Pricing Game for Time Mute in Femto–Macro Coexistent Networks

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***Abstract*—In heterogenous networks, intercell interference coordination (ICIC) is a big challenge. This paper presents an analytical framework to evaluate power control and time mute schemes in closed access femto and macro coexistent networks. We use stochastic geometry to model the downlink scenario and derive the coverage probability of indoor macro users and femto users. The optimal operating parameters for altruistic power control and time mute schemes are achieved. Considering the selfishness of the owners of femtos, we formulate the two-tier interference coordination as pricing games and obtain the closed-form of Nash equilibria. Simulation results demonstrate the influence of different parameters on the performance of ICIC schemes and show that when target SINR ≥ 3 dB, the time mute scheme outperforms the power control scheme in handling the indoor macro user coverage problem.**

***Index Terms*—Femtocells, CSG, almost blank subframe, game theory.**

I. Introduction

THE rapid increase of mobile data activity motivates the development of new cellular technologies and topologies. An interesting trend of cellular evolution is femtocells [1]. Femto base stations (FBSs) are small, inexpensive and low-power. They are consumer deployed and connected to the cellular operator network through wired broadband connection. The deployment of FBSs can improve indoor coverage and offload traffic from macro BSs (MBSs). Different from WiFi access points, FBSs achieve these aspects with commercial cellular standards and licensed spectrum. Moreover, the owners are capable of manipulating the FBSs, such as setting them to closed/open access, or controlling other operating parameters.

A big challenge for femtocell deployment is the less predictable and more complicated intercell interference. In [2], the authors show that the overall interference conditions are not exacerbated when the FBSs are open access and users select the strongest cells. However, in the case of closed access, when a macro user (MUE) gets close to an active FBS, it will see severe interference in the downlink. Due to the extremely poor channel condition, the user cannot connect to any cell and hence is in outage [3]. Different tools have been proposed to counter the coverage problem in OFDMA femtocells including power control [4], time mute [5], frequency partitioning [6], precoding [7], and subband scheduling [8]. An example of the time mute scheme is almost blank subframe (ABS), which has been proposed by 3GPP members to combat co-channel cross-tier interference in heterogeneous networks [9]. The rationale of time mute scheme in femtocell is muting some subframes of femto tier and scheduling the vulnerable macro users in these subframes. Hence, the channel conditions of the macro users are improved in these muting time slots.

To the best of our knowledge, only a few works in the literature have been dedicated to the time mute scheme in femtocell. Simulation results of utilizing time mute in closed access femto and macro networks are illustrated in [10]–[12]. Detecting methods of vulnerable macro users in time mute scheme are evaluated in [11], and a coordinated framework for ABS in LTE-Advanced system is presented in [12]. There has been little work done on the theoretical analysis of time mute scheme. In [5], the authors studied the required number of ABS to guarantee the outage throughput of macro users. Different from [5] where only the number of ABS is analyzed, in this paper, we investigate both the active time ratio and transmit power of FBSs, and theoretically compare the performance of power control and time mute approaches.

We first derive the coverage probability of indoor macro user and femto user in closed femto and macro co-existent networks based on the stochastic geometry [13], [14]. Secondly, we analyze the altruistic interference mitigation of a FBS to guarantee a minimum coverage probability of indoor macro users, and derive the optimal transmit power and active time ratio of the FBS. Thirdly, we consider the case where the owner of a FBS is capable of deciding the interference leakage to indoor macro users according to the reward from the operator of MBSs. By formulating the two-tier interference coordination problem as pricing games, we obtain the closed-form Nash equilibria, which reveal the stable working parameters of FBSs and the payment of the MBS operator. Simulation results verify the derivations and show that time mute scheme performs better than power control scheme in dealing with the indoor coverage problem.

The rest of this paper is outlined as follows. Section II introduces the system model based on stochastic geometry. In Section III, we derive the coverage probability of indoor macro user and femto user. In Section IV, we use coverage probability

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to analyze the altruistic interference mitigation from FBS. In Section V, the two-tier interference coordination problem is formulated as pricing games and the closed-form of NEs are given. Simulation results are demonstrated and discussed in Section VI. Finally, conclusion is given in Section VII.

II. System Model

We consider a two-tier heterogenous network with MBSs and closed FBSs in tier 1 and 2, respectively. The two tiers share the same physical transmission resources. Each tier is specified by a set of operating parameters, including transmit power, active time ratio, required outage throughput, etc. Assume that each user can only be served by a BS belonging to its accessible tier, and users served by the same BSs are scheduled on orthogonal transmission resources, i.e., there is no intra-cell interference. We consider an arbitrary user anywhere in the network, and no matter this user is a macro user or a femto user, it is in a room with a FBS. We focus on the scenario where macro users are indoor, since macro users are most vulnerable to the interference from FBSs in this case.

The locations of the BSs in tier i are assumed to be given by a homogeneous Poisson Point Process (PPP) Φ_i on the plane with intensity λ_i (units of BSs per m^2). Thus, the number of BSs $N_i(B)$ in any finite region B is Poisson distributed with mean $\lambda_i \times area(B)$,

$$P\{N_i(B)=n\} = e^{-\lambda_i \times area(B)} \frac{[\lambda_i \times area(B)]^n}{n!}. \quad (1)$$

Since femto BSs are consumer deployed, the PPP model is naturally fit for them. For macro BSs, it has been shown in [15] that a fully random placement gives a lower bound for SINR distribution, while a deterministic gird deployment produces an upper bound, and a real-world macro BSs placement lies in between. The use of PPP model is widely accepted since it offers a tool to analyze the large scale network theoretically.

We assume a single antenna transceiver at both BS and user, and do not consider coordinated multi-point transmission and advanced receiver processing. The received power at a user located at a distance of r from a BS b of tier i is given by,

$$y_{i,b} = \frac{h_{i,b}}{r^{\delta_i}}, \quad (2)$$

where $\delta_i > 2$ is the pathloss exponent, $h_{i,b}$ is the attenuation in power due to fading on the link, and the effect of transmit power, antenna gain, etc. $\{h_{i,b}\}$ are independent distribution over all BSs in the two tiers, and for the sake of tractability, we do not model shadow fading, and assume that all links follow Rayleigh distribution, and $h_{i,b}$ obeys exponential distribution with $E[h_{i,b}] = \mu_i$,

$$f_{h_{i,b}}(x) = \frac{1}{\mu_i} \exp\left(-\frac{x}{\mu_i}\right). \quad (3)$$

Denote $\{\mu_i\} = \{\mu_{1,1}, \mu_{2,0}, \mu_{2,2}\}$ as the set of channel gain, where $\mu_{i,k}$ is associated with the channel gain from a BS in tier i to the observed user, and k corresponds to the number of walls between the transmitter and the observed user.

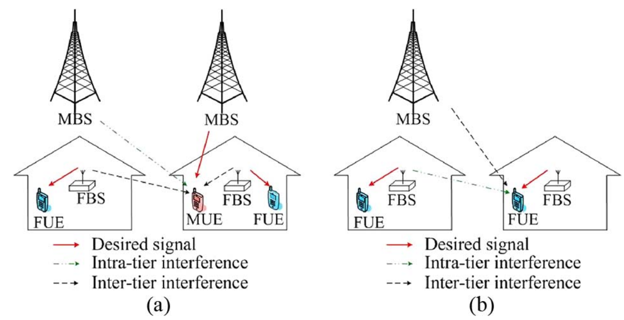

Fig. 1. Receiving signals of indoor macro users and femto users. (a) Indoor macro user. (b) Femto user.

III. Coverage Probability

In this section, we derive coverage probability of macro and femto users. Coverage probability is the probability that a user has a signal-to-interference-plus-noise (SINR) higher than an outage threshold. The coverage probability is also the complementary cumulative distribution function (CCDF) of SINR.

A. Indoor Macro User

Denote $r_{i,b}$ as the distance from BS b in tier i to the observed user. Assume that a user is served by the nearest BS b_0 in its accessible tier i.

$$b_0 = arg \min_b r_{i,b}. \quad (4)$$

Denote $r = r_{1,b_0}$ as the distance between the observed macro user and its serving macro BS. The SINR of a macro user on a subcarrier is given by,

$$\gamma_{MUE} = \frac{h_{1,b_0} r^{-\delta_1}}{Z_r}, \quad (5)$$

where Z_r is interference plus noise.

As can be seen from Fig. 1(a), for an indoor macro user, there exists a dominant interferer $\tilde{b}$, i.e., the femto BS located in the same room with the macro user. Assume that the distance between femto BS $\tilde{b}$ and the observed macro user $r_{2,\tilde{b}} = R$. Then,

$$Z_r = h_{2,\tilde{b}} R^{-\delta_2} + \kappa \sum_{b \in \Phi_1 \setminus b_0} h_{1,b} r_{1,b}^{-\delta_1} + \sum_{b \in \Phi_2 \setminus \tilde{b}} h_{2,b} r_{2,b}^{-\delta_2} + \sigma^2. \quad (6)$$

The first term in the right-hand side of (6) is the interference from FBS $\tilde{b}$. As we can see from Fig. 1(a), there is no external wall between FBS $\tilde{b}$ and the observed user, and thus $E[h_{2,\tilde{b}}] = \mu_{2,0}$. The second term represents the interference received from intra-tier Φ_1, where κ is the interference leakage coefficient of other MBSs. Since the signal from any MBS(serving or interfering MBS) goes through one external wall before arriving at the observed user, we assume that $\{h_{1,b \in \Phi_1}\}$ are i.i.d., and $E[h_{1,b}] = \mu_{1,1}$. The third term stands for the interference from FBSs located in other rooms. There exist two external walls between any FBS $b \in \Phi_2 \setminus \tilde{b}$ and the observed user. Hence, we assume that $\{h_{2,b \in \Phi_2 \setminus \tilde{b}}\}$ are i.i.d., and $E[h_{2,b \in \Phi_2 \setminus \tilde{b}}] = \mu_{2,2}$. The last term σ^2 is the variance of a zero-mean circularly symmetric complex Gaussian noise.

