MIMO Interference Cancelation via Network Formation Game

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***Abstract*—This paper considers the cooperative maximization of mutual information in the MIMO Gaussian interference channel in a fully distributed manner via game theory. Null shaping constraints are enforced in the design of transmit covariance matrices to enable interference mitigation among links. The transmit covariance matrices leading to the Nash Equilibrium (NE) are derived, and the existence and uniqueness of the NE is analyzed. The formation of the cooperative sets, that represent the cooperation relationship among links, is considered as network formation games. We prove that the proposed network formation (NF) algorithm converges to a Nash Equilibrium. Simulation results show that the proposed NF algorithm enhances the sum rate of the system apparently even at low signal-to-noise ratio region and/or with small number of transmit antennas.**

I. Introduction

The multiple-input multiple-out (MIMO) interference channel [1] is a mathematical model applicable to many communication systems where multiple links share the same communication medium. Two typical examples are the MIMO cellular systems and the MIMO ad hoc network. In this model, cross link interference greatly impacts the transmission rate, and thus, how to suppress/cancel it is crucial.

Recently, distributed precoding matrix design in the MIMO interference systems has drawn great attentions. One way to deal with this problem is to employ interference alignment (IA) [2-4]. However, IA-based approach requires global channel state information (CSI), which is hard to acquire in practice. Another avenue to deal with the problem is the game-theoretic approach [5-12]. Scutari et al [5][6] formulated the problem as a non-cooperative game, designed an iterative waterfilling (IWF) algorithm, and analyzed the existence and uniqueness of the Nash Equilibrium (NE). This IWF algorithm requires no cross-link CSI and is easy to implement, but the efficiency of the NE is restricted due to the selfishness of the players and the lack of interference coordination. A simple way to improve the efficiency of the NE is to introduce pricing [7] for interference management[8][9]. Cooperative games can also be used to formulate interference coordination. Ye et al [10] designed a practical suboptimal algorithm for finding the Nash Bargaining (NB) solution in MIMO interference system. In [11], Ho et al designed the precoding vectors by combining egostic and altruistic beamforming vectors. This idea has been shown to achieve pareto boundary in two-player MISO interference systems [12]. The Pareto boundary for multi-player MIMO interference channels was characterized in [13].

Unlike the above works, we consider a new approach of interference coordination, where null shaping constraints are enforced in the design of transmit covariance matrices to enable interference mitigation among different links. In [14], the null shaping constraints are used in cognitive radio scenario to limit the interference of secondary users to the primary users unrequitedly, without cooperation among themselves. In our work, the null shaping constraints are imposed multilaterally. For each transceiver, there is a pair of outgoing cooperative set and incoming cooperative set, which stands for the links alleviating interference to this link and the links profiting from null shaping constraints enforced on this link, respectively.

In this paper, we first formulate the rate maximization problem as a cooperative game by fixing the outgoing cooperative set and incoming cooperative set. Then the solution leading to the NE is derived, and the uniqueness and existence of the NE is analyzed. Thirdly, we formulate the formation of cooperative sets as network formation games [15], provide corresponding algorithm and investigate the stability of the proposed algorithm. Simulation results show that the game with null shaping constraints among links improves the probability of uniqueness of the NE, compared to the non-cooperative game. The proposed network formation algorithm improves the sum rate of the system significantly even at low SNR region and/or with small number of transmit antennas.

The rest of this paper is outlined as follows. Section II reviews the noncooperative rate maximization game in MIMO interference systems. In Section III, we present the cooperative multi-link MIMO transmission with transmitter null shaping, derive the solution leading to NE, and investigate the condition of the uniqueness and existence of the NE. In Section IV, we model the problem of cooperative sets formation as network formation games, and propose distributed algorithm for it. Simulation results are demonstrated and analyzed in Section V. Finally, concluding remarks are given in Section VI.

II. NONCOOPERATIVE RATE MAXIMIZATION GAME

We consider a vector Gaussian interference channel comprising K MIMO links that share the same physical resources. In link q, the transmitter and receiver are equipped with n_{T_q}

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and n_{R_q} antennas respectively. The transmission over link q can be described by the baseband signal model

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{x}_q + \sum_{r \neq q} \mathbf{H}_{rq}\mathbf{x}_r + \mathbf{n}_q, \quad (1)$$

where $\mathbf{x}_q \in \mathbb{C}^{n_{T_q} \times 1}$ is the transmitted signal vector by source q, $\mathbf{n}_q \in \mathbb{C}^{n_{R_q} \times 1}$ is a zero-mean circularly symmetric complex Gaussian noise vector with nonsingular covariance matrix $\mathbf{R}_{n_q}$. $\mathbf{H}_{qq} \in \mathbb{C}^{n_{R_q} \times n_{T_q}}$ is the channel matrix of link q, $\mathbf{H}_{rq} \in \mathbb{C}^{n_{R_q} \times n_{T_r}}$ is the cross-channel matrix between source r and destination q. We assume that $\mathbf{H}_{rq} \sim \mathcal{CN}(0, \eta_{rq}^2 \mathbf{I})$ $(r, q \in \{1, 2, .., K\})$ with $\eta_{rq} = \sqrt{\kappa / d_{rq}^{\alpha}}$, where κ is the path loss constant, d_{rq} is the distance between transmitter r and receiver q, and α is the path loss exponent.

