}$  only depends on one user's waveform  $\mathbf{u}_k$ , and thus the waveform design structure is decoupled in the uplink with the solution given by the generalized eigenvalue problem [21].

By exploiting the fact that the SINR achievable regions are the same [10] for the two dual problems, we develop an iterative algorithm to solve  $\mathcal{P}_{Rate}^{DL}$  by first solving  $\mathcal{P}_{Rate}^{UL}$ . It is now well-known [11] that for given SINR targets  $\{\gamma_k\}_{k=1}^K$ , the minimum required total power for the downlink and its virtual uplink are the same. On the other hand, given a sumpower constraint  $P_{max}$ , the achievable SINR region is the same for both the downlink and its virtual uplink. Therefore, the solution for  $\mathcal{P}_{Rate}^{UL}$  is also the solution for  $\mathcal{P}_{Rate}^{DL}$ . Because the transmit waveforms  $\{\mathbf{u}_j\}_{j=1}^K$  and  $\mathcal{P}_{Rate}^{DL}$  cannot be directly solved, the proposed algorithm iterates between computing the waveforms  $\{\mathbf{u}_j\}_{j=1}^K$  and solving for the uplink power vector  $\mathbf{q}$ . After the iteration for virtual uplink is completed, the downlink power vector  $\mathbf{p}$  is then calculated using the waveforms  $\{\mathbf{u}_j\}_{j=1}^K$  and the virtual uplink power vector  $\mathbf{q}$ .

Given a fixed power allocation, the optimal waveform design of  $\{\mathbf{u}_j\}_{j=1}^K$  can be directly derived by leveraging the uplink-downlink duality. Based on this, we can then focus on the design of power allocation. We propose a power allocation algorithm to be employed in the iterative sum rate optimization algorithm. Due to the non-convexity of the problem, to obtain the global optimum in general requires exhaustive search. The proposed algorithm can attain a sub-optimum that is very close to the global optimum in terms of weighted sum rate performance and thus much better than traditional methods such as zero-forcing and time-reversal waveforms. In the following two subsections, we describe the waveform design and the power allocation algorithm in detail.

# B. Individual Waveform Design

The SINR<sup>UL</sup><sub>k</sub> in (7) can also be written as

$$\operatorname{SINR}_{k}^{\operatorname{UL}} = \frac{q_{k} \mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(1)} \mathbf{u}_{k}}{\mathbf{u}_{k}^{H} \left( q_{k} \mathbf{R}_{k}^{(0)} + \sum_{j \neq k} q_{j} \mathbf{R}_{j} + \sigma^{2} \mathbf{I} \right) \mathbf{u}_{k}}, \quad (8)$$

where only  $\mathbf{u}_k$  is involved and thus SINR<sub>k</sub><sup>UL</sup> can be optimized by choosing  $\mathbf{u}_k$  to be the principle eigenvector of the generalized eigenvalue problem,

$$q_k \mathbf{R}_k^{(1)} \mathbf{u}_k = \text{SINR}_k^{\text{UL}} \left( q_k \mathbf{R}_k^{(0)} + \sum_{j \neq k} q_j \mathbf{R}_j + \sigma^2 \mathbf{I} \right) \mathbf{u}_k, \quad (9)$$

This SINR-maximizing waveform turns out to be the MMSE waveform

$$\mathbf{u}_{k}^{\text{MMSE}} = c_{k}^{\text{MMSE}} \left( \sum_{j=1}^{K} q_{j} \mathbf{R}_{j} + \sigma^{2} \mathbf{I} \right)^{-1} \mathbf{H}_{k}^{(1)H}.$$
(10)

Here,  $c_k^{\text{MMSE}}$  is a constant such that the norm of  $\mathbf{u}_k^{\text{MMSE}}$  is normalized to unit. This can be easily verified by substituting (10) into (9), and the corresponding eigenvalue can be obtained as  $\text{SINR}_k^{\text{UL}} = \mathbf{H}_k^{(1)} \left( q_k \mathbf{R}_k^{(0)} + \sum_{j \neq k} q_j \mathbf{R}_j + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}_k^{(1)H}$ .

# C. Power Allocation: Iterative SINR Waterfilling

Given fixed  $\{\mathbf{u}_j\}_{j=1}^K$ , the problem  $\mathcal{P}_{\text{Rate}}^{\text{UL}}$  becomes solving the power allocation vector  $\mathbf{q}$  given a sum power constraint  $P_{\text{max}}$ . It can be verified that this problem is non-convex so the global optimal solution is difficult to search. Instead, our objective of the power allocation algorithm is to efficiently obtain a near-optimal solution.

We propose a new power allocation algorithm called *Iterative SINR Waterfilling*. The key feature of the proposed algorithm is that, instead of directly allocating the power  $\{q_k\}_{k=1}^K$ , we first allocate the SINRs  $\{\gamma_k\}_{k=1}^K$  to maximize the weighted sum rate under the sum power constraint. And then with the allocated SINRs, the power allocation of  $\{q_k\}_{k=1}^K$  can be easily established. The conversion to SINR waterfilling changes the objective function to be convex and the feasible region to be non-convex. In the following, it will be seen that such conversion can better capture the structure of interference. The SINR is expressed in terms of the power by

$$\gamma_k = \text{SINR}_k^{\text{UL}} = \frac{\mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k q_k}{\mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k q_k + \sum_{j \neq k} \mathbf{u}_k^H \mathbf{R}_j \mathbf{u}_k q_j + \sigma^2}.$$
(11)

Let **D** be a diagonal matrix with  $[\mathbf{D}]_{kk} = \gamma_k / \mathbf{u}_k^H \mathbf{R}_k^{(1)} \mathbf{u}_k$ , and

$$\left[\mathbf{\Phi}\right]_{kj} = \begin{cases} \mathbf{u}_j^H \mathbf{R}_k \mathbf{u}_j, & k \neq j \\ \mathbf{u}_k^H \mathbf{R}_k^{(0)} \mathbf{u}_k, & k = j \end{cases}$$
(12)

On the other hand, rewriting (11), we can represent the power allocation vector  $\mathbf{q}$  in terms of  $\{\gamma_k\}_{k=1}^K$  by

$$\mathbf{q} = \left(\mathbf{I} - \mathbf{D}\boldsymbol{\Phi}^T\right)^{-1}\mathbf{D}\boldsymbol{\sigma},\tag{13}$$

where  $\sigma$  is a  $K \times 1$  vector of all elements equal to  $\sigma^2$ . With (13), the power allocations  $\{q_k\}_{k=1}^K$  can be obtained from the SINR targets  $\{\gamma_k\}_{k=1}^K$ .