Since the observed user is served by the closest BS, no BS can be closer than r. The probability density function (pdf) of r can be derived using the fact that no BS exists in the area πR^2,

$$P[r > R] = P[\text{No BS in the area } \pi R^2] = e^{-\lambda_i \pi R^2}. \quad (7)$$

Hence, the cumulative distribution function (CDF) is $F_r(R) = P[r < R] = 1 - e^{-\lambda_i \pi R^2}$ and the pdf can be written as [15],

$$f_r(r) = \frac{\mathrm{d}F_r(r)}{\mathrm{d}r} = e^{-\lambda_i \pi r^2} 2\pi\lambda_i r. \quad (8)$$

With the pdf in (8), we can derive the coverage probability of an indoor macro user as follows.

Corollary 1.1: The coverage probability of an indoor macro user on the shared channel in the two-tier heterogenous network model can be expressed as (9), shown at the bottom of the page, where T is target SINR, and

$$\rho(y,\delta) = y^{2/\delta} \int_{y^{-2/\delta}}^{\infty} \frac{1}{1+u^{\delta/2}} \mathrm{d}u. \quad (10)$$

Proof:

$$\begin{aligned}
\mathbb{P}^{MS} &= \int_0^\infty P[\gamma_{MUE} > T] f_r(r) \mathrm{d}r \\
&= \int_0^\infty P\left[\frac{h_{1,b_0} r^{-\delta_1}}{Z_r} > T\right] e^{-\lambda_1 \pi r^2} 2\pi\lambda_1 r \,\mathrm{d}r \\
&= \int_0^\infty P\left[h_{1,b_0} > T r^{\delta_1} Z_r\right] e^{-\lambda_1 \pi r^2} 2\pi\lambda_1 r \,\mathrm{d}r. \quad (11)
\end{aligned}$$

Considering that MBSs cannot locate indoor, a non-homogeneous PPP model is more accurate to model the locates of MBSs. However, to make the analysis traceable and thus give some insights, a homogeneous PPP model is generally used in the literature such as [5] and [15]. When a homogeneous PPP is used, the integration limits are from 0 to infinity, since a MBS can be arbitrarily close to the observed MUE.

Since h_{1,b_0} obeys exponential distribution, we can get

$$\begin{aligned}
P\left[h_{1,b_0} > T r^{\delta_1} Z_r\right] &= \mathbb{E}_{Z_r}\left[\exp\left(-\frac{1}{\mu_{1,1}} T r^{\delta_1} Z_r\right)\right] \\
&= \mathcal{L}_{Z_r}\left(\frac{1}{\mu_{1,1}} T r^{\delta_1}\right). \quad (12)
\end{aligned}$$

Using the definition of the Laplace transform, and taking the expectation over Φ, we get (13), shown at the bottom of the page, where step (a) follows from the assumption that $\{h_{i,b}\}$ are independent and the locations of BSs are generated independently from PPP $\Phi = \{\Phi_1, \Phi_2\}$, step (b) is due to the distribution $h_{i,b} \sim \mathrm{Exp}(1/\mu_i)$, and step (c) follows from the probability generating functional of the PPP [14]. The integration limits associated with tier 1 are from r to ∞ since the nearest interfering MBS is at least at a distance of r. d_{minF} is the restricted minimal distance from the observed user and FBSs located in other rooms.

$$\mathbb{P}^{MS} = \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\frac{T}{\mu_{1,1}} v^{\frac{\delta_1}{2}} \sigma^2 - \pi\lambda_1 v\left(1 + \rho(\kappa T, \delta_1) + \frac{\lambda_2}{\lambda_1}\left(\frac{\mu_{2,2}}{\mu_{1,1}} T v^{\frac{(\delta_1-\delta_2)}{2}}\right)^{2/\delta_2} / \mathrm{sinc}(2/\delta_2)\right)\right)}{1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\frac{\delta_1}{2}} R^{-\delta_2}} \mathrm{d}v \quad (9)$$

$$\begin{aligned}
\mathcal{L}_{Z_r}(s) &= \mathbb{E}_{Z_r}[e^{-sZ_r}] \\
&= \mathbb{E}_\Phi\left[e^{-s\sigma^2 - sh_{2,\tilde{b}} R^{-\delta_2}} \prod_{b\in\Phi_1\backslash b_0} e^{-s\kappa h_{1,b} r_{1,b}^{-\delta_1}} \prod_{b\in\Phi_2\backslash\tilde{b}} e^{-sh_{2,b} r_{2,b}^{-\delta_2}}\right] \\
&\overset{a}{=} e^{-s\sigma^2} \mathbb{E}_{h_{2,\tilde{b}}}\left[e^{-sh_{2,\tilde{b}} R^{-\delta_2}}\right] \mathbb{E}_\Phi\left[\prod_{b\in\Phi_1\backslash b_0} \mathbb{E}_{h_{1,b}}\left[e^{-s\kappa h_{1,b} r_{1,b}^{-\delta_1}}\right] \prod_{b\in\Phi_2\backslash\tilde{b}} \mathbb{E}_{h_{1,b}}\left[e^{-sh_{2,b} r_{2,b}^{-\delta_2}}\right]\right] \\
&\overset{b}{=} \frac{e^{-s\sigma^2}}{1 + s\mu_{2,0} R^{-\delta_2}} \mathbb{E}_\Phi\left[\prod_{b\in\Phi_1\backslash b_0} \frac{1}{1 + s\kappa\mu_{1,1} r_{1,b}^{-\delta_1}} \prod_{b\in\Phi_2\backslash\tilde{b}} \frac{1}{1 + s\mu_{2,2} r_{2,b}^{-\delta_2}}\right] \\
&\overset{c}{=} \frac{e^{-s\sigma^2}}{1 + s\mu_{2,0} R^{-\delta_2}} \exp\left(-2\pi\lambda_1 \int_r^\infty \left(1 - \frac{1}{1 + s\kappa\mu_{1,1} v^{-\delta_1}}\right) v \,\mathrm{d}v\right) \times \exp\left(-2\pi\lambda_2 \int_{d_{minF}}^\infty \left(1 - \frac{1}{1 + s\mu_{2,2} v^{-\delta_2}}\right) v \,\mathrm{d}v\right) \quad (13)
\end{aligned}$$

Substituting $s = \frac{T}{\mu_{1,1}} r^{\delta_1}$ into (13), we have

$$\mathcal{L}_{Z_r}\left(\frac{1}{\mu_{1,1}} T r^{\delta_1}\right) = \frac{e^{-\frac{T}{\mu_{1,1}} r^{\delta_1} \sigma^2}}{1 + T r^{\delta_1} R^{-\delta_2} \frac{\mu_{2,0}}{\mu_{1,1}}} \times \exp\left(-2\pi\lambda_1 \int_r^\infty \left(\frac{1}{1 + \frac{1}{\kappa T}\left(\frac{v}{r}\right)^{\delta_1}}\right) v \, dv\right) \times \exp\left(-2\pi\lambda_2 \int_{d_{minF}}^\infty \left(\frac{1}{1 + \frac{\mu_{1,1}}{\mu_{2,2}} \frac{1}{T} \frac{v^{\delta_2}}{r^{\delta_1}}}\right) v \, dv\right). \quad (14)$$

Denote $m_{21} = \mu_{2,2}/\mu_{1,1} T r^{(\delta_1 - \delta_2)}$, and employ changes of variables $u_1 = \left(\frac{v}{r}(\kappa T)^{-1/\delta_1}\right)^2$ and $u_2 = \left(\frac{v}{r} m_{21}^{-1/\delta_2}\right)^2$. Then we can obtain

$$\mathcal{L}_{Z_r}\left(\frac{1}{\mu_{1,1}} T r^{\delta_1}\right) = \frac{e^{-\frac{T}{\mu_{1,1}} r^{\delta_1} \sigma^2}}{1 + T r^{\delta_1} R^{-\delta_2} \frac{\mu_{2,0}}{\mu_{1,1}}} \times \exp\left(-\pi r^2 \left(\lambda_1 \rho(\kappa T, \delta_1) \lambda_2 m_{21}^{2/\delta_2} G_{\delta_2}\left(\frac{d_{minF}^2}{r^2} m_{21}^{-2/\delta_2}\right)\right)\right) \quad (15)$$

where,

$$G_\delta(y) = \int_y^\infty \frac{1}{1 + u^{\delta/2}} \, du. \quad (16)$$

For simplicity, we assume that there is no other restriction on the minimal distance from observed macro user and femto BSs $b \in \Phi_2 \setminus \tilde{b}$, that is, $d_{minF} = 0$. By applying the results [16], [17, Eq. (3.222.2)] and [17, Eq. (8.391), (9.131.1)],

$$G_\delta(y) = \int_y^\infty \frac{1}{1 + u^{\frac{\delta}{2}}} \, du = \begin{cases} \frac{2\pi/\delta}{\sin(2\pi/\delta)}, & y = 0, \\ \frac{2}{\delta - 2} \frac{y}{1 + y^{\delta/2}} F\left(1, 1; 2 - \frac{2}{\delta}; \frac{1}{1 + y^{\delta/2}}\right), & y > 0, \end{cases} \quad (17)$$

where $F(a, b; c, z) = 1 + \sum_{n=1}^\infty \frac{z^n}{n!} \prod_{m=0}^{n-1} \frac{(a+m)(b+m)}{c+m}$ is hypergeometric function. By combining (11), (12), (15), and (17), and letting $v = r^2$, we can obtain the expression in (9) ■

The expression of (9) seems complicated, however, each part has clearly physical meaning. In the numerator, the first term $\exp(-\frac{T}{\mu_{1,1}} v^{\frac{\delta_1}{2}} \sigma^2)$ represents the noise, the second term $\exp(-\pi\lambda_1 v)$ is associated with the pdf of variable r, i.e., $f_r(r)$, the third term $\exp(-\pi\lambda_1 v \rho(\kappa T, \delta_1))$ stands for the intra-tier interference, and the last term $\exp(-\pi\lambda_2 v (\frac{\mu_{2,2}}{\mu_{1,1}} T v^{\frac{(\delta_1 - \delta_2)}{2}})^{2/\delta_2} / \mathrm{sinc}(2/\delta_2))$ corresponds to the interference from FBSs located in other rooms. The term $(1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\frac{\delta_1}{2}} R^{-\delta_2})$ in the denominator stands for the interference from dominant interfering FBS.