The second term in the right handside of (1), $\sum_{r \neq q} \mathbf{H}_{rq}\mathbf{x}_r$, represents the co-channel interference received by qth destination. We assume that the co-channel interference from other links to destination q is unknown and treated as noise. We assume the slow fading channels, i.e., the channels are fixed during a symbol transmission. Moreover, we assume perfect CSI at both transmitter and receiver sides, and each receiver can perfectly measure the covariance matrix of the noise together with co-channel interference generated by other links.

With the above assumption, the maximum information rate on link q can be expressed as [1]:

$$R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log \det(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{Q}_q), \quad (2)$$

where $\mathbf{Q}_q = \mathbb{E}[\mathbf{x}_q \mathbf{x}_q^H]$ is the Hermitian positive semi-definite (PSD) transmit covariance matrix of the transmitted vector $\mathbf{x}_q$, i.e. $\mathbf{Q}_q \succeq 0$, and

$$\mathbf{R}_{-q}(\mathbf{Q}_{-q}) = \mathbf{R}_{n_q} + \sum_{r \neq q} \mathbf{H}_{rq} \mathbf{Q}_r \mathbf{H}_{rq}^H \quad (3)$$

is the interference-plus-noise covariance matrix observed by user q, $\mathbf{Q}_{-q} \triangleq (\mathbf{Q}_r)_{r \neq q}$ is the set of all links' covariance matrices by removing the link q. The transmission of each link is power limited, i.e.,

$$\mathrm{Tr}(\mathbf{Q}_q) \leqslant P_q. \quad (4)$$

Given the above setup, the problem can be formulated as a strategic noncooperative game:

$$(\mathcal{G}): \quad \begin{array}{ll} \max\limits_{\mathbf{Q}_q} & R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{s.t.} & \mathbf{Q}_q \in \mathcal{Q}_q \end{array} \quad \forall q \in \Omega \quad (5)$$

where $\Omega \triangleq \{1, .., K\}$ is the set of players (i.e., the links), $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is the payoff function of play q defined in (2), and $\mathcal{Q}_q$ is the set of valid strategies (the covariance matrices) of player q, defined as

$$\mathcal{Q}_q \triangleq \{\mathbf{Q} \in \mathbb{C}^{n_{T_q} \times n_{T_q}}: \quad \mathbf{Q} \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{Q}) \leqslant P_q\}. \quad (6)$$

In the non-cooperative game $\mathcal{G}$, each player competes with each other selfishly by choosing his strategy, the transmit covariance matrix $\mathbf{Q}_q$, to maximize his own information rate $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ defined in (2), subject to the average transmit power constraint in (4). A Nash Equilibrium is reached when each user, given the strategy profiles of others, does not get any rate increase by unilaterally changing his own strategy[7]. The transmit covariance matrix leading to the NE can be found via IWF as [5][6]:

$$\mathbf{Q}_q^{\star} = \mathbf{U}_q(\mu_q \mathbf{I} - \mathbf{D}_q^{-1})^{+} \mathbf{U}_q^H, \quad (7)$$

where $\mathbf{U}_q \mathbf{D}_q \mathbf{U}_q^H = \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq}$ is the eigenvalue decomposition (EVD) of $\mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq}$, $\mathbf{U}_q$ is the unitary matrix of eigenvectors, $\mathbf{D}_q$ is a diagonal matrix of eigenvalues, and μ_q denotes the power level given by IWF.

III. MULTI-LINK MIMO TRANSMISSION WITH TRANSMITTER NULL SHAPING

The strength of desired signal and noise plus cross-link interference are the two main factors affacting the transmission rate. In the non-cooperative game $\mathcal{G}$, players choose their strategies by maximizing the useful signal power, without considering the cross-link interference, due to which the transmission rate is impaired. To further improve the performance in MIMO interference channel system, we propose a new cooperative game based approach in this paper, where null shaping constraints are enforced in the design of transmit covariance matrices.