Then the weighted sum rate optimization problem in terms of  $\{\gamma_k\}_{k=1}^K$  can be reformulated as

$$\max_{\gamma_1,\dots,\gamma_K} \sum_{k=1}^{K} \alpha_k \log\left(1+\gamma_k\right), \tag{14}$$

s.t. 
$$\mathbf{1}^T \left( \mathbf{I} - \mathbf{D} \boldsymbol{\Phi}^T \right)^{-1} \mathbf{D} \boldsymbol{\sigma} \le P_{\max},$$
 (15)

$$\rho\left(\mathbf{D}\boldsymbol{\Phi}^{T}\right) < 1,\tag{16}$$

where  $\rho(\cdot)$  denotes the spectral radius. Inequality (15) denotes the sum power constraint in terms of  $\{\gamma_k\}_{k=1}^K$ . The feasibility condition (16) and the constraint that the obtained power  $\{q_k\}_{k=1}^K$  are all non-negative are equivalent to each other. The detailed proof can be found in [22, Theorem 2]. One direction can be shown by observing that in (13),  $(\mathbf{I} - \mathbf{D} \Phi^T)^{-1} = \sum_{i=0}^{\infty} (\mathbf{D} \Phi^T)^i$  if  $\rho(\mathbf{D} \Phi^T) < 1$  (cf. [23, p.301]), and the matrix  $\mathbf{D} \Phi^T$  is element-wise positive.

According to the Karush-Kuhn-Tucker (KKT) conditions, the optimum  $\gamma_k$  must satisfy

$$\gamma_k = \left(\frac{\alpha_k}{\lambda t_k} - 1\right)^+,\tag{17}$$

$$\mathbf{1}^{T} \left( \mathbf{I} - \mathbf{D} \boldsymbol{\Phi}^{T} \right)^{-1} \mathbf{D} \boldsymbol{\sigma} = P_{\max}, \qquad (18)$$

$$p\left(\mathbf{D}\boldsymbol{\Phi}^{T}\right) < 1,$$
 (19)

where  $\lambda$  is the KKT multiplier and

$$t_{k} = \frac{\mathbf{u}_{k}^{H} \mathbf{R}_{k}^{(1)} \mathbf{u}_{k}}{\gamma_{k}^{2}} \mathbf{1}^{T} \left( \mathbf{I} - \mathbf{D} \boldsymbol{\Phi}^{T} \right)^{-1} \mathbf{D} \mathbf{e}_{k} \mathbf{e}_{k}^{T} \\ \times \left( \mathbf{I} - \mathbf{D} \boldsymbol{\Phi}^{T} \right)^{-1} \mathbf{D} \boldsymbol{\sigma},$$
(20)

and  $\mathbf{e}_k$  is the *k*th column of a  $K \times K$  identity matrix. The term  $t_k$  is a function of  $\{\gamma_k\}_{k=1}^K$ , i.e., it implicitly captures the interference introduced by the SINR allocation. Next, in order to solve  $\lambda$ , we show the monotonicity of  $\lambda$  in the left hand side of (18) and (19).

*Lemma 1: Let*  $\Lambda$  *be a square diagonal matrix with positive diagonal elements, and* **S** *be a square matrix with positive elements. Then*  $\rho(\Lambda S) \leq \rho(\Lambda)\rho(S)$ .

*Proof:* Let **x** and **y** be the eigenvectors corresponding to the maximum eigenvalues of  $\Lambda S$  and  $\Lambda^{1/2}S\Lambda^{-1/2}$ , respectively, with  $\|\mathbf{x}\| = 1$ , and  $\|\mathbf{y}\| = \|\Lambda^{1/2}\mathbf{x}\|$ . We have  $\|\mathbf{y}\|^2 \le \rho(\Lambda) \|\mathbf{x}\|^2$ . Then,

$$\rho(\mathbf{\Lambda}\mathbf{S}) = \mathbf{x}^{T}(\mathbf{\Lambda}\mathbf{S})\mathbf{x} \le \rho(\mathbf{\Lambda}^{1/2}\mathbf{S}\mathbf{\Lambda}^{-1/2})\|\mathbf{y}\|^{2} \le \rho(\mathbf{S})\rho(\mathbf{\Lambda}).$$
(21)

Proposition 1:  $\rho(\mathbf{D}\mathbf{\Phi}^T)$  is monotonically decreasing with  $\lambda$ .  $\mathbf{1}^T (\mathbf{I} - \mathbf{D}\mathbf{\Phi}^T)^{-1} \mathbf{D}\boldsymbol{\sigma}$  is also monotonically decreasing with  $\lambda$  if  $\rho(\mathbf{D}\mathbf{\Phi}^T) < 1$ .

*Proof:* Assume  $\hat{\lambda} > \lambda$ . From (17), we have  $\hat{\gamma}_k \leq \gamma_k$  and  $\rho(\hat{\mathbf{D}}\mathbf{D}^{-1}) \leq 1$ . With Lemma 1,

$$\rho(\hat{\mathbf{D}}\boldsymbol{\Phi}^{T}) = \rho(\hat{\mathbf{D}}\mathbf{D}^{-1}\mathbf{D}\boldsymbol{\Phi}^{T}) \le \rho(\hat{\mathbf{D}}\mathbf{D}^{-1})\rho(\mathbf{D}\boldsymbol{\Phi}^{T}) \le \rho(\mathbf{D}\boldsymbol{\Phi}^{T}).$$
(22)

Thus,  $\rho(\mathbf{D}\mathbf{\Phi}^T)$  is monotonically decreasing with  $\lambda$ .

If  $\rho(\mathbf{D}\boldsymbol{\Phi}^T) < 1$ , then  $(\mathbf{I} - \mathbf{D}\boldsymbol{\Phi}^T)^{-1} = \sum_{r=0}^{\infty} (\mathbf{D}\boldsymbol{\Phi}^T)^r$ (cf. [23, p.301]). We have

$$\mathbf{1}^{T} \left( \mathbf{I} - \mathbf{D} \boldsymbol{\Phi}^{T} \right)^{-1} \mathbf{D} \boldsymbol{\sigma} = \mathbf{1}^{T} \sum_{r=0}^{\infty} \left( \mathbf{D} \boldsymbol{\Phi}^{T} \right)^{r} \mathbf{D} \boldsymbol{\sigma}$$
$$\geq \mathbf{1}^{T} \sum_{r=0}^{\infty} \left( \hat{\mathbf{D}} \boldsymbol{\Phi}^{T} \right)^{r} \hat{\mathbf{D}} \boldsymbol{\sigma}.$$
(23)

Thus,  $\mathbf{1}^T (\mathbf{I} - \mathbf{D} \mathbf{\Phi}^T)^{-1} \mathbf{D} \boldsymbol{\sigma}$  is also monotonically decreasing with  $\lambda$  if  $\rho(\mathbf{D} \mathbf{\Phi}^T) < 1$ .