When a macro user gets close to a FBS, the interference from dominant interfering FBS $\tilde{b}$ becomes extremely strong, and FBS $\tilde{b}$ may need to decrease its transmit power to keep the nearby macro user being served. As the transmit power of FBS $\tilde{b}$ decreases, the value of $\mu_{2,0}$ diminishes, leading to an increase of $\mathbb{P}^{MS}$.

If FBSs perform time muting to mitigate the interference to the observed macro user, and assume that there is no interference leakage from FBSs to macro users on the dedicated channel (e.g., muted time slots), then we can get the coverage probability of the observed macro user on the dedicated channel $\mathbb{P}^{MD}$ by substituting $\mu_{2,0} = \mu_{2,2} = 0$ into (9). If we further assume that noise is much smaller than the interference, i.e., $\sigma^2/Z_r \to 0$, $\mathbb{P}^{MD}$ can be simplified as follows.

Corollary 1.2: Assume that $\sigma^2/Z_r \to 0$, and there is no interference leakage from FBSs to macro users on the dedicated channel, the coverage probability of indoor macro users on the dedicated channel is

$$\mathbb{P}_{\delta_1}^{MD} = \frac{1}{(1 + \rho(\kappa T, \delta_1))}. \quad (18)$$

Proof: Substituting $\sigma^2 = 0$ and $\mu_{2,0} = \mu_{2,2} = 0$ into (9), we can easily get the coverage probability of indoor macro users on the dedicated channel as in (18). ■

When the pathloss exponents of the two tiers are the same, i.e., $\delta_1 = \delta_2 = \delta$, the expression of $\mathbb{P}^{MS}$ in (9) can be further simplified in the following Corollary.

Corollary 1.3: Assume that $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$, the coverage probability of indoor macro users on the shared channel can be simplified as:

$$\mathbb{P}_\delta^{MS} = \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\pi\lambda_1 v \left(1 + \rho(\kappa T, \delta) + \frac{\lambda_2}{\lambda_1} \frac{\left(\frac{\mu_{2,2}}{\mu_{1,1}} T\right)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\right)\right)}{1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\delta/2} R^{-\delta}} \, dv. \quad (19)$$

If we further assume that the interference from FBSs located in other rooms is very small compared to the rest of interference, then the coverage probability can be approximately written as:

$$\mathbb{P}_{\delta,\mathrm{apx}}^{MS} = \pi\lambda_1 \int_0^\infty \frac{\exp(-\pi\lambda_1 v (1 + \rho(\kappa T, \delta)))}{1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\delta/2} R^{-\delta}} \, dv. \quad (20)$$

B. Femto User

For a femto user (FUE), its serving BS is the femto BS b_0 in the same room. Since the area of a room is rather small compared to the observed area, for simplicity, we assume that the distance between any user (indoor MUE/FUE) and its nearest FBS is the same, i.e., $r_{2,b_0} = R$. Then, the SINR of the femto user on a subcarrier is,

$$\gamma_{FUE} = \frac{h_{2,b_0} R^{-\delta_2}}{Z_R} \quad (21)$$

where Z_R is interference plus noise,

$$Z_R = \sum_{b \in \Phi_2 \setminus b_0} h_{2,b} r_{2,b}^{-\delta_2} + \sum_{b \in \Phi_1} h_{1,b} r_{1,b}^{-\delta_1} + \sigma^2. \quad (22)$$

With (21), we can derive the coverage probability of a FUE as follows in Corollary 2.1.

Corollary 2.1: The coverage probability of a femto user on the shared channel in the two-tier heterogenous network model of Section II is expressed as (23), shown at the bottom of the page.

Proof:

$$\begin{aligned}\mathbb{P}^{FS} &= P[\gamma_{FUE} > T] \\ &= P\left[h_{2,b_0} > TR^{\delta_2} Z_R\right] \\ &= \mathcal{L}_{Z_R}\left(\frac{1}{\mu_{2,0}} TR^{\delta_2}\right), \end{aligned} \tag{24}$$

where, the last equality follows from the exponential distribution of random variable h_{2,b_0}, and

$$\begin{aligned}\mathcal{L}_{Z_R}(s) &= E_\Phi\left[e^{-s\sigma^2} \prod_{b\in\Phi_2\backslash b_0} e^{-sh_{2,b} r_{2,b}^{-\delta_2}} \prod_{b\in\Phi_1} e^{-sh_{1,b} r_{1,b}^{-\delta_1}}\right] \\ &= e^{-s\sigma^2} E_\Phi\left[\prod_{b\in\Phi_2\backslash b_0} E_{h_{2,b}}\left[e^{-sh_{2,b} r_{2,b}^{-\delta_2}}\right] \times \prod_{b\in\Phi_1} E_{h_{1,b}}\left[e^{-sh_{1,b} r_{1,b}^{-\delta_1}}\right]\right] \\ &= e^{-s\sigma^2} E_\Phi\left[\prod_{b\in\Phi_2\backslash b_0} \frac{1}{1+s\mu_{2,2} r_{2,b}^{-\delta_2}} \prod_{b\in\Phi_1} \frac{1}{1+s\mu_{1,1} r_{1,b}^{-\delta_1}}\right] \\ &= e^{-s\sigma^2} \exp\left(-2\pi\lambda_2 \int_{d_{minF}}^{\infty} \left(1-\frac{1}{1+s\mu_{2,2} v^{-\delta_2}}\right) v \mathrm{d}v\right) \\ &\quad \times \exp\left(-2\pi\lambda_1 \int_{d_{minM}}^{\infty} \left(1-\frac{1}{1+s\mu_{1,1} v^{-\delta_1}}\right) v \mathrm{d}v\right) \end{aligned} \tag{25}$$

where d_{minM} is the restricted minimal distance between the observed FUE and MBSs, and d_{minF} is the restricted minimal distance between the observed user and interfering FBSs.

Denote $m_{22} = \frac{\mu_{2,2}}{\mu_{2,0}} T$ and $m_{12} = \frac{\mu_{1,1}}{\mu_{2,0}} TR^{(\delta_2-\delta_1)}$. Substituting $s = \frac{T}{\mu_{2,0}} R^{\delta_2}$ into (25), we have

$$\begin{aligned}&\mathcal{L}_{Z_r}\left(\frac{T}{\mu_{2,0}} R^{\delta_2}\right) \\ &= e^{-\frac{T}{\mu_{2,0}} R^{\delta_2}\sigma^2} \exp\left(-2\pi\lambda_2 \int_{d_{minF}}^{\infty} \left(\frac{1}{1+\frac{\mu_{2,0}}{\mu_{2,2}}\frac{1}{T}\left(\frac{v}{R}\right)^{\delta_2}}\right) v \,\mathrm{d}v\right) \\ &\quad \times \exp\left(-2\pi\lambda_1 \int_{d_{minM}}^{\infty} \left(\frac{1}{1+\frac{\mu_{2,0}}{\mu_{1,1}}\frac{1}{T}\frac{v^{\delta_1}}{R^{\delta_2}}}\right) v \,\mathrm{d}v\right) \\ &= \exp\Bigg(-\frac{T}{\mu_{2,0}} R^{\delta_2}\sigma^2 - \pi R^2\Bigg(\lambda_2 m_{22}^{2/\delta_2} G_{\delta_2}\left(\frac{d_{minF}^2}{R^2} m_{22}^{-2/\delta_2}\right) \\ &\quad + \lambda_1 m_{12}^{2/\delta_1} G_{\delta_1}\left(\frac{d_{minM}^2}{R^2} m_{12}^{-2/\delta_1}\right)\Bigg)\Bigg). \end{aligned} \tag{26}$$

For simplicity, we assume that there is no other restriction on the minimal distances, that is, $d_{minM} = d_{minF} = 0$. Since a FUE is served by the FBS located in the same room, it is possible that an interfering FBS/MBS locates close to the observed FUE, at a distance of less than the serving distance R. Substituting (17) into (26) leads to the closed-form expression of the coverage probability as in (23). ■

If a FBS adopts power control to mitigate interference to a macro user, $\mu_{2,0}$ decreases, and $\mathbb{P}^{FS}$ degenerates. Corollary 2.1 can be further simplified under the assumption that $\delta_1 = \delta_2 = \delta$ and $\sigma^2/Z_r \to 0$, as in the following Corollary.