A. Problem Formulation

Let $\mathcal{C}_q$ be the incoming cooperative set of links that eliminate co-channel interference to link q, and $\mathcal{N}_q$ be the outgoing cooperative set of links that profit from null shaping constraints imposed on link q. To enable the interference cancelation at the transmitter q, the number of antennas should satisfy

$$\sum_{r \in \mathcal{N}_q} n_{R_r} + n_{R_q} \leqslant n_{T_q}. \quad (8)$$

Given the cooperative sets $\mathcal{C}_q$ and $\mathcal{N}_q$ for each player, the maximum information rate on link q can be expressed as

$$\begin{aligned} R_q^c(\mathbf{Q}_q(\mathcal{N}_q), \mathbf{Q}_{-\mathcal{C}_q}) = \\ \log \det(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_{-\mathcal{C}_q}^{-1}(\mathbf{Q}_{-\mathcal{C}_q}) \mathbf{H}_{qq} \mathbf{Q}_q(\mathcal{N}_q)), \end{aligned} \quad (9)$$

where

$$\mathbf{R}_{-\mathcal{C}_q}(\mathbf{Q}_{-\mathcal{C}_q}) = \mathbf{R}_{n_q} + \sum_{r \notin \mathcal{C}_q} \mathbf{H}_{rq} \mathbf{Q}_r \mathbf{H}_{rq}^H \quad (10)$$

is the interference-plus-noise covariance matrix observed by user q, $\mathbf{Q}_{-\mathcal{C}_q} \triangleq (\mathbf{Q}_r)_{r \notin \mathcal{C}_q}$ is the set of covariance matrices of links that is not in $\mathcal{C}_q$. Compared to (2), the source of interferers diminishes with cost of sacrificing spatial degrees of freedom to help others.

The valid strategies set of player q through cooperation is defined as

$$\begin{aligned} \mathcal{Q}_q^c \triangleq \{\mathbf{Q} \in \mathbb{C}^{n_{T_q} \times n_{T_q}}: \mathbf{Q} \succeq \mathbf{0}, \mathrm{Tr}(\mathbf{Q}) \leqslant P_q, \\ \mathbf{H}_{qr} \mathbf{Q} \mathbf{H}_{qr}^H = \mathbf{0}, \forall r \in \mathcal{N}_q\}. \end{aligned} \quad (11)$$

Compared to the valid strategies set of non-cooperative game in (6), additional null constraints corresponding to set $\mathcal{N}_q$ are enforced in (11).

Given the rate function in (9) and the constraints in (11), the cooperative transmission problem can be formulated as a cooperative game:

$$(\mathcal{G}^c): \quad \begin{array}{ll} \max\limits_{\mathbf{Q}_q} & R_q^c(\mathbf{Q}_q(\mathcal{N}_q), \mathbf{Q}_{-\mathcal{C}_q}) \\ \text{s.t.} & \mathbf{Q}_q \in \mathcal{Q}_q^c \end{array} \quad \forall q \in \Omega \qquad (12)$$

In the cooperative game $\mathcal{G}^c$, the player cooperates with each other by canceling the interference to other links, due to which all players benefit from interference mitigation among links. The player chooses the transmit covariance matrix $\mathbf{Q}_q$ by maximizing his own information rate $R_q^c(\mathbf{Q}_q, \mathbf{Q}_{-q})$ with the null shaping constraints.

B. NE of the Cooperative Game $\mathcal{G}^c$

To investigate the Nash equilibria of the proposed cooperative game $\mathcal{G}^c$, we first introduce some notations. Let $\mathbf{U}_r^\perp$ be the semi-unitary matrix generated from the null shaping constraint at transmitter r, and $\tilde{\mathbf{H}}_{rq} = \mathbf{H}_{rq}\mathbf{U}_r^\perp$ be the modified channel from transmitter r to receiver q. The Nash equilibria of the proposed cooperative game $\mathcal{G}^c$ is shown in the following theorem.

Theorem 1 : All the Nash equilibria of the cooperative game $\mathcal{G}^c$ are the solutions to the following fixed-point equations:

$$\mathbf{Q}_q^{c\star} = \mathbf{U}_q^\perp \tilde{\mathbf{U}}_q(\tilde{\mu}_q \mathbf{I} - \tilde{\mathbf{D}}_q^{-1})^+ \tilde{\mathbf{U}}_q^H \mathbf{U}_q^{\perp H}, \forall q \in \Omega \qquad (13)$$

where $\tilde{\mathbf{U}}_q\tilde{\mathbf{D}}_q\tilde{\mathbf{U}}_q^H = \tilde{\mathbf{H}}_{qq}^H \mathbf{R}_{-\mathcal{C}_q}^{-1}(\mathbf{Q}_{-\mathcal{C}_q})\tilde{\mathbf{H}}_{qq}$ is the eigenvalue decomposition. $\tilde{\mathbf{U}}_q$ is the unitary matrix of eigenvectors, $\tilde{\mathbf{D}}_q$ is a diagonal matrix of eigenvalues, and $\tilde{\mu}_q$ denotes the power level given by IWF.