TABLE I Iterative SINR Waterfilling

(i) Given <b>q</b> , initialize $\gamma_k$ with (11).
(ii) Loop:
1. Calculate $t_k$ using (20).
2. Bisection search $\lambda$ with (17)-(19), i.e.,
(a) Set bisection upper bound $\lambda^{\max} = \max_k \alpha_k / t_k$ ,
and lower bound $\lambda^{\min} = \delta > 0$ .
(b) Loop:
Set $\lambda = \frac{1}{2}(\lambda^{\max} + \lambda^{\min}).$
Compute $\gamma_k = \left(\frac{\alpha_k}{\lambda t_k} - 1\right)^+$ .
If $\rho(\mathbf{D}\mathbf{\Phi}^T) < 1$ then
If $\mathbf{I}^T (\mathbf{I} - \mathbf{D} \mathbf{\Phi}^T)^{-1} \mathbf{D} \boldsymbol{\sigma} < P_{\max}$ then
$\lambda^{\max} = \lambda.$
else
$\lambda^{\min} = \lambda.$
else
$\lambda^{\min} = \lambda.$
Until $ 1^T (\mathbf{I} - \mathbf{D} \mathbf{\Phi}^T)^{-1} \mathbf{D} \boldsymbol{\sigma} - P_{\max}  < \epsilon.$
3. With $\gamma_k$ obtained in last step, compute <b>q</b> by (13).
<b>Until q</b> converges or the max. number of iterations is reached.
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ITERATIVE WEIGHTED SUM RATE OPTIMIZATION ALGORITHM FOR SINGLE DATA STREAM

(i) Initialize $q_k = P_{\max}/K$ .
(ii) <b>Loop</b> (uplink optimization):
1. Calculate $\{\mathbf{u}_j\}_{j=1}^K$ by (10).
2. Calculate q using Iterative SINR Waterfilling.
<b>Until q</b> and $\{\mathbf{u}_j\}_{j=1}^K$ converges or the max. number of iterations
is reached.
(iii) Compute $\gamma_k$ by (11).
(iv) Obtain downlink power vector $\mathbf{p}$ by (24).

Since the  $\gamma_k$  in (17),  $\rho(\mathbf{D}\Phi^T)$ , and  $\mathbf{1}^T (\mathbf{I} - \mathbf{D}\Phi^T)^{-1} \mathbf{D}\sigma$ are all monotonic with  $\lambda$ , the bisection search can be applied to efficiently compute the  $\lambda$  such that the power constraint is satisfied. In the one dimensional bisection search, the initial upper bound of  $\lambda$  can be set as  $\max_k \alpha_k / t_k$  since the SINR targets  $\{\gamma_k\}_{k=1}^K$  are all zero for  $\lambda$  higher than this value. The lower bound can be set as a small positive number, which corresponds to very large values of  $\{\gamma_k\}_{k=1}^K$ .

Eqn. (17) is a waterfilling-like solution with a feasibility constraint (19) and a nonlinear power constraint (18). The  $t_k$ can be considered as a modification term to the water level due to the effect of the interference. In solving the optimum  $\gamma_k$ , we can first fix  $t_k$ , and then SINR target  $\gamma_k$  is found by using bisection search for  $\lambda$  and substituting  $\lambda$  into (17). The new  $\gamma_k$  is then used to update  $t_k$  as in (20). The procedure is repeated until convergence. The proposed Iterative SINR Waterfilling is summarized in Table I.

We can incorporate a memory term for  $\gamma_k$  to slow down the update and the convergence can be improved. In the *n*th iteration, the  $\gamma_k(n)$  can be calculated by  $\gamma_k(n) = \beta \gamma_k^{\text{new}}(n) + (1-\beta)\gamma_k(n-1)$ , where  $\gamma_k^{\text{new}}(n)$  is the one obtained after the bisection search and  $\beta$  is the forgetting factor with  $0 < \beta < 1$ .

#### D. Iterative Sum Rate Optimization

The iterative sum rate optimization algorithm iterates between calculating the waveforms  $\{\mathbf{u}_j\}_{j=1}^K$  using (10) and the power allocation  $\mathbf{q}$  using Table I in the virtual uplink. The iterative algorithm is not guaranteed to converge. However, very fast convergence is almost always observed in the numerical simulation. When the algorithm converges, the obtained solution is a fixed point of (17)-(20), i.e., the solution satisfies the KKT conditions. In case it does not converge or it takes a long time to converge, the algorithm stops when the maximum number of iterations is reached. The solution obtained in each iteration is always feasible regardless of convergence. Hence, after convergence or the maximum number of iterations is reached, we can compute the corresponding achievable SINR targets  $\{\gamma_k\}_{k=1}^K$  and the downlink power allocation **p** can then be obtained similar to (13), i.e.,

$$\mathbf{p} = (\mathbf{I} - \mathbf{D}\boldsymbol{\Phi})^{-1} \mathbf{D}\boldsymbol{\sigma}.$$
 (24)

The proposed algorithm for the weighted sum rate optimization algorithm is summarized in Table II. After convergence or maximum number of iterations is reached, we take the variables obtained at the last iteration as the solution. The performance may be better if the iterative algorithm keeps track of all passing solutions and chooses the best solution when the maximum number of iterations is reached. However, keeping track of all passing solutions requires a heavy overhead of space complexity but does not contribute much to the averaged performance due to the rareness of the non-converging cases. We have conducted simulations and verified that the performance difference is not perceivable. Hence, concerning the complexity and performance tradeoff, we choose to use the variables obtained at the last iteration instead of keeping track of all passing solutions.

The accuracy of using the virtual uplink to compute the solution of the downlink is commented as follows. Given fixed transmit waveforms  $\{u_j\}_{j=1}^K$ , the power allocation problems to minimize the required sum power in the uplink and the downlink for achieving certain SINR targets are dual problems [10], [11]. As a consequence, the achievable weighted sum rates of the uplink and the downlink under the same sum power constraint are exactly the same. The solution in the uplink can be transformed into the downlink using (13), where the SINR targets are calculated by the uplink powers using (11), to achieve exactly identical SINRs and thus exactly the same weighted sum rate.

# IV. MULTIUSER MIMO DOWNLINK WITH MULTIPLE DATA STREAMS

In MIMO time-reversal systems where multiple data streams are transmitted to each user, the transmit waveforms of different data streams have a significant impact on the achievable rates of all users. The proposed Iterative SINR Waterfilling can only work for systems with single data streams. In this section, we first describe the system model and then also develop an iterative algorithm for the waveform design.