Corollary 2.2: Assume that the pathloss exponents $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$, the coverage probability in Corollary 2.1 can be approximated as:

$$\mathbb{P}_\delta^{FS} = \exp\left(-\pi R^2\left(\lambda_2 \frac{(\mu_{2,2}/\mu_{2,0}T)^{2/\delta}}{\mathrm{sinc}(2/\delta_2)} + \lambda_1 \frac{(\mu_{1,1}/\mu_{2,0}T)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\right)\right). \tag{27}$$

If we further assume that the intra-tier interference, i.e., the interference from other FBSs can be neglected compared to the interference from macro tier. Then, the coverage probability can be further simplified as:

$$\mathbb{P}_{\delta,\mathrm{apx}}^{FS} = \exp\left(-\pi R^2 \lambda_1 \frac{(\mu_{1,1}/\mu_{2,0}T)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\right). \tag{28}$$

IV. Altruistic Interference Coordination

In this section, we consider the case that a FBS altruistically mitigates interference to indoor macro users according to the statistic requirement of the operator of MBSs, e.g., coverage probability of macro users. The operating parameters of a FBS are set statically or semi-statically. Compared to the case that a FBS dynamically adjusts parameters according to instant channel condition of macro users, the static or semi-static scheme has lower computation complexity, and requires less message exchange among BSs.

In our paper, we intend to investigate the outage problem of a macro user when it gets close to a FBS. For simplicity, we consider a special scenario where macro users are indoor and suffer strong interference from FBSs in the downlink. We further assume that there is only one dominant interfering FBS for each macro user, which is proved to be true in our model in Section VI. Based on this assumption, the interference from other FBSs can be neglected, and each FBS sets individual operating parameters according to the existence of nearby macro users. In this section, The *downlink power control* and *time mute with power control* schemes are analyzed and compared.

$$\mathbb{P}^{FS} = \exp\left(-\frac{T}{\mu_{2,0}} R^{\delta_2}\sigma^2 - \pi R^2\left(\lambda_2 \frac{(\mu_{2,2}/\mu_{2,0}T)^{2/\delta_2}}{\mathrm{sinc}(2/\delta_2)} + \lambda_1 \frac{\left(\mu_{1,1}/\mu_{2,0}TR^{(\delta_2-\delta_1)}\right)^{2/\delta_1}}{\mathrm{sinc}(2/\delta_1)}\right)\right) \tag{23}$$

A. Downlink Power Control

Consider equal power allocation on each resource block, and denote P_2 $(0 < P_2 \leq 1)$ as the normalized transmit power on each resource block in a FBS. To mitigate cross-tier interference to the nearby macro user, the FBS decreases P_2 to guarantee a minimal throughput or coverage probability of indoor macro users. Here, we choose coverage probability as the quality of service (QoS) to be guaranteed. Denote $C^{FS}(P_2)$ as the average throughput on a resource block of shared channel,

$$\begin{aligned} C^{FS}(P_2) &= \int_T^\infty \log(1+y) f_\gamma(y)\,\mathrm{d}y, \\ &= \int_T^\infty \log(1+y)\left(-\frac{\mathrm{d}\mathbb{P}^{FS}(y,P_2)}{\mathrm{d}y}\right)\mathrm{d}y, \\ &= \int_T^\infty \frac{\mathbb{P}^{FS}(y,P_2)}{1+y}\,\mathrm{d}y + \mathbb{P}^{FS}(T,P_2)\log(1+T). \end{aligned} \tag{29}$$

The second term in last line of (29) is the average throughput at the outage threshold, which is also called outage throughput.

Similar to [5], we consider the throughput of FBSs in the worst-case scenario as the objective function, which is the outage throughput $C_{out}^{FS}(P_2) = \mathbb{P}^{FS}(P_2)\log(1+T)$. Given the BSs density of two tiers $\{\lambda_1,\lambda_2\}$, the set of channel gain $\{\mu_{1,1},\mu_{2,0},\mu_{2,2}\}$, target SINR T, and the threshold of coverage probability of indoor macro users $p_c^M (p_c^M < \mathbb{P}^{MD})$, the altruistic power control scheme can be formulated as follows.

$$\begin{aligned} &\max_{P_2} \quad C_{out}^{FS}(P_2) \\ &\text{s.t.} \begin{cases} \mathbb{P}^{MS}(P_2) \geq p_c^M, \\ 0 \leq P_2 \leq 1. \end{cases} \end{aligned} \tag{30}$$

The coverage probability in the constraint is averaged over all possible realizations of the point processes. Although an operator may be more interested in a single realization of the point process in his network, this formulation gives the operator some insight into the behavior of interference coordination.

The solution to the optimization problem in (30) is shown in the following theorem.

Theorem 1: Assume that the pathloss exponents $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$, the solution of altruistic power control is

$$P_2 = \min\{1, \bar{P}_2\} \tag{31}$$

where $\bar{P}_2$ is the solution to the following equation,

$$\pi\lambda_1 \int_{v>0} \frac{\exp\left(-\pi\lambda_1 v\left(1+\rho(\kappa T,\delta)\right)\right)}{1+P_2\frac{\mu_{2,0}}{\mu_{1,1}}T v^{\delta/2}R^{-\delta}}\,\mathrm{d}v = p_c^M. \tag{32}$$

Proof: Since $C_{out}^{FS}(P_2)$ is a non-decreasing function of P_2, $\bar{P}_2$ is obtained when the first constraint in (30) is achieved with equality, i.e., $\mathbb{P}_{\delta,\text{apx}}^{MS}(P_2) = p_c^M$. To make $\bar{P}_2$ tractable, we use the approximate expression in (20). Then $\bar{P}_2$ is the solution to (32). ∎

We cannot obtain a closed-form solution for $\bar{P}_2$, but fortunately, the coverage probability $\mathbb{P}_{\delta,\text{apx}}^{MS}$, is an decreasing function of P_2, and there exists only one solution. Since the range of P_2 is finite, the solution can be found numerically.

B. Time Mute With Power Control

Denote β_2 $(0 < \beta_2 \leq 1)$ as the active time ratio of a FBS. Then the percentage of shared and dedicated time slots for the nearby macro users are β_2 and $1-\beta_2$ respectively. The normalized power allocated on each resource block is denoted as P_2. In altruistic time mute scheme, a FBS adjusts the parameters (β_2, P_2) to guarantee a minimum coverage probability of indoor macro users p_c^M $(p_c^M < \mathbb{P}^{MD})$.

Given the BSs density of two tiers $\{\lambda_1,\lambda_2\}$, the set of channel gain $\{\mu_{1,1},\mu_{2,0},\mu_{2,2}\}$, target SINR T, and the threshold of coverage probability of indoor macro users p_c^M, and consider the outage throughput as the objective function of a FBS, the altruistic time mute with power control scheme can be formulated as follows.

$$\begin{aligned} &\max_{\beta_2,P_2} \quad \beta_2 C_{out}^{FS}(P_2) \\ &\text{s.t.} \begin{cases} (1-\beta_2)\mathbb{P}^{MD} + \beta_2\mathbb{P}^{MS}(P_2) \geq p_c^M, \\ 0 \leq P_2 \leq 1, \\ 0 \leq \beta_2 \leq 1. \end{cases} \end{aligned} \tag{33}$$

Before completely solving the optimization problem in (33), we first introduce a Lemma as follows.

Lemma 1: Assume that the pathloss exponents $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$. Define $g_1(\beta_2,P_2) = (1-\beta_2)\mathbb{P}_\delta^{MD} + \beta_2\mathbb{P}_{\delta,\text{apx}}^{MS}(P_2) - p_c^M$. If $g_1(1,1) = \mathbb{P}_{\delta,\text{apx}}^{MS}(1) - p_c^M < 0$, then the optimal solutions to the optimization problem in (33) are

$$(\beta_2,P_2) = \begin{cases} \left(\frac{\mathbb{P}_\delta^{MD}-p_c^M}{\mathbb{P}_\delta^{MD}-\mathbb{P}_{\delta,\text{apx}}^{MS}(1)}, 1\right), & \text{if } g_2(1) > 0, \\ (1,\bar{P}_2), & \text{if } g_2(\bar{P}_2) < 0, \\ \left(\frac{\mathbb{P}_\delta^{MD}-p_c^M}{\mathbb{P}_\delta^{MD}-\mathbb{P}_{\delta,\text{apx}}^{MS}(\hat{P}_2)}, \hat{P}_2\right), & \text{if } g_2(\hat{P}_2) = 0. \end{cases} \tag{34}$$

where $\bar{P}_2$ is defined in (32), and $g_2(P_2)$ is a discriminant function, defined as follows.

$$g_2(P_2) = \frac{\partial C_{out}^{FS}}{\partial P_2}\triangle\mathbb{P}_\delta^M(P_2) + C_{out}^{FS}(P_2)\frac{\partial \mathbb{P}_{\delta,\text{apx}}^{MS}}{\partial P_2}. \tag{35}$$

where $\triangle\mathbb{P}_\delta^M(P_2) = \mathbb{P}_\delta^{MD} - \mathbb{P}_{\delta,\text{apx}}^{MS}(P_2)$, and $\hat{P}_2$ is the solution to $g_2(P_2) = 0$.

The function $g_2(P_2)$ decides the sign of the derivative of $C_{out}^{FS}(P_2)/\triangle\mathbb{P}_\delta^M(P_2)$ with respect to P_2. The numerator $C_{out}^{FS}(P_2)$ is the outage throughput of a FBS when $\beta_2 = 1$, and the denominator $\triangle\mathbb{P}_\delta^M(P_2)$ represents the difference of the coverage probability of macro users caused by time muting. As we can see later from the proof of Lemma 1, this ratio is equivalent to the objective function of a FBS. The $g_2(P_2) > 0$ implies that $C_{out}^{FS}(P_2)/\triangle\mathbb{P}_\delta^M(P_2)$ grows with the increase of P_2. Hence, $P_2 = 1$ is the optimal solution. And $g_2(P_2) < 0$ indicates that $C_{out}^{FS}(P_2)/\triangle\mathbb{P}_\delta^M(P_2)$ decreases as P_2 grows, and thus $P_2 = \bar{P}_2$ is the optimal solution. $P_2 = \hat{P}_2$ is the critical point of the objective function.