Proof: Denote $\mathbf{H}_{q\mathcal{N}_q} = [\mathbf{H}_{ql_1}^T, .., \mathbf{H}_{ql_{|\mathcal{N}_q|}}^T]^T, l_i \in \mathcal{N}_q$ as the aggregated channel matrix of links in set $\mathcal{N}_q$. Transmitter q does not cause interference to the links in set $\mathcal{N}_q$, which means that $\mathbf{Q}_q$ lies in the null space of $\mathbf{H}_{q\mathcal{N}_q}$, i.e., $\mathbf{H}_{q\mathcal{N}_q}\mathbf{Q}_q = \mathbf{0}$. Thus all the solutions to (12) can be written as

$$\mathbf{Q}_q^{c\star} = \mathbf{U}_q^\perp \tilde{\mathbf{Q}}_q^{c\star} \mathbf{U}_q^{\perp H}, \qquad (14)$$

where $\mathbf{U}_q^\perp \in \mathbb{C}^{n_{T_q} \times r_{\mathbf{U}_q^\perp}}$ is the semi-unitary matrix orthogonal to $\mathbf{H}_{q\mathcal{N}_q}$, with $r_{\mathbf{U}_q^\perp} \triangleq rank(\mathbf{U}_q^\perp) = n_{T_q} - r(\mathbf{H}_{q\mathcal{N}_q})$, $\{\tilde{\mathbf{Q}}_q^{c\star} \in \mathbb{C}^{r_{\mathbf{U}_q^\perp} \times r_{\mathbf{U}_q^\perp}}\}$ are the Nash Equilibria of the following lower-dimensional game

$$\begin{array}{ll} \max\limits_{\tilde{\mathbf{Q}}_q \succeq \mathbf{0}} & \log\det(\mathbf{I} + \mathbf{U}_q^{\perp H}\mathbf{H}_{qq}^H \tilde{\mathbf{R}}_{-\mathcal{C}_q}^{-1}(\tilde{\mathbf{Q}}_{-\mathcal{C}_q})\mathbf{H}_{qq}\mathbf{U}_q^\perp \tilde{\mathbf{Q}}_q(\mathcal{N}_q)) \\ \text{s.t.} & \text{Tr}(\tilde{\mathbf{Q}}_q) \leqslant P_q \end{array} \qquad (15)$$

for all $q \in \Omega$, where

$$\begin{aligned} \tilde{\mathbf{R}}_{-\mathcal{C}_q}(\tilde{\mathbf{Q}}_{-\mathcal{C}_q}) &= \mathbf{R}_{-\mathcal{C}_q}(\mathbf{Q}_{-\mathcal{C}_q}) \\ &= \mathbf{R}_{n_q} + \sum_{r \notin \mathcal{C}_q} \mathbf{H}_{rq}\mathbf{U}_r^\perp \tilde{\mathbf{Q}}_r \mathbf{U}_r^{\perp H}\mathbf{H}_{rq}^H. \end{aligned} \qquad (16)$$

The solutions to (15) are the fixed-points of the following nonlinear equations[5][6]:

$$\tilde{\mathbf{Q}}_q^{c\star} = \tilde{\mathbf{U}}_q(\tilde{\mu}_q \mathbf{I} - \tilde{\mathbf{D}}_q^{-1})^+ \tilde{\mathbf{U}}_q^H, \forall q \in \Omega \qquad (17)$$

where $\tilde{\mathbf{U}}_q\tilde{\mathbf{D}}_q\tilde{\mathbf{U}}_q^H = \mathbf{U}_q^{\perp H}\mathbf{H}_{qq}^H \tilde{\mathbf{R}}_{-\mathcal{C}_q}^{-1}(\tilde{\mathbf{Q}}_{-\mathcal{C}_q})\mathbf{H}_{qq}\mathbf{U}_q^\perp$ is the eigenvalue decomposition. $\tilde{\mathbf{U}}_q$ is the unitary matrix of eigenvectors, $\tilde{\mathbf{D}}_q$ is a diagonal matrix of eigenvalues, and $\tilde{\mu}$ denotes the power level given by IWF. Substituting (17) into (14), leads to the desired structure in (13).