#### A. System Model

The transmitter is now equipped with  $N_t$  transmit antennas. Each of the K users has  $N_{r,k}$  receive antennas. The transmitter is transmitting  $M_k$  data streams to user k. The  $N_{r,k} \times 1$  receive signal vector of the kth user at time m,  $\mathbf{y}_k[m]$ , can be written as  $\mathbf{y}_k[m] = \sum_l \mathbf{H}_k[m-l]\mathbf{s}[l] + \mathbf{n}_k[m]$ , where the  $N_t \times 1$  vector  $\mathbf{s}[m]$  is the transmit signal at time m and the  $N_{r,k} \times N_t$  matrices  $\{\mathbf{H}_k[m]\}_{m=0}^{L-1}$  denote the MIMO channel impulse response of user k at time m. We assume each channel is L-tap. In a matrix form, the receive signal vector of the kth user is given by

$$\mathbf{y}_{k} = \mathbf{H}_{k}\mathbf{s} + \mathbf{n}_{k}$$
$$= \mathbf{H}_{k}\left(\mathbf{U}_{k}\sqrt{\mathbf{P}_{k}}\mathbf{x}_{k} + \sum_{j \neq k}\mathbf{U}_{j}\sqrt{\mathbf{P}_{j}}\mathbf{x}_{j}\right) + \mathbf{n}_{k}, \quad (25)$$

where  $\mathbf{y}_k = [\mathbf{y}_k^T[1], ..., \mathbf{y}_k^T[2L-1]]^T \in \mathbb{C}^{(2L-1)N_{r,k}}$ , and the  $M_k \times 1$  vector  $\mathbf{x}_k$  comprises  $M_k$  data streams intended for user k. The matrix  $\mathbf{U}_k = [\mathbf{U}_k^T[1], ..., \mathbf{U}_k^T[L]]^T \in \mathbb{C}^{LN_t \times M_k}$  is the transmit waveform for user k. The diagonal matrix  $\mathbf{P}_k = diag\{p_{k1}, ..., p_{kM_k}\}$  is the power allocated to the  $M_k$  data streams of user k.  $\mathbf{n}_k \in \mathbb{C}^{(2L-1)N_{r,k}}$  denotes the additive white Gaussian noise and each element of  $\mathbf{n}_k$  is with zero mean and variance  $\sigma_k^2$ . The channel  $\mathbf{H}_k \in \mathbb{C}^{(2L-1)N_{r,k} \times LN_t}$  is a block-Toeplitz matrix in which each sub-block  $\mathbf{H}_k[m] \in \mathbb{C}^{N_{r,k} \times N_t}$  is the channel matrix of receiver k at time m, i.e.,

$$\mathbf{H}_{k} = \begin{bmatrix} \mathbf{H}_{k}[1] & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{H}_{k}[2] & \mathbf{H}_{k}[1] & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}_{k}[1] \end{bmatrix}, \quad (26)$$

In the MIMO time-reversal system, users perform the single-tap detection by considering only the receive signal vector at time L, i.e.,  $\mathbf{y}_k[L]$ . Let  $\mathbf{H}_k^{(l)}$  denote the *l*th sub-block row of  $\mathbf{H}_k$ , e.g.,  $\mathbf{H}_k^{(L)} = [\mathbf{H}_k[L], ..., \mathbf{H}_k[1]]$ . After processing  $\mathbf{y}_k[L]$  with receive filter  $\mathbf{V}_k$ , the complete characterization of the signal, ISI and IUI is given by

$$\begin{aligned} \hat{\mathbf{x}}_{k}(L) &= \mathbf{V}_{k}^{H} \mathbf{y}_{k}[L] \\ &= \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(L)} \mathbf{U}_{k} \sqrt{\mathbf{P}_{k}} \mathbf{x}_{k}(L) \\ &+ \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(L)} \left( \sum_{j \neq k} \mathbf{U}_{j} \sqrt{\mathbf{P}_{j}} \mathbf{x}_{j}(L) \right) \\ &+ \sum_{l \neq L} \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(l)} \left( \sum_{j} \mathbf{U}_{j} \sqrt{\mathbf{P}_{j}} \mathbf{x}_{j}(l) \right) + \mathbf{V}_{k}^{H} \mathbf{n}_{k}[L]. \end{aligned}$$

$$(27)$$

Assume that user k only decodes its own current symbol  $\mathbf{x}_k(L)$  and considers the interferences (IUI and ISI) as noise. Then the rate of user k is given as

$$R_k^{\rm DL} = \log \det \left( \mathbf{I} + \mathbf{V}_k^H \mathbf{H}_k^{(L)} \mathbf{U}_k \mathbf{P}_k \mathbf{U}_k^H \mathbf{H}_k^{(L)H} \mathbf{V}_k \mathbf{X}_k^{-1} \right),$$
(28)

where the superscript DL denotes downlink and the interference matrix

$$\mathbf{X}_{k} = \sigma_{k}^{2} \mathbf{V}_{k}^{H} \mathbf{V}_{k} + \sum_{l \neq L} \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(l)} \mathbf{U}_{k} \mathbf{P}_{k} \mathbf{U}_{k}^{H} \mathbf{H}_{k}^{(l)H} \mathbf{V}_{k}$$
$$+ \sum_{j \neq k} \sum_{l} \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(l)} \mathbf{U}_{j} \mathbf{P}_{j} \mathbf{U}_{j}^{H} \mathbf{H}_{k}^{(l)H} \mathbf{V}_{k}.$$
(29)

The second term of (29) is the ISI of user k, and the third term is the IUI from other users' signals.

In the following, we will jointly design the transmit waveforms of the K users  $\mathbf{U} = [\mathbf{U}_1, ..., \mathbf{U}_K]$  and power allocation  $\mathbf{P} = diag \{\mathbf{P}_1, ..., \mathbf{P}_K\}$  to maximize the weighted sum rate  $\sum_{k=1}^{K} \alpha_k R_k^{\text{DL}}$  subject to a total power constraint  $P_{\text{max}}$ , i.e.,

$$\mathcal{P}_{\text{Rate}}^{\text{DL}}: \max_{\mathbf{P}, \mathbf{U}} \sum_{k=1}^{K} \alpha_k R_k^{\text{DL}}$$
  
s.t.  $\operatorname{tr}(\mathbf{P}) \leq P_{\max},$  (30)

where  $\alpha_k$  denotes the rate weighting coefficient for user k.