Proof: The function $g_1(\beta_2,P_2)$ represents the first constraint in (33). When $g_1(1,1) < 0$, the optimal solution is achieved when $g_1(\beta_2, P_2) = 0$, i.e.,

$$\beta_2 = \frac{\mathbb{P}_\delta^{MD} - p_c^M}{\mathbb{P}_\delta^{MD} - \mathbb{P}_{\delta,\text{apx}}^{MS}(P_2)}. \tag{36}$$

Substituting (36) into (33), we can rewrite (33) as a single variable optimization problem:

$$\min_{P_2} \quad -\frac{C_{out}^{FS}(P_2)}{\triangle \mathbb{P}_\delta^M(P_2)}$$
$$\text{s.t.} \begin{cases} 0 \le P_2 \le 1, \\ \mathbb{P}_{\delta,\text{apx}}^{MS}(P_2) \le p_c^M. \end{cases} \tag{37}$$

The Lagrange function of (37) is

$$L = -\frac{C_{out}^{FS}(P_2)}{\triangle \mathbb{P}_\delta^M(P_2)} - \omega_1(-P_2+1) - \omega_2(-\mathbb{P}_\delta^{MS}(P_2)+p_c^M) - \omega_3 P_2. \tag{38}$$

The KKT conditions of (37) is,

$$\begin{cases} \frac{\mathrm{d}L}{\mathrm{d}P_2} = -\frac{g_2(P_2)}{\left(\triangle \mathbb{P}_\delta^M(P_2)\right)^2} + \omega_1 + \omega_2 \frac{\mathrm{d}\mathbb{P}_{\delta,\text{apx}}^{MS}}{\mathrm{d}P_2} - \omega_3 = 0, \\ \omega_1(-P_2+1) = 0, \\ \omega_2(-\mathbb{P}_{\delta,\text{apx}}^{MS}(P_2) + p_c^M) = 0, \\ \omega_3 P_2 = 0, \\ -P_2 + 1 \ge 0, \\ -\mathbb{P}_c^{MS}(P_2) + p_c^M \ge 0, \\ P_2 \ge 0. \end{cases} \tag{39}$$

By solving (39), we have:

$$(P_2, \omega_1, \omega_2, \omega_3)$$
$$= \begin{cases} \left(1, \frac{g_2(1)}{\left(\triangle \mathbb{P}_\delta^M(1)\right)^2}, 0, 0\right), & \text{if } g_2(1) > 0, \\ \left(\bar{P}_2, 0, \frac{g_2(\bar{P}_2)}{\left(\triangle \mathbb{P}_\delta^M(\bar{P}_2)\right)^2} / \frac{\mathrm{d}\mathbb{P}_{\delta,\text{apx}}^{MS}}{\mathrm{d}P_2}, 0\right), & \text{if } g_2(\bar{P}_2) < 0, \\ (\hat{P}_2, 0, 0, 0), & \text{if } g_2(\hat{P}_2) = 0, \\ \left(0, 0, 0, -\frac{g_2(0)}{\left(\triangle \mathbb{P}_\delta^M(0)\right)^2}\right), & \text{if } g_2(0) < 0. \end{cases} \tag{40}$$

Since $P_2 = 0$ is not an efficient solution for FBS, we take the first three solutions into account. By combining (36) and (40), we have the desired structure in Lemma 1. ■

From (34), we can see that there are three possible solutions to the optimization problem (33): the first one is time mute without power control, the second solution is the same as power control scheme, and the third one is time mute with power control. The optimal solution depends on the value of $g_2(P_2)$. In the following Lemma 2, a sufficient condition is given to guarantee $g_2(P_2) > 0$.

Lemma 2: Assume that $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$. If $\mathbb{P}_{\delta,\text{apx}}^{MS}(1) < \frac{k_1}{1+k_1}\mathbb{P}_\delta^{MD}$, then $g_2(P_2) > 0$, where

$$k_1 = \pi R^2 \lambda_1 \frac{(\mu_{1,1}/\mu_{2,0}T)^{2/\delta}}{\text{sinc}(2/\delta)}. \tag{41}$$

Proof:

$$g_2(P_2) = \frac{\partial C_{out}^{FS}}{\partial P_2} \triangle \mathbb{P}_\delta^M(P_2) + C_{out}^{FS}(P_2) \frac{\partial \mathbb{P}_{\delta,\text{apx}}^{MS}}{\partial P_2}$$
$$= \log(1+T) \exp\left(-k_1 P_2^{-2/\delta}\right)$$
$$\times \left(\frac{2k_1}{\delta} P_2^{-2/\delta-1} \triangle \mathbb{P}_\delta^M(P_2) + \frac{\partial \mathbb{P}_{\delta,\text{apx}}^{MS}}{\partial P_2}\right). \tag{42}$$

The sign of $g_2(P_2)$ is determined by the third term, $(\frac{2k_1}{\delta} P_2^{-2/\delta-1} \triangle \mathbb{P}_\delta^M(P_2) + \frac{\partial \mathbb{P}_{\delta,\text{apx}}^{MS}}{\partial P_2})$, which we denote as $\bar{g}_2(P_2)$.

Substituting (20) into $\bar{g}_2(P_2)$, and denoting $\rho(\kappa T, \delta)$ as ρ and

$$k_2(v) = \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\delta/2} R^{-\delta} \tag{43}$$

for short, we have

$$\bar{g}_2(P_2) = \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\pi\lambda_1 v(1+\rho)\right) k_2(v)}{\left(1+P_2 k_2(v)\right)^2}$$
$$\times \left(\frac{2k_1}{\delta} P_2^{-2/\delta}\left(1+P_2 k_2(v)\right) - 1\right) \mathrm{d}v. \tag{44}$$

From (44), we can see that $\bar{g}_2(P_2) > 0$, when $P_2 \le (2k_1/\delta)^{\frac{\delta}{2}}$.

On the other hand, $\bar{g}_2(P_2)$ can also be expressed as

$$\bar{g}_2(P_2) = \frac{2k_1}{\delta} P_2^{-2/\delta-1} \left(\mathbb{P}_\delta^{MD} - \mathbb{P}_{\delta,\text{apx}}^{MS}(P_2)\right)$$
$$- \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\pi\lambda_1 v(1+\rho)\right) k_2(v)}{\left(1+P_2 k_2(v)\right)^2} \mathrm{d}v$$
$$= \frac{2}{\delta P_2}\left(\left(k_1 P_2^{-2/\delta} - \left(k_1 P_2^{-2/\delta}+1\right)\mathbb{P}_c^{MS}(P_2)\right.\right.$$
$$\left. + \pi\lambda_1 \int_{v>0} \frac{\exp\left(-\pi\lambda_1 v(1+\rho)\right)}{1+P_2 k_2(v)} \pi\lambda_1(1+\rho) v \, \mathrm{d}v\right)$$
$$> \frac{2}{\delta P_2}\left((k_1 P_2^{-2/\delta} \mathbb{P}_\delta^{MD} - (k_1 P_2^{-2/\delta}+1)\mathbb{P}_{\delta,\text{apx}}^{MS}(P_2)\right). \tag{45}$$

Define $\tilde{g}_2(P2) = k_1 P_2^{-2/\delta} \mathbb{P}_\delta^{MD} - (k_1 P_2^{-2/\delta}+1)\mathbb{P}_{\delta,\text{apx}}^{MS}(P_2)$. Fig. 2 illustrates the relationship between $\tilde{g}_2(P2)$ and P_2 when $P_2 > (2k_1/\delta)^{\frac{\delta}{2}}$. From the expression of $\tilde{g}_2(P2)$, we know that it is mainly affected by P_2, wall penetration L_{ow} and target SINR T. As we can see from Fig. 2, when $P_2 > (\frac{2k_1}{\delta})^{\frac{\delta}{2}}$, $\tilde{g}_2(P2)$ is a decreasing function of P_2, regardless of wall penetration.

Since $\tilde{g}_2(P2)$ is a decreasing function of P_2 and $\mathbb{P}_{\delta,\text{apx}}^{MS}(1) < \frac{k_1}{1+k_1}\mathbb{P}_\delta^{MD}$, we have $\tilde{g}_2(P_2) \ge \tilde{g}_2(1) > 0$, when $1 \ge P_2 > (2k_1/\delta)^{\frac{\delta}{2}}$. ■

The condition $\mathbb{P}_{\delta,\text{apx}}^{MS}(1) < \frac{k_1}{1+k_1}\mathbb{P}_\delta^{MD}$ can also be written as $\triangle \mathbb{P}_\delta^M(1) > 1/k_1 \mathbb{P}_{\delta,\text{apx}}^{MS}(1)$. This condition means that the improvement of coverage probability of macro users caused

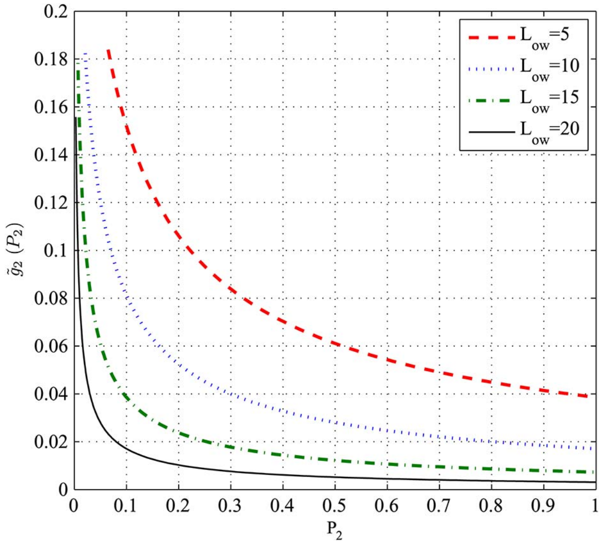

Fig. 2. $\tilde{g}_2(P2)$ with regard to P_2 and wall penetration (when $P_2 > (\frac{2k_1}{\delta})^{\frac{\delta}{2}}$), $\delta = 4$, T = 3 dB.

by closing the dominant FBS is greater than $1/k_1$ times of $\mathbb{P}^{MS}_{\delta,\text{apx}}(1)$. Based on Lemma 1 and Lemma 2, the optimal solutions of altruistic time mute with power control scheme for indoor environment are given in the following Theorem 2.