■

Here, we show an approach to obtain $\mathbf{U}_q^\perp$. Perform singular value decomposition of $\mathbf{H}_{q\mathcal{N}_q} = \mathbf{U}_{q\mathcal{N}_q}\mathbf{D}_{q\mathcal{N}_q}\mathbf{V}_{q\mathcal{N}_q}^H$. Since $\mathbf{H}_{q\mathcal{N}_q}$ is a fat matrix, there are at least $(n_{T_q} - \sum_{l_i \in \mathcal{N}_q} n_{R_{l_i}})$ zero singular values. The right singular matrix $\mathbf{V}_{q\mathcal{N}_q}$ is composed of two parts $\mathbf{V}_{q\mathcal{N}_q} = [\mathbf{V}_{q\mathcal{N}_q}^e, \mathbf{V}_{q\mathcal{N}_q}^0]$, where $\mathbf{V}_{q\mathcal{N}_q}^e \in \mathbb{C}^{n_{T_q} \times r(\mathbf{H}_{q\mathcal{N}_q})}$ is made up of the right singular vectors associated with non-zero singular values. Denote $\mathbf{H}_{q\mathcal{N}_q}^\perp = \mathbf{I} - \mathbf{V}_{q\mathcal{N}_q}^e \mathbf{V}_{q\mathcal{N}_q}^{eH}$ as the orthogonal projection of matrix $\mathbf{H}_{q\mathcal{N}_q}$, i.e. its column vectors spans the null space of $\mathbf{H}_{q\mathcal{N}_q}$. To obtain the unitary matrix associated with $\mathbf{H}_{q\mathcal{N}_q}^\perp$, eigenvalue decomposition is performed

$$\mathbf{U}_{qG}\mathbf{D}_{qG}\mathbf{U}_{qG}^H = (\mathbf{I} - \mathbf{V}_{q\mathcal{N}_q}^e \mathbf{V}_{q\mathcal{N}_q}^{eH})^H(\mathbf{I} - \mathbf{V}_{q\mathcal{N}_q}^e \mathbf{V}_{q\mathcal{N}_q}^{eH}), \qquad (18)$$

where $\mathbf{U}_{qG}$ is the unitary matrix associated with $\mathbf{H}_{q\mathcal{N}_q}^\perp$, and $\mathbf{U}_q^\perp$ is composed of eigenvectors corresponding to the largest n_{R_q} eigenvalues, i.e., the first n_{R_q} columns of unitary matrix $\mathbf{U}_{qG}$.

C. Conditions for the Existence and Uniqueness of the NE

In this subsection, we analyze the existence and uniqueness of the NE of the cooperative game $\mathcal{G}^c$.

Theorem 2 (Existence): In the cooperative game $\mathcal{G}^c$, there always exists a NE, for any given cooperative sets, channel matrices and power constraints.

Proof: In game $\mathcal{G}^c$, the payoff functions is quasi-concave and the valid strategy sets are convex compact, thus there always exists a NE for any modified channel matrices and power constraints [6]. ■

To analyze the condition for the uniqueness of the NE of the cooperative game $\mathcal{G}^c$, we introduce a nonnegative matrix $\mathbf{S}^c \in \mathbb{C}^{K \times K}$ as follows

$$[\mathbf{S}^c]_{qr} \triangleq \begin{cases} \rho(\tilde{\mathbf{H}}_{rq}^H \tilde{\mathbf{H}}_{qq}^{-H} \tilde{\mathbf{H}}_{qq}^{-1} \tilde{\mathbf{H}}_{rq}), & \text{if} \quad r \neq q \\ 0, & \text{otherwise} \end{cases} \qquad (19)$$

where $\rho(\mathbf{A})$ is the spectral radius of a matrix $\mathbf{A}$. With the matrix $\mathbf{S}^c$ in (19), we can obtain sufficient conditions for the uniqueness of the NE of the proposed cooperative game $\mathcal{G}^c$ as shown in the following theorem.

Theorem 3 (Uniqueness): The NE of the cooperative game $\mathcal{G}^c$ is unique if

$$\rho(\mathbf{S}^c) < 1 \qquad (20)$$

Proof: A sufficient condition for the uniqueness of the NE is that the mapping is contraction with regards to some norm [16, Prop.1.1(a)]. Applying the Theorem 5-6 in [6], we can obtain the sufficient conditions for the uniqueness of NE of $\mathcal{G}^c$ as in (20). ■

IV. Cooperative Sets Formation Using Network Formation Games

In previous section, we discuss the formulation and properties of the proposed cooperative game when the cooperative sets are given. However, how to acquire the cooperative set is very important. In this section, we formulate the cooperative set formation as network formation games, and design the corresponding distributed algorithm.

We first introduce some basic concepts of network formation games. A directed graph $G^D(S)$ is defined to be a pair (S, A^D), where S is a non-empty finite set of vertices, and A^D is a collection of directed arcs of the graph. Let a_{qr} be a directed arc from vertex q to vertex r. The set of incoming arcs of vertex q is defined as $A_q^{in} = \{a_{rq} \in A^D | r \in S, q \in S\}$, and the set of outing arcs of vertex q is defined as $A_q^{out} = \{a_{qr} \in A^D | r \in S, q \in S\}$. Denote $A_q^D = A_q^{in} \cup A_q^{out}$ as the directed arcs of vertex q. The set of vertices that are origins/destinations of the incoming/outgoing arcs of vertex q is denoted as $vtx(A_q^{in})$ and $vtx(A_q^{out})$ respectively.