#### B. Uplink-Downlink Duality for Multiple Data Streams

In (28) and (29), all the waveforms  $\{\mathbf{U}_j\}_{j=1}^K$  are involved in  $R_k$ , so the waveform design is complicated in the downlink. With the uplink-downlink duality for multiple data streams [12], the downlink optimal waveform can be found in the virtual uplink with fixed power allocation. The sum rate optimization problem in the virtual uplink is constructed as follows.

$$\mathcal{P}_{\text{Rate}}^{\text{UL}}: \quad \max_{\mathbf{Q},\mathbf{U}} \sum_{k=1}^{K} \alpha_k R_k^{\text{UL}}$$
  
s.t.  $\operatorname{tr}(\mathbf{Q}) \leq P_{\max}$  (31)

where  $\mathbf{Q} = diag \{\mathbf{Q}_1, ..., \mathbf{Q}_K\}$  is the power allocation in the virtual uplink, the downlink transmit waveform U is equivalent to the uplink receive waveform, and the uplink transmission rate for user k is

$$R_{k}^{\mathrm{UL}} = \log \det \left( \mathbf{I} + \mathbf{U}_{k}^{H} \mathbf{H}_{k}^{(L)H} \mathbf{V}_{k} \mathbf{Q}_{k} \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(L)H} \mathbf{U}_{k} \mathbf{Y}_{k}^{-1} \right),$$
(32)

where the superscript UL denotes the virtual uplink, and the interference matrix

$$\mathbf{Y}_{k} = \sigma_{k}^{2} \mathbf{U}_{k}^{H} \mathbf{U}_{k} + \sum_{l \neq L} \mathbf{U}_{k}^{H} \mathbf{H}_{k}^{(l)H} \mathbf{V}_{k} \mathbf{Q}_{k} \mathbf{V}_{k}^{H} \mathbf{H}_{k}^{(l)} \mathbf{U}_{k} + \sum_{j \neq k} \sum_{l} \mathbf{U}_{k}^{H} \mathbf{H}_{k}^{(l)H} \mathbf{V}_{j} \mathbf{Q}_{j} \mathbf{V}_{j}^{H} \mathbf{H}_{k}^{(l)} \mathbf{U}_{k}.$$
(33)

By exploiting the fact that under MMSE receive filtering the SINR achievable regions of the two dual problems are the same for multiple data streams [12], we develop an iterative algorithm to compute the transmit waveform U and the uplink power Q in the virtual uplink, and calculate the receive waveform V and the downlink power P in the downlink. In the following two subsections, we describe the waveform design and the power allocation algorithm in detail.

#### C. Individual Waveform Design for Multiple Data Streams

As mentioned in Section IV-B, under MMSE receive filtering the SINR achievable regions of the two dual problems are the same [12]. Therefore, in this subsection we briefly introduce the MMSE receive filter.

Given the power allocation matrix  $\mathbf{P}$  and transmit waveform  $\mathbf{U}$ , the MMSE receive filter for the downlink can be derived as

$$\mathbf{V}_{k} = \left(\mathbf{H}_{k}^{(L)}\mathbf{U}_{k}\mathbf{P}_{k}\mathbf{U}_{k}^{H}\mathbf{H}_{k}^{(L)H} + \mathbf{X}_{k}\right)^{-1}\mathbf{H}_{k}^{(L)}\mathbf{U}_{k}\sqrt{\mathbf{P}_{k}}.$$
(34)

Similarly, for the virtual uplink, given the power allocation  $\mathbf{Q}$  and transmit filter  $\mathbf{V}$ , the MMSE receive filter is given by

$$\mathbf{U}_{k} = \left(\mathbf{H}_{k}^{(L)H}\mathbf{V}_{k}\mathbf{Q}_{k}\mathbf{V}_{k}^{H}\mathbf{H}_{k}^{(L)} + \mathbf{Y}_{k}\right)^{-1}\mathbf{H}_{k}^{(L)H}\mathbf{V}_{k}\sqrt{\mathbf{Q}_{k}}.$$
(35)

## D. Power Allocation for Multiple Data Streams: Iterative Power Waterfilling

We introduce the proposed power allocation algorithm for multiple data streams. This algorithm is the multiple-datastream extension of the *modified iterative waterfilling* in [20]. For multiple data streams, we cannot obtain the power allocation vector by allocating the SINR targets since there may be multiple solutions satisfying the same SINR targets. Thus, we directly allocate the power allocation vector.

Given the transmit waveforms  $U_k$ , the power allocation problem can be written as

$$\max_{\{\mathbf{P}_{k}\}} \sum_{k=1}^{K} \alpha_{k} R_{k}$$
  
s.t. 
$$\sum_{k=1}^{K} \operatorname{tr}(\mathbf{P}_{k}) \leq P_{\max}, \mathbf{P}_{k} \geq 0, \forall k.$$
 (36)

Taking derivative on the Lagrangian with respect to  $p_{kl}$ ,  $1 \le l \le M_k$ , we have

$$\frac{\alpha_k}{z_{kl}^{-1} + p_{kl}} - t_{kl} = \lambda - \mu_{kl}.$$
 (37)

where

$$z_{kl} = \mathbf{e}_{l}^{T} \boldsymbol{\Phi}_{k,k,L}^{H} \left( \mathbf{X}_{k} + \sum_{m=1,m\neq l}^{L_{k}} p_{km} \boldsymbol{\Phi}_{k,k,L} \mathbf{e}_{m} \mathbf{e}_{m}^{T} \boldsymbol{\Phi}_{k,k,L}^{H} \right)^{-1} \times \boldsymbol{\Phi}_{k,k,L} \mathbf{e}_{l},$$
(38)

and

$$t_{kl} = \alpha_k \sum_{i \neq L} \mathbf{e}_l^T \mathbf{\Phi}_{k,k,i}^H \left( \mathbf{X}_k + \mathbf{\Phi}_{k,k,L} \mathbf{P}_k \mathbf{\Phi}_{k,k,L}^H \right)^{-1} \\ \times \mathbf{\Phi}_{k,k,L} \mathbf{P}_k \mathbf{\Phi}_{k,k,L}^H \mathbf{X}_k^{-1} \mathbf{\Phi}_{k,k,i} \mathbf{e}_l \\ + \sum_{j \neq k} \alpha_j \sum_i \mathbf{e}_l^T \mathbf{\Phi}_{j,k,i}^H \left( \mathbf{X}_j + \mathbf{\Phi}_{j,j,L} \mathbf{P}_j \mathbf{\Phi}_{j,j,L}^H \right)^{-1} \\ \times \mathbf{\Phi}_{j,j,L} \mathbf{P}_j \mathbf{\Phi}_{j,j,L}^H \mathbf{X}_j^{-1} \mathbf{\Phi}_{j,k,i} \mathbf{e}_l,$$
(39)

where the  $M_k \times M_k$  matrix  $\mathbf{\Phi}_{k,j,i}$  is defined as  $\mathbf{V}_k^H \mathbf{H}_k^{(i)} \mathbf{U}_j$ .