Theorem 2: Assume that $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$. If $\triangle\mathbb{P}^M_\delta(1) > 1/k_1\mathbb{P}^{MS}_{\delta,\text{apx}}(1)$, the optimal solutions of (33) are

$$(\beta_2, P_2) = \begin{cases} (1,1), & \text{if } \mathbb{P}^{MS}_{\delta,\text{apx}}(1) \geq p^M_c \\ \left(\frac{\mathbb{P}^{MD}_\delta - p^M_c}{\mathbb{P}^{MD}_\delta - \mathbb{P}^{MS}_{\delta,\text{apx}}(1)}, 1\right), & \text{if } \mathbb{P}^{MS}_{\delta,\text{apx}}(1) < p^M_c. \end{cases} \quad (46)$$

Proof:

1) If $g_1(1,1) \geq 0$

Since the objective function is an increasing function of both β_2 and P_2, (1,1) is the optimal solution.

2) If $g_1(1,1) < 0$

From Lemma 2, we have $g_2(P_2) > 0$, when $\mathbb{P}^{MS}_{\delta,\text{apx}}(1) < \frac{k_1}{1+k_1}\mathbb{P}^{MD}_\delta$. Then from Lemma 1, $g_2(P_2) > 0$ implies that $(\beta_2, P_2) = (\frac{\mathbb{P}^{MD}_\delta - p^M_c}{\mathbb{P}^{MD}_\delta - \mathbb{P}^{MS}_{\delta,\text{apx}}(1)}, 1)$ is the optimal solution of (33). ■

The condition $\triangle\mathbb{P}^M_\delta(1) > 1/k_1\mathbb{P}^{MS}_{\delta,\text{apx}}(1)$ is a sufficient condition that makes time mute without power control the optimal solution for FBS in altruistic transmission. In Section VI, we will examine the environment where this condition is satisfied.

V. Interference Coordination Games

In previous section, we consider the situation where a FBS altruistically guarantees the coverage probability of nearby macro users as required by the operator of MBSs. In this section, we assume that a FBS is rational and capable of determining the interference leakage to the macro users, and the operator pays for the rate loss of FUEs. Since the FBS and MBSs share channels, they generally belong to the same operator. Each month, for example, the operator charges a FBS for using the core network. The fee is the least if the FBS is in altruistic mode, and the highest if the FBS is in incoordinate mode. For each period, the fee is initialized to the highest, and decreases according to the rate loss of its FUEs and the reward from the operator. For simplicity, we assume that $\delta_1 = \delta_2 = \delta$, and utilize the approximate expressions of coverage probability of both MUEs and FUEs in this section.

A. Power Control via Pricing Game

Suppose that a FBS sets the transmit power to P_2, and the operator of MBSs pays for the rate loss of FUEs with a unit price η_F. Let player 1 be the operator, and player 2 be the owner of the FBS. Given the player set $\Omega = \{1,2\}$, we define the utility of player i as

$$v_1(\Omega, \eta_F, P_2) = \alpha_M \log(1+T)\mathbb{P}^{MS}_{\delta,\text{apx}}(P_2) - \eta_F\left(C^{FS}_{out}(1) - C^{FS}_{out}(P_2)\right) \quad (47)$$

$$v_2(\Omega, \eta_F, P_2) = -\alpha_F C^{FS}_{out}(1) + \alpha_F C^{FS}_{out}(P_2) + \eta_F\left(C^{FS}_{out}(1) - C^{FS}_{out}(P_2)\right) \quad (48)$$

where α_M and α_F are the unit prices of outage throughput that operator charges MUEs and the owner of the FBS respectively. The utility of player 1 consists of two terms, where the first term $\alpha_M \log(1+T)\mathbb{P}^{MS}_{\delta,\text{apx}}(P_2)$ is the fee that operator charges MUEs for providing the service, and the second term $\eta_F(C^{FS}_{out}(1) - C^{FS}_{out}(P_2))$ is the cost that operator pays for the help of the FBS. The utility of player 2 is composed of three terms, where the first term $\alpha_F C^{FS}_{out}(1)$ is the initial fee that the owner of FBS pays to the operator, the second term $\alpha_F C^{FS}_{out}(P_2)$ is the benefit from transmission, and the third term $\eta_F(C^{FS}_{out}(1) - C^{FS}_{out}(P_2))$ is the reward from operator.

Assume that the owner of a FBS will not let the outage throughput be lower than a threshold c_F, no matter how much the operator pays.

Given the BSs density of two tiers $\{\lambda_1, \lambda_2\}$, the set of channel gain $\{\mu_{1,1}, \mu_{2,0}, \mu_{2,2}\}$, target SINR T and FBS outage throughput threshold c_F, the interference coordination between the operator and owner of a FBS can be formulated as a strategic game $\mathcal{G}_P$ [18]:

Player 1:

$$\max_{\eta_F} \quad \alpha_M \log(1+T)\mathbb{P}^{MS}_\delta(P_2) - \eta_F\left(C^{FS}_{out}(1) - C^{FS}_{out}(P_2)\right)$$
$$\text{s.t.} \quad \eta_F \geq 0. \quad (49)$$

Player 2:

$$\max_{P_2} \; -\alpha_F C^{FS}_{out}(1) + \alpha_F C^{FS}_{out}(P_2) + \eta_F\left(C^{FS}_{out}(1) - C^{FS}_{out}(P_2)\right)$$
$$\text{s.t.} \begin{cases} C^{FS}_{out}(P_2) \geq c_F, \\ 0 \leq P_2 \leq 1. \end{cases}$$

The strategies of player 1 and player 2 are η_F and P_2 respectively. The following Theorem shows the NEs of $\mathcal{G}_P$.

Theorem 3: The Nash equilibria of $\mathcal{G}_P$ are

$$(\eta_F, P_2) = \begin{cases} (\alpha_F, \tilde{P}_2), \\ (\eta_F < \alpha_F, 1). \end{cases} \tag{50}$$

where $\tilde{P}_2$ is the solution to $\mathbb{P}^{FS}_{\delta,\text{apx}}(P_2)\log(1+T) = c_F$, and can be written by the following closed form,

$$\tilde{P}_2 = \left(\frac{k_1}{-\log\left(\frac{c_F}{\log(1+T)}\right)} \right)^{\delta/2}. \tag{51}$$

Proof: Assume that the strategies of the players are initialized to $(\eta_F^{(1)}, P_2^{(1)} > \tilde{P}_2)$, and $\tilde{P}_2$ is defined in (51).

If $\eta_F^{(1)} > \alpha_F$, then $v_2(\Omega, \eta_F, P_2)$ becomes a decreasing function of P_2. Hence, player 2 deviates and chooses $P_2 = \tilde{P}_2$, leading to $(\eta_F^{(2)}, P_2^{(2)}) = (\eta_F^{(1)}, \tilde{P}_2)$. Then for player 1, his utility $v_1(\Omega, \eta_F, P_2)$ becomes a decreasing function of η_F. Hence, player 1 will change his strategy and set $\eta_F^{(3)} = \alpha_F$, leading to $(\eta_F^{(3)}, P_2^{(3)}) = (\alpha_F, \tilde{P}_2)$. After that the two players will not deviate, and $(\alpha_F, \tilde{P}_2)$ is a NE of game $\mathcal{G}_P$.

If $\eta_F^{(1)} < \alpha_F$ and $P_2^{(1)} < 1$, then $v_2(\Omega, \eta_F, P_2)$ becomes an increasing function of P_2, and player 2 will deviate and choose the strategy $P_2^{(2)} = 1$, leading to $(\eta_F^{(2)}, P_2^{(2)}) = (\eta_F^{(1)}, 1)$. After that the two players will not deviate, and $(\eta_F < \alpha_F, 1)$ is also a NE of game $\mathcal{G}_P$. ■

From Theorem 3, we know that there are two NEs of game $\mathcal{G}_P$. When a MBS detects a macro user suffering strong interference from a FBS, the operator provides high price to persuade the FBS to help the macro user, and the two players achieve the NE $(\eta_F, P_2) = (\alpha_F, \tilde{P}_2)$, where the operator rewards and charges the owner of FBS with the same unit price $\eta_F = \alpha_F$, and the FBS deceases the outage throughput to the threshold c_F. To guarantee that the operator benefits from FBS coordination, i.e., $v_1(\Omega, \alpha_F, \tilde{P}_2) > v_1(\Omega, \eta_F < \alpha_F, 1)$, the operator should design α_M and α_F with the constraint $\alpha_M(\mathbb{P}^{MS}_{\delta,\text{apx}}(\tilde{P}_2) - \mathbb{P}^{MS}_{\delta,\text{apx}}(1)) > \alpha_F(\mathbb{P}^{FS}_{\delta,\text{apx}}(1) - \mathbb{P}^{FS}_{\delta,\text{apx}}(\tilde{P}_2))$.