The set of possible outgoing arcs from vertex q to other vertices is denoted as $\bar{A}_q^{out} = \{a_{qr} | r \in S, q \in S\}$, while the set of possible incoming arcs from other vertices to vertex q is denoted as $\bar{A}_q^{in} = \{a_{rq} | r \in S, q \in S\}$. The set of all possible directed arcs between vertex q and other vertices is denoted as $\bar{A}_q^D = \bar{A}_q^{in} \cup \bar{A}_q$.

Given a directed graph $G^D = (\Omega, A^D)$, we denote $x_q(G^D)$ as the payoff that player $q \in \Omega$ can receive when graph G^D is in place. The value set, i.e., the mapping V, can be defined as follows:

$$V(G^D) = \{\mathbf{x}(G^D) \in \mathbb{R}^{|\Omega|} \mid \forall q \in \Omega, x_q(G^D) = v_q(A^D)\} \tag{21}$$

where $v_q(A^D)$ is given by

$$v_q(A^D) = R_q^c(\mathbf{Q}_q(vtx(A_q^{out})), \mathbf{Q}_{-vtx(A_q^{in})}) \tag{22}$$

Using (21), the cooperative set formation can be modeled as a (Ω, A^D, V) network formation game. With such a formulation, the outgoing cooperative set $\mathcal{N}_q = vtx(A_q^{out})$, and the incoming cooperative set $\mathcal{C}_q = vtx(A_q^{in})$.

In order to present a network formation algorithm, we borrow the concept of potential function from [17]: An exact potential function Φ is a function that maps every strategy vector $\mathbf{s} = (s_1, s_2, .., s_M)$ to some real value and satisfies the following conditions: If $s'_q \neq s_q$ is an alternate strategy for player q, and $\mathbf{s}' = (\mathbf{s}_{-q}, s'_q)$, then $\phi(\mathbf{s}) - \phi(\mathbf{s}') = v_q(\mathbf{s}) - v_q(\mathbf{s}')$.

This definition implies that each player's individual interest is aligned with the group's interest, since each change in the utility function of each player directly represents the same change in the potential function. If players act sequentially, and choose best response strategies or at least improve their utilities (better response strategies), given the most recent actions of the other players, then the game will converge to a NE regardless of the order of players and the initial condition of the game [17].

Algorithm 1
The proposed NF algorithm

Initial State
At the beginning, the network starts with noncooperative state, $A_{init}^D = \emptyset$.
Phase 1 Neighbor Discovery:
Each player detects its strongest interferers and marks them.
Phase 2 Network Formation:
repeat
Each player q investigates potential change of incoming arcs by checking the value of potential function in (23).
Once the value is improved:
a) Player q updates its set of incoming arcs A_q^{in}.
b) Related players update their sets of outgoing arcs A_r^{out}.
until converges to a Nash Equilibrium.
$\{\mathcal{N}_q\} = \{vtx(A_q^{out})\}$, $\{\mathcal{C}_q\} = \{vtx(A_q^{in})\}$.
Phase 3 MIMO Transmission with Transmitter Null Shaping:
Transmit covariance matrices are designed by the cooperative game $\mathcal{G}^c$ with the cooperative sets $\{\mathcal{N}_q\}$ and $\{\mathcal{C}_q\}$ formed in Phase 2.

Define the potential function ϕ_q as

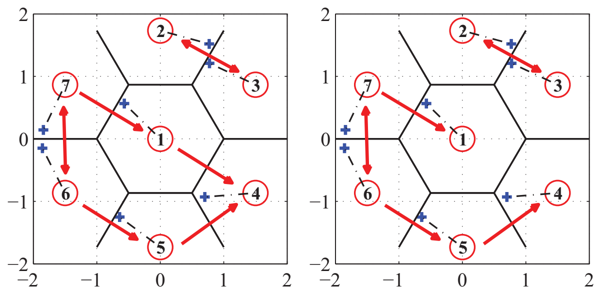
$$\phi_q(A_q'^D, A_{-q}'^D) = v_q(A_q'^D, A_{-q}'^D) - \sum_{r \neq q} \left(v_r(A_q^D, A_{-q}^D) - v_r(A_q'^D, A_{-q}'^D) \right). \tag{23}$$

This expression reflects the intention to maximize the player's own payoff, but subtracting the potential negative effect over other players.

Now, we propose a network formation (NF) algorithm composed of three phases: neighbor discovery, network formation with potential function, and MIMO transmission with transmitter null shaping. In Phase 1, each player discovers its strongest interferers who it may send interference mitigation requests to. Then in Phase 2, each player investigates the possibility of changing its set of directed arcs by pairwise negotiation with the rest of players. Once its potential function in (23) is improved (better response strategy), a distributed decision is made to update its set of directed arcs with the consent of the related players. We assume that players perform directed arcs modification sequentially with random orders. For simplicity and without loss of generality, we assume that each player only investigates the possibility of changing its incoming arcs in its turn. Finally, in Phase 3, players transmit data cooperatively in MIMO interference channel, imposing null shaping constraints formed in Phase 2 on the transmitters.