According to the Karush-Kuhn-Tucker (KKT) conditions for (36), the optimum  $p_{kl}$  satisfies

$$p_{kl} = \left(\frac{\alpha_k}{\lambda + t_{kl}} - z_{kl}^{-1}\right)^+,\tag{40}$$

$$\sum_{k=1}^{K} \sum_{l=1}^{L_k} p_{kl} \le P_{\max}.$$
(41)

From the complementary slackness, either  $\sum_{k=1}^{K} \sum_{l=1}^{L_k} p_{kl} = P_{\max}, \lambda > 0 \text{ or } \sum_{k=1}^{K} \sum_{l=1}^{L_k} p_{kl} < P_{\max}, \lambda = 0$  should be satisfied. Since  $\lambda$  is monotonic with respect to  $\sum_{k,l} p_{kl}$ , we can first check whether  $\sum_{k=1}^{K} \sum_{l=1}^{L_k} p_{kl} > P_{\max}$  is satisfied for  $\lambda = 0$ . If so, the value of  $\lambda$  satisfying  $\sum_{k,l} p_{kl} = P_{\max}$  can be obtained via a one dimensional

 TABLE III

 Iterative Power Waterfilling for Multiple Data Streams

(i) Given P (ii) Loop: 1. Calculate  $t_{kl}$  and  $z_{kl}$  using (39) and (38). 2. Bisection search  $\lambda$  with (40) and (41), i.e., If  $\sum_{k,l} \left( \alpha_k t_{kl}^{-1} - z_{kl}^{-1} \right)^+ < P_{\max}$ 

If 
$$\sum_{k,l} (\alpha_k t_{kl} - z_{kl}) < P_{\max}$$
 then  
 $p_{kl} = (\alpha_k t_{kl}^{-1} - z_{kl}^{-1})^+$ .  
else  
(a) Set bisection upper bound  $\lambda^{\max} = \max_{k,l} \{\alpha_k z_{kl} - t_{kl}\}$ ,  
and lower bound  $\lambda^{\min} = \delta > 0$ .  
(b) Loop:  
Set  $\lambda = \frac{1}{2}(\lambda^{\max} + \lambda^{\min})$ .  
Compute  $p_{kl} = (\frac{\alpha_k}{\lambda + t_{kl}} - z_{kl}^{-1})^+$ .  
If  $\sum_{k,l} p_{kl} < P_{\max}$  then  
 $\lambda^{\min} = \lambda$ .  
Until  $|\sum_{k,l} p_{kl} - P_{\max}| < \epsilon$ .  
Until  $|\sum_{k,l} p_{kl} - P_{\max}| < \epsilon$ .

#### TABLE IV Iterative Weighted Sum Rate Optimization Algorithm for Multiple Data Streams

(i) Initialize Q<sub>k</sub> = Pmax ∑<sub>j</sub> M<sub>j</sub> I<sub>M<sub>k</sub></sub>, U<sub>k</sub> = some random matrix.
(ii) Loop :

Calculate V by (34).
Calculate Q using *Iterative Power Waterfilling*.
Calculate U by (35).
Calculate P using *Iterative Power Waterfilling*.

Until (U, Q, V, P) converges or the max. number of iterations is reached.

bisection search, where the upper bound of  $\lambda$  can be set as  $\max_{k,l} \{\alpha_k z_{kl} - t_{kl}\}$ , and we choose a small positive value for the lower bound. Similar procedures can be done for the case when  $\lambda = 0$  and  $\sum_{k,l} p_{kl} < P_{\max}$ . The proposed Iterative Power Waterfilling is summarized in Table III.

#### E. Iterative Sum Rate Optimization for Multiple Data Streams

For multiple data streams, the sum rate optimization algorithm iterates between the virtual uplink (U and Q) and downlink (V and P). When computing one of (U, Q, V, P), the other three variables are considered constant. Table III is applied for calculating the power allocation P, and the algorithm for computing Q is similar. Different from the proposed algorithm for single data stream (Table II), where the receive filter is simply a scalar and does not need to be updated, for multiple data streams the calculation of P or U relies on V, and the calculation of Q or V relies on U. Therefore, the algorithm has to iterate between the virtual uplink and the downlink. After convergence or maximum number of iterations is reached, we take the variables obtained at the last iteration as the solution and compute the achievable sum rate accordingly. The algorithm is summarized in Table IV.

The global optimum of a non-convex problem can be obtained by exhaustive search which, however, requires prohibitively high computational complexity. The solution of the proposed iterative waveform design is suboptimal since we



Fig. 2. Sum rate performance comparison for a 2-user system with L = 8, D = 2,  $\alpha_1 = \alpha_2 = 1$ , and  $M_1 = M_2 = 1$ .

tradeoff the optimality with complexity. Simulation results show that such sub-optima can still achieve a much better weighted sum rate performance than traditional methods such as Block-Diagonalization (BD) [24] and ZF.

#### V. NUMERICAL SIMULATION

In this section, we use numerical simulations to demonstrate the performance of the proposed iterative sum rate optimization algorithms. In the simulation, each path of the channel is assumed to be an i.i.d. complex Gaussian random variable with zero mean and variance of  $\frac{1}{2L}$  per dimension.

The amount of ISI depends on the symbol rate. Thus, we introduce the decimation ratio D, which represents the ratio of the symbol duration to the signal sampling duration. Each element in y is a signal sample, and the data symbols are transmitted every D signal samples. Clearly, higher D results in less ISI but lower symbol rate. In other words, one symbol induces ISI to at most  $\lfloor 2(L-1)/D \rfloor$  other symbols. Therefore, with decimation ratio D, the channel matrix **H** can be decimated by keeping only  $\lfloor 2(L-1)/D \rfloor + 1$  rows and deleting the other rows for simplicity.