If a FBS does not cause serve interference to macro users, the reward price is low, leading to the NE $(\eta_F, P_2) = (\eta_F < \alpha_F, 1)$.

B. Time Mute via Pricing Game

We assume that a FBS adjusts active time ratio β_2 to control interference to indoor MUEs, and the operator pays for the rate loss of FUEs with a unit price η_F. Let player 1 be the operator, and player 2 be the owner of a FBS. Given the player set $\Omega = \{1, 2\}$, we define the utility of player i as

$$v_1(\Omega, \eta_F, \beta_2) = \alpha_M \log(1+T)\mathbb{P}^M_\delta(1) - \eta_F(1-\beta_2)C^{FS}_{out}(1), \tag{52}$$

$$v_2(\Omega, \eta_F, \beta_2) = -\alpha_F C^{FS}_{out}(1) + \alpha_F \beta_2 C^{FS}_{out}(1) + \eta_F(1-\beta_2)C^{FS}_{out}(1), \tag{53}$$

where $\mathbb{P}^M_\delta(P_2) = (1-\beta_2)\mathbb{P}^{MD}_\delta + \beta_2 \mathbb{P}^{MS}_{\delta,\text{apx}}(P_2)$ is the average coverage probability of macro tier. The utility of player 1 consists of two terms, where the first term $\alpha_M \log(1+T)\mathbb{P}^M_\delta(1)$ is the fee that operator charges MUEs for providing the service, and the second term $\eta_F(1-\beta_2)C^{FS}_{out}(1)$ is the cost that operator pays for the help of FBS. The utility of player 2 is composed of three terms, where the first term $\alpha_F C^{FS}_{out}(1)$ is the initial fee that the owner of FBS pays to the operator, the second term $\alpha_F \beta_2 C^{FS}_{out}(1)$ is the benefit from transmission, and the third term $\eta_F(1-\beta_2)C^{FS}_{out}(1)$ is the reward from operator.

Assume that player 2 maintains a minimal outage throughput c_F, regardless of the reward from player 1. Given the BSs density of two tiers $\{\lambda_1, \lambda_2\}$, the set of channel gain $\{\mu_{1,1}, \mu_{2,0}, \mu_{2,2}\}$, target SINR T and FBS outage throughput threshold c_F, the two-tier interference coordination can be formulated as a strategic game $\mathcal{G}_T$:

Player 1:

$$\max_{\eta_F} \quad \alpha_M \log(1+T)\mathbb{P}^M_\delta(1) - \eta_F(1-\beta_2)C^{FS}_{out}(1) \quad \text{s.t.} \quad \eta_F > 0 \tag{54}$$

Player 2:

$$\max_{\beta_2} \quad -\alpha_F C^{FS}_{out}(1) + \alpha_F \beta_2 C^{FS}_{out}(1) + \eta_F(1-\beta_2)C^{FS}_{out}(1)$$

$$\text{s.t.} \begin{cases} \beta_2 C^{FS}_{out}(1) \geq c_F \\ 0 < \beta_2 \leq 1 \end{cases}$$

The strategies of player 1 and player 2 are η_F and β_2 respectively. The NEs of $\mathcal{G}_T$ are shown in Theorem 4.

Theorem 4: The Nash equilibria of $\mathcal{G}_T$ are

$$(\eta_F, \beta_2) = \begin{cases} \left(\alpha_F, \frac{c_F}{C^{FS}_{out}(1)}\right), \\ (\eta_F < \alpha_F, 1). \end{cases} \tag{55}$$

The proof of Theorem 4 is similar to that of Theorem 3. There are also two NEs of $\mathcal{G}_T$. If a macro user detects a dominant interfering FBS, the operator is willing to highly reward the FBS, and the two players achieve the NE $(\eta_F, \beta_2) = (\alpha_F, c_F/C^{FS}_{out}(1))$. This NE represents that the operator rewards and charges the owner of FBS with the same unit price $\eta_F = \alpha_F$, and the FBS deceases the outage throughput to the threshold c_F. To guarantee that the operator benefits from FBS coordination, i.e., $v_1(\Omega, \alpha_F, c_F/C^{FS}_{out}(1)) > v_1(\Omega, \eta_F < \alpha_F, 1)$, the operator should design α_M and α_F satisfying the constraint $\alpha_M \triangle \mathbb{P}^M_\delta(1) > \alpha_F \mathbb{P}^{FS}_\delta(1)$.

For a FBS that has not been identified as dominant interferer by any macro user, the reward price is low, leading to the NE $(\eta_F, P_2) = (\eta_F < \alpha_F, 1)$.

VI. Simulation Results

In this section, the aforementioned theorems are verified through Monte Carlo simulations. In the simulation, the observed macro user is located at the center of the observed area, and the FBS inside the same room is in the northeast of it, at a distance of R. The number of macro/femto BSs are random variables generated by Poisson distribution, and the locations of the macro/femto BSs are uniformly distributed in the observed area. Fig. 3 shows a snapshot of the locations of the observed macro user and BSs. The main parameters of simulations are set according to Table I.

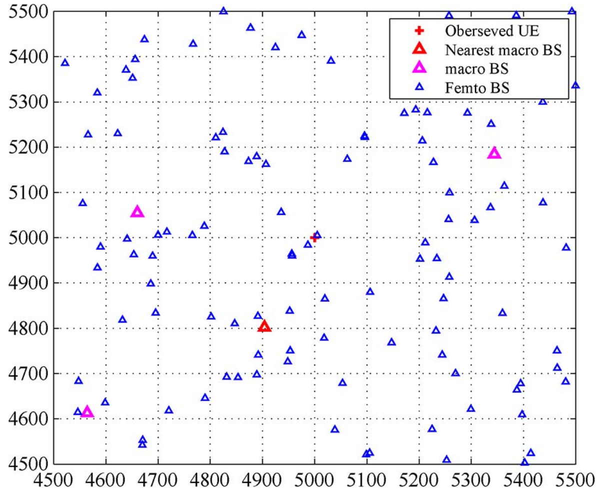

Fig. 3. Snapshot of the locations of the observed macro user and BSs within the area of [4500 m 5500 m] × [4500 m 5500 m].

TABLE I
PARAMETERS SETUP

Parameter	Value
Observed area	10×10 km^2
Room area	10×10 m^2
Distance between a user and FBS in the same room	$R = 7.07$ m
BSs Density	4/km^2 (macros), 100/km^2(femtos)
Max Tx Power	46 dBm (macros), 20 dBm (femtos)
Antennas	1Tx, 1Rx (both macros and femtos)
Antenna gains	14 dB (macros), 5 dB (femtos)
Bandwidth	10 MHz (600 subcarriers)
Pathloss exponent	δ =3,4
Pathloss	$L = 15.3 + 10\delta \log_{10}(r)$, r in m
External wall penetration loss	$L_{ow} = 5$ dB, 20 dB
Intra-tier 1 leakage	$\kappa = 0.1 \sim 0.7$

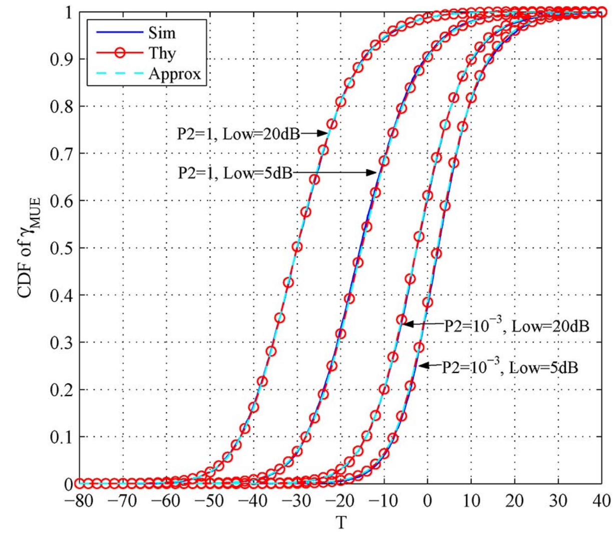

Fig. 4. CDF of γ_{MUE} with regard to wall penetration and transmit power of FBS, $\delta = 3$, and $\kappa = 0.3$.

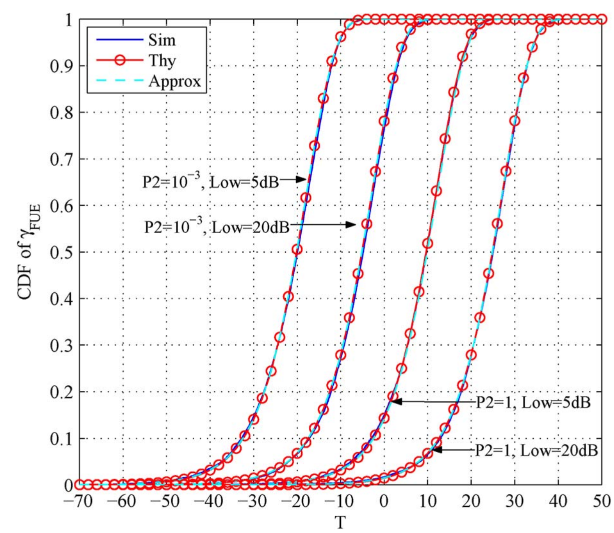

Fig. 5. CDF of γ_{FUE} with regard to wall penetration and transmit power of FBS, $\delta = 3$, and $\kappa = 0.3$.