According to the conclusion of potential game in [18, Th 19.11-19.12], the proposed network formation algorithm is guaranteed to converge as shown in the following theorem.

Theorem 4: The proposed network formation algorithm always converges to a pure Nash Equilibrium with any initial graph and operation order.



(a) GCG (b) NF

Fig. 1. Snapshot of cooperative sets formation resulting from GCG and proposed NF algorithms with $(n_{T_q}, n_{R_q}) = (6, 2)$, $d = 0.8$ and $P/\sigma^2 = 10$dB.

TABLE I
AVERAGE LINK RATE

Algorithm	NF	GCG	IWF
Rate Per Link	3.9175	3.8707	3.1962

V. SIMULATION RESULTS AND ANALYSIS

For simulation, we consider a MIMO mutil-cell cellular network, consisting seven hexagonal cells, with full frequency reuse. In each cell, there is one base station (BS) and one user equipment (UE). Each UE is randomly distributed with the normalized distance $d \in [0.2, 1)$ to its serving BS. The distance between two adjacent BS is 1km. The elements of the channel matrix $\mathbf{H}_{rq}$ are generated as circularly symmetric complex Gaussian variable. $\mathbf{H}_{rq} \sim \mathcal{CN}(0, \eta_{rq}^2 \mathbf{I})$, with $\eta_{rq} = \sqrt{1/d_{rq}^{\alpha}}$, and the path loss exponent $\alpha = 3$.

In Fig. 1, we randomly deploy the users with the normalized distance $d = 0.8$, the number of transmit/receiver antennas is $(n_{T_q}, n_{R_q}) = (6, 2)$ and $P/\sigma^2 = 10$dB. Fig. 1 shows the features of the proposed NF algorithms. The cooperative sets generated from the proposed NF algorithm can be interpreted as a directed graph. Grand cooperative graph (GCG) represents the most altruistic cooperation, where players accept as many interference mitigation requests as they can support, i.e. condition (8) is satisfied. The GCG is shown here as a reference, since the cooperative sets formed by the proposed NF algorithm are a subset of that of GCG. Table 1 displays the average transmission rates using different algorithms. The proposed NF algorithm achieves higher average rate than that of IWF, since interference mitigation are performed in the network.

In Fig. 2, we compare the probability of the uniqueness of the NE of noncooperative game $\mathcal{G}$ and cooperative game $\mathcal{G}^c$. We simulate different numbers of the transmit/receiver antennas $(n_{T_q}, n_{R_q}) = \{(4, 2), (6, 2), (8, 2)\}$, with $P/\sigma^2 = 5$dB. Here, we use the simplest GCG to represent the cooperative game. As shown in Fig. 2, the probability of uniqueness of the NE of both games $\mathcal{G}$ and $\mathcal{G}^c$ with GCG decreases as UE are away from the BS, corresponding to an increase of intercell interference. This confirms to the definition of matrix $\mathbf{S}$ in (19) and the sufficient condition for the uniqueness of the NE given in Theorem 3. Fig. 2 also shows that, increasing the antennas at the transmitter side leads to a grow of uniqueness. The cooperative game $\mathcal{G}^c$ with GCG has higher probability of uniqueness of NE, compared to the noncooperative game $\mathcal{G}$, since the power of interference channels are weakened due to cross-link cooperation. The probability difference grows with the number of antennas at the transmitter, since more spatial degrees of freedom are available for interference mitigation.

In Fig. 3, we show the average rate achieved per link with regard to different SNR (P/σ^2) levels. The results are averaged over random positions of UEs with normalized distance $d = 0.7$ and random realization of the channel matrix. As seen from Fig. 3, the average rates of all the proposed

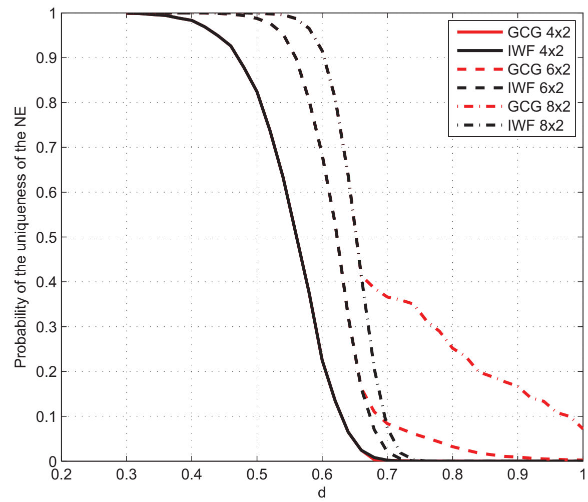


Fig. 2. Probability of uniqueness of the NE of noncooperative $\mathcal{G}$ and cooperative $\mathcal{G}^c$ with grand cooperative graph, $(n_{T_q}, n_{R_q}) = \{(4, 2), (6, 2), (8, 2)\}$ and $P/\sigma^2 = 5$dB.