For example, if L = 3 and D = 2, the decimated **H** then becomes

$$\mathbf{H} = \begin{bmatrix} h[0] & 0 & 0\\ h[2] & h[1] & h[0]\\ 0 & 0 & h[2] \end{bmatrix}.$$

Figure 2 shows the sum rate performance of a 2-user system with L = 8, D = 2,  $\alpha_1 = \alpha_2 = 1$ . Each rate is averaged over 1000 channel realizations. TR denotes the traditional time-reversal filter, i.e.,  $\mathbf{u}_k^{\text{TR}} = c_k^{\text{TR}} \mathbf{H}_k^{(1)H}$ , where  $c_k^{\text{TR}}$  is a normalization constant such that  $\|\mathbf{u}_k^{\text{TR}}\|_2 = 1$ ; ZF denotes the zero-forcing waveform, i.e.,  $\mathbf{u}_k^{\text{ZF}} = c_k^{\text{ZF}} ([\mathbf{H}_1^T, \dots, \mathbf{H}_K^T]^T)^{\dagger} \tilde{\mathbf{e}}_k$ , where  $(\cdot)^{\dagger}$  denotes the Moore-Penrose pseudo-inverse operator, and  $\tilde{\mathbf{e}}_k = [\mathbf{0}^T, \mathbf{0}^T, \dots, \mathbf{0}^T, \mathbf{e}_L^T, \mathbf{0}^T, \dots, \mathbf{0}^T]^T$ , which is a  $K(2L-1) \times 1$  vector with its kth vector as  $\mathbf{e}_L$ . Here with a slight abuse of notation, we denote  $\mathbf{e}_L$  to be the *L*th column of a  $(2L-1) \times (2L-1)$  identity matrix. The **0** denotes a  $(2L-1) \times 1$  all zero vector. Therefore,  $\tilde{\mathbf{e}}_k$  has only one nonzero value at its ((2L-1)(k-1)+L)th element.  $c_k^{\mathrm{ZF}}$  is chosen to normalize the norm of  $\mathbf{u}_k^{\mathrm{ZF}}$  to be 1.

We compare the proposed Iterative SINR Waterfilling with equal power allocation and optimal power allocation in Figure 2. For the proposed algorithms, the forgetting factor  $\beta$  is set to be 1/K. The maximum iteration number of Iterative SINR Waterfilling is set to be 20. In this paper, since we focus on demonstrating the performance advantage of the proposed power allocation scheme, some parameters of the proposed algorithms, such as the maximum number of iterations and the forgetting factor  $\beta$ , are empirically chosen and the performance is already promising. Thus, we do not aim to further optimize these parameters. The equal power allocation is to split the total power equally to each user, i.e.,  $p_k = P_{\text{max}}/K$ . The optimal power allocation is simulated by exhaustive search of the discretized power variables, where the number of discrete levels of each power variable is set as  $10^3$ . The exhaustive search requires very high computational complexity, which is exponentially increasing in the number of variables as the number of discrete levels increases.

From the figure, the proposed power allocation can improve the performance of equal power allocation for all waveform designs, since the proposed Iterative SINR Waterfilling is able to find sub-optima by taking into consideration the channel gains. The improvement for the MMSE waveform is especially significant at high power region. The MMSE waveform with the proposed Iterative SINR Waterfilling performs almost the same as the globally-optimal power allocation. We also observe that even with the MMSE waveform, which is optimal given any power allocation for single data stream, the equal power allocation still saturates at high power region.

Note that since the sub-optimal waveforms TR and ZF do not change under different power allocation, these methods do not require iterations between the waveform design and power allocation. For the MMSE with equal power allocation, since the power allocation remains the same, the MMSE waveform does not need to be updated accordingly. Therefore, these methods are not iterative and thus require lower computational complexity compared to the proposed algorithm, which has two levels of iterations.

It is well-known [25] that since TR only maximizes the received signal power without considering the interference, it saturates at a lower rate, as shown in both figures. ZF cancels the interference but sacrifices the received signal power resulting in worse performance at low power region. MMSE can strike a balance between the two by reducing the interference including ISI and IUI, while keeping a high received signal power.

In Figure 3, the proposed Iterative SINR Waterfilling is compared with the convex approximation using geometric programming (GP) [26], which approximates the rate function  $log(1 + SINR_k)$  as  $log(SINR_k)$  in high SINR regime. With such an approximation, the weighted sum rate function can be shown to be a posynomial and the optimization problem becomes a geometric program, which can be optimally solved via standard convex programming techniques. In the figure, for K = 2, L = 8, and D = 3, since the interference is low and SINR<sub>k</sub>  $\gg$  1, the sum rate optimization problem can



Fig. 3. Sum rate performance comparison of the proposed algorithm in Table II and the convex approximation using geometric programming (GP).



Fig. 4. Convergence behaviors of the proposed sum rate optimization algorithm.

be well approximated using the convex objective function, and the performance of the proposed method is very close to the globally optimal solution of the approximated convex optimization problem. For K = 4, L = 8, D = 4, and K = 4, L = 8, D = 3, the higher interference from more users causes more performance degradation to the GP method. This is because the approximate objective function  $\sum_k \log(SINR_k)$ can be seen as the proportional fairness criterion for SINRs and it deters some  $SINR_k$  from being very small and significantly decreasing the approximate objective function. On the contrary, the original sum rate  $\sum_k \log(1 + \text{SINR}_k)$  is not impaired as much if some  $SINR_k$  are small, because most power can be allotted to other users with lower interference and still makes good contribution to the sum rate. In other words, if some users' interference is high, the original sum rate maximization can abandon these users and allocate most power to the others. Such a consequence cannot arise in the GP method. Hence, only when the interference is low for all users, the sum rate optimization problem can be well approximated with the convex objective function.

The performance gap between the proposed method and



Fig. 5. Sum rate performance for difference maximum numbers of iterations.

the GP method becomes larger as  $P_{\rm max}/\sigma^2$  increases. This seems not to comply with the intuition that the GP method can obtain higher accuracy of approximation with high  $P_{\rm max}/\sigma^2$ . Instead, the GP approximation is less accurate when the available power is higher since the interference is also higher. When  $P_{\rm max}/\sigma^2$  is low, the noise is more dominant than the interference, so the interference mitigation from power allocation has less prominent influence on the sum rate. As  $P_{\rm max}/\sigma^2$  increases, the interference also increases. In a high interference scenario, the proposed algorithm can make better use of the available power compared with the GP method, which is based on a less accurate approximation. Therefore, the resulting advantage of the proposed algorithm is more significant as  $P_{\rm max}/\sigma^2$  increases. In this figure, we can also observe that the performance gap for K = 4, L = 8, D = 3between the two algorithms is larger than the gap for K = 4, L = 8, D = 4 since the GP method allocates power based on a less accurate approximation when the interference is higher. Comparing between K = 4, L = 8, D = 3 and K = 2, L = 8, D = 3, the proposed algorithm can achieve a better sum rate performance when K increases, whereas GP instead performs worse, which is again due to the ineffective approximation.