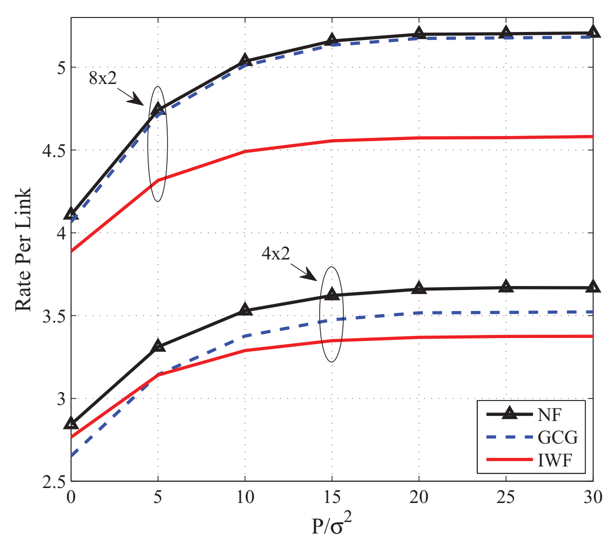


Fig. 3. Average rate of the IWF, GCG and NF algorithms with regard to different P/σ^2. $(n_{T_q}, n_{R_q}) = \{(4, 2), (8, 2)\}$ and $d = 0.7$.

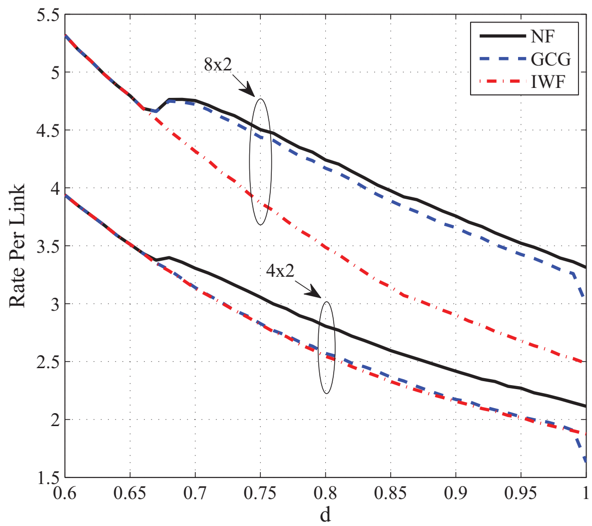


Fig. 4. Average rate of IWF, GCG and NF algorithms with regard to d (inter-cell interference). $(n_{T_q}, n_{R_q}) = \{(8, 2), (4, 2)\}$, and $P/\sigma^2 = 5$dB.

algorithms and IWF algorithm increase as P/σ^2 grows and finally reach their upper bounds. Increasing the antennas at the transmitter leads to higher average rate for all algorithms. When the number of antennas at the transmitter is 8, the simple GCG scheme achieves almost the same rate as the proposed NF algorithm. When the number of antennas at the transmitter is 4, GCG scheme performs worse than IWF at low SNR region. Fortunately, the proposed NF algorithm is apparently superior to the IWF with the different SNR and numbers of antennas at the transmit side.

In Fig. 4, we simulate the influence of strength of inter-cell interference to the proposed algorithms at low SNR ($P/\sigma^2 = 5$dB). The results are averaged over random positions of UEs with the normalized distance d varying form 0.6 to 1, and random realization of the channel matrix. It is observed that, the average transmission rate of IWF, GCG and NF algorithms generally decrease as users are away from their serving BSs. For the NF algorithm, there is an apparent rise around $d = 0.66$, since cross-link cooperation is started. Then the curves gradually go down but always have higher rate than other algorithms. The GCG scheme approaches NF when the antennas at the transmitter is 8, but is no superior to IWF when the antennas at the transmitter is 4, and there is a sudden drop near $d = 1$, due to excessive cooperation.

VI. Conclusion

In this paper, we consider the cooperative maximization of mutual information in the MIMO Gaussian interference channel via game theory. Null shaping constraints are enforced in the design of transmit covariance matrices to enable interference mitigation among links. The transmit covariance matrices leading to the NE are derived, and the existence and uniqueness of the NE is analyzed. We define cooperative sets to stand for the cooperation relationship among links, and the formation of the cooperative sets is formulated as network formation games. The proposed NF algorithm converges to a Nash Equilibrium. Simulation results show that the proposed NF algorithm enhances the sum rate of the system apparently even at low SNR region and/or with small number of transmit antennas.

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