Figure 4 shows a typical convergence behavior of the proposed sum rate optimization algorithm (Table II). Monotonicity and very fast convergence are almost always observed (typically about 3 to 12 iterations). The proposed sum rate optimization algorithms with different maximum numbers of iterations are compared in Figure 5. It can be seen that the sum rate performance is improved with more iterations. The improvement is more significant for smaller maximum numbers of iterations and becomes less noticeable for higher maximum numbers of iterations. We have performed extensive (10,000 channel realizations) simulations to inspect the convergence of the proposed algorithm. Over 99% of them converge within 100 iterations, while the remaining less than 1% converge more slowly. Note that we define the convergence as the rate improvement between two consecutive iterations being within  $10^{-6}$ , i.e.,  $|(R^{(n+1)} - R^{(n)})/R^{(n)}| < 10^{-6}$ . We observed that for those cases with slow convergence, the rate still monotonically increases but the increase is just too slow to converge within 100 iterations. Since we assume L-



Fig. 6. Sum rate performance comparison for different decimation ratio D using the proposed algorithm in Table II. The performance is normalized by D.



Fig. 7. Sum rate performance comparison for a 2-user system with  $N_t = 6$ ,  $N_{r,1} = N_{r,2} = 2$ , L = 2, and  $M_1 = M_2 = 2$ .

path multipath channel with each path being a Gaussian, the complexity to locate the peculiarity of these channels is very high.

In Figure 6, we compare the sum rate performance with different decimation ratio D. Note that for fair comparison, the performance is normalized by 1/D which reflects the frequency of channel usage. For smaller D, the transmission is conducted more frequently but severer interference may occur due to the ISI. Similarly for higher D, the ISI is reduced but the channel is utilized less frequently. From the figure, we can see that at low SNR region, D = 1 attains the highest normalized performance since at low SNR, the ISI is less prominent and the channel utilization is more important to the normalized sum rate. On the other hand, at high SNR, the ISI has a dominant effect and higher D can provide a better normalized sum rate performance despite less frequent channel usage.

Figure 7 shows the sum rate performance of a 2-user system with L = 2,  $\alpha_1 = \alpha_2 = 1$ ,  $N_t = 6$ ,  $N_{r,1} = N_{r,2} = 2$ ,



Fig. 8. Comparison of the two proposed algorithms with different number of users. L = 4,  $N_t = 1$ ,  $N_{r,k} = 1$ ,  $\forall k$ ,  $M_k = 1$ ,  $\forall k$ , and  $P_{\max}/\sigma^2 = 15$  (dB).



Fig. 9. Comparison of the two proposed algorithms.  $K = 4, L = 2, N_t = 1, N_{r,k} = 1, \forall k$ , and  $M_k = 1, \forall k$ .

and  $M_1 = M_2 = 2$ . The proposed algorithm (Table IV) is compared with BD [24] and ZF. For BD, the signal space of each user is orthogonal to each other, i.e.,  $\mathbf{U}_k$  is in the null space of ISI and IUI. Thus, in order for BD to find a feasible solution, the simulation parameters are chosen to satisfy  $LN_t - (2L-1) \sum_{j \neq k} N_{r,j} - (2L-1)N_{r,k} \ge M_k, \forall k$ . As to ZF, the signal space of each data stream is orthogonal to each other. Hence, ZF also has similar constraint on the dimensions.

We compare the Iterative Power Waterfilling as in Section IV-D with equal power allocation in Figure 7. The equal power allocation is to split the total power equally to each data stream, i.e.,  $\mathbf{P}_k = \frac{P_{\text{max}}}{\sum_j M_j} \mathbf{I}_{M_k}$ . From the figure, it is clear that the proposed power allocation outperforms equal power allocation for MMSE, BD, and ZF. It is well-known that interference cancellation based methods, such as BD and ZF, suffer from the noise enhancement and thus result in worse performance than MMSE.

We compare the two proposed power allocation algorithms for single data stream with different number of users in Figure 8. The parameters are chosen as  $N_t = 1$ ,  $N_{r,k} = 1$ ,  $\forall k$ ,



Fig. 10. Comparison of the proposed algorithm and equal power allocation for sum rate versus channel uncertainty.

L = 4, and  $\alpha_k = 1, \forall k$ . From the figure, Iterative SINR Waterfilling outperforms the Iterative Power Waterfilling when the number of users is large. Figure 9 shows that Iterative SINR Waterfilling can achieve superior sum rate at high SNR, where the parameters are chosen as K = 4, L = 2,  $N_t = 1$ ,  $N_{r,k} = 1, \forall k$ , and  $\alpha_k = 1, \forall k$ . From Figures 8 and 9, it can be seen that Iterative SINR Waterfilling outperforms Iterative Power Waterfilling in the scenario of high interference. Intuitively, the SINR targets have direct influence on the sum rate and allocating the SINR can better capture the impact of interference compared to allocating the power.

In Figure 10, the proposed Iterative SINR Waterfilling is compared with equal power allocation. The channel uncertainty model for the kth user at time m is given by  $h_k[m] =$  $h_k[m] + e_k[m]$ , where  $\hat{h}_k[m]$  denotes the estimated channel coefficient,  $h_k[m]$  denotes the true channel with variance  $\sigma_h^2$ , and  $e_k[m]$  is the estimation error with variance  $\sigma_e^2$ . In this figure, we can see that when the channel uncertainty is small, the proposed method can still outperform the equal power allocation. As the channel uncertainty increases, the benefit of the proposed method over the equal power allocation reduces, since the proposed method relies on the perfect channel information to allocate the available power. When the channel uncertainty is very high, the equal power allocation performs better because the proposed method allocates the power according to the coefficients almost uncorrelated to the true channel.

Finally, we note that although we cannot prove the proposed iterative algorithms converge to the global optimum, the simulation results show that the proposed Iterative SINR Waterfilling still results in comparable performance to that of the globally-optimal power allocation and thus outperforms other traditional methods.

# VI. CONCLUSION

In this paper, we explored the weighted sum rate optimization problem by transmit waveform design for the MIMO time-reversal multiuser downlink communication systems where the receiver processing is based on a single sample. The waveform design problem is shown to have a structure similar to the downlink beamforming problem with a self-interfering term induced by the ISI. In order to tackle the problem, we proposed a new power allocation scheme called Iterative SINR Waterfilling which, instead of directly allocating the power, the SINRs are first allocated to maximize the weighted sum rate. With the allocated target SINRs, the corresponding power allocation can be easily determined. For multiple data streams, Iterative Power Waterfilling is further proposed. Iterative algorithms alternately optimize the transmit waveform and the power allocation for each user. Both of the proposed sum rate optimization algorithms significantly outperform other traditional approaches such as zero-forcing and time-reversal waveforms. We also demonstrated that Iterative SINR Waterfilling outperforms Iterative Power Waterfilling in the scenario of high interference, e.g., large number of users or high SNR region. With the MMSE waveform, Iterative SINR Waterfilling is shown to achieve near-optimal performance by comparing with exhaustively-searched global optimum.

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