In Fig. 4, we verify the coverage probability of indo or macro users on shared channel. Since the coverage probability is the complementary CDF of SINR, here, we use CDF to indirectly prove it. The simulation results are collected from 10 000 random realizations of BS locations and channels. We simulate different external wall penetration $L_{ow} = \{5 \text{ dB}, 20 \text{ dB}\}$ and normalized transmit power of FBS $P_2 = \{10^{-3}, 1\}$, with $\delta = 3$, $\kappa = 0.3$. "Thy" and "Approx" stand for the derived coverage probability in (19) and (20) respectively. In Fig. 4, we can see that both the exact and approximate curves fit well with the simulation results. The reduction of P_2 makes the CDF move right, due to the mitigation of interference from femto tier. As L_{ow} grows, the CDF curve moves left, since the signal from serving macro BS gets weaker. The perfect match of the approximate curve implies that the interference from FBSs in $\Phi_2 \setminus \tilde{b}$ can be neglected. The reason is that all rooms are separated in our model, and the signal from a FBS located in other room undergoes two external walls before it reaches the observed macro user, and thus is very weak.

In Fig. 5, the coverage probability of femto users on shared channel is verified. We make 10 000 random realizations of BS locations and channels to generate the simulation results. Different external wall penetration $L_{ow} = \{5 \text{ dB}, 20 \text{ dB}\}$ and normalized transmit power of femto BSs $P_2 = \{10^{-3}, 1\}$ are evaluated, with $\delta = 3$ and $\kappa = 0.3$. "Thy" and "Approx" represent the derived coverage probability of femto users in (27) and (28) respectively. As can be seen from Fig. 5, the theoretic curves coincide with the simulation ones. The reduction of P_2 makes the CDF move left, since signal from serving femto BS gets weaker. The increase of L_{ow} let the CDF curve move right, due to the reduction of interference from outside. The reason that the approximate curves match with the exact ones is the same as the explanation of Fig. 4.

In Fig. 6, we evaluate the sufficient condition in Theorem 2. Specifically, we compare the values of $\triangle\mathbb{P}_{\delta}^{M}(1)$ and $1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$. If $\triangle\mathbb{P}_{\delta}^{M}(1) > 1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$, then the condition in Theorem 2 is satisfied. We simulate different intra-tier 1 leakage $\kappa = \{0.3, 0.5\}$, pathloss exponent $\delta = \{3, 4\}$, and target SINR $T \in [-2, 10]$, with external wall penetration $L_{ow} = 20$ dB. Here, we do not consider the case $\kappa > 0.5$, since the intra-tier 1 interference is reduced by soft frequency reuse [19],

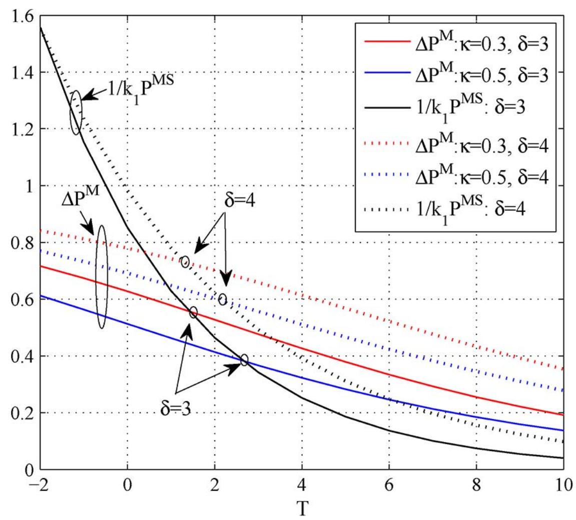

Fig. 6. $\triangle\mathbb{P}_{\delta}^{M}(1)$ and $1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$ with regard to target SINR, $L_{ow} = 20$ dB.

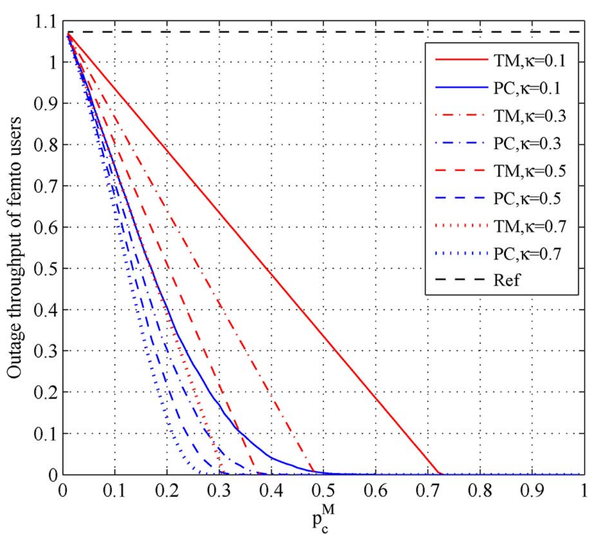

Fig. 7. Outage throughput of FUEs in altruistic mode with regard to p_c^M. $T =$ 3 dB, and $L_{ow} = 20$ dB.

coordinated multi-point transmission/reception [20], or other techniques, in order to achieve inter-cell quasi-orthogonality in Long Term Evolution-Advanced (LTE-Advanced) system [21]. For the case $\kappa > 0.5$, the benefit from FBS reducing transmit power or active time ratio is also limited. It's better to reduce the intra-tier 1 interference before asking FBS for help. As we can see from Fig. 6, both $\triangle\mathbb{P}_{\delta}^{M}(1)$ and $1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$ decrease as T grows. With the decreasing of T, $1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$ can be larger than 1, but $\triangle\mathbb{P}_{\delta}^{M}(1)$ is always less than 1, since it is the difference between two probability. The cross point of the two curves is between 1 dB and 3 dB. This indicates that when $T \geq 3$ dB, the sufficient condition in Theorem 2 is satisfied, and time mute without power control is the optimal solution for FBS to mitigate interference to macro users. When $T < 3$ dB, especially $1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1) > 1$, $g_2(P_2)$ may be smaller than 0, and power control can be the optimal solution.

In Fig. 7, we compare the performance of altruistic power control (PC) and time mute (TM) schemes with regard to the target coverage probability of indoor MUEs. Target SINR is set to $T = 3$ dB, and $L_{ow} = 20$ dB. Table II demonstrates the optimal operating parameters for PC and TM schemes when $\kappa = 0.1$. As can be seen from Fig. 7, the increase of p_c^M leads to the decease of outage throughput of FUEs. The reason is that a FBS has to further decrease P_2 or β_2, in order to guarantee a higher coverage probability of indoor macro user. Fig. 6 shows that the condition $\triangle\mathbb{P}_{\delta}^{M}(1) > 1/k_1\mathbb{P}_{\delta,\text{apx}}^{MS}(1)$ is satisfied when $T = 3$ dB. According to Theorem 2, TM scheme performs better than PC scheme when the condition is satisfied. As we can see from Fig. 7, the curves of TM scheme achieve higher outage throughput for FUE, compared to those of PC scheme. From Table II, we can also see that P_2 of PC decreases more quickly than β_2 of TM. As a result, the simulation results coincide with Theorem 2. When $\mathbb{P}_{\delta}^{MD} < p_c^M$, operating parameters of FBS become zero, implying that there is no solution for FBS. In this case the coverage probability is limited by intra-tier 1 interference.

In Fig. 8, we demonstrate the dynamic processes to achieve the NEs of $\mathcal{G}_T$, where player 1 is the operator of MBSs and player 2 is the owner of a FBS that causes strong downlink interference to a nearby macro user. We assume that the target SINR T is a constant, $v_i/\log(1+T)$ is illustrated for simplicity. The unit prices for MUE and FUE are $\alpha_M = 2$, $\alpha_F = 1$ respectively, which guarantees that the operator benefits from FBS coordination. The active time ratio associated with c_F is $\beta_{2,th} = c_F/C_{out}^{FS}(1) = 0.4$, and $\kappa = 0.2$, $\delta = 3$. The region surrounded by blue lines stands for the case failing to trigger coordination, while the area encompassed by red lines represents the coordinative case. The definitions of the boundaries are displayed in the legend. As we can seen from Fig. 8, there exist two NEs of $\mathcal{G}_T$. The solid and dash lines represent the processes to achieve the NEs in the coordinative and incoordinate cases respectively. It takes three steps to achieve the NE in coordinative case and two steps to achieve the NE in incoordinate case. These processes are similar to the processes given in the proof of Theorem 3. The utilities of player 2 at the two NEs are both zero, but with different meanings. If the NE $(\eta_F, \beta_2) = (\eta_F < \alpha_F, 1)$ is obtained, the outage throughput of femto users is $C_{out}^{FS}(1)$, and the owner of the FBS pays the operator $\alpha_F C_{out}^{FS}(1)$ for using the core network; If the NE $(\eta_F, \beta_2) = (\alpha_F, \beta_{2,th})$ is achieved, the outage throughput of femto users is $\beta_{2,th}C_{out}^{FS}(1)$, and the operator only charges the owner of the FBS $\alpha_F\beta_{2,th}C_{out}^{FS}(1)$.

In Fig. 9, we evaluate the coverage probability of MUEs at coordinative NE of the interference coordination pricing games. Target SINR is set to $T = 3$ dB, and $L_{ow} = 20$ dB. From Fig. 9, we can see that as the outage throughput threshold c_F decreases, the coverage probability of MUEs grows. And the increase of intra-tier 1 leakage reduces $\mathbb{P}_{\delta}^{MD}$, and thus the growing speed of coverage probability of MUEs declines. However, compared to the PC scheme, TM scheme provides higher coverage probability for MUEs.

VII. Conclusion

In this paper, we considered the interference coordination in closed access femto and macro co-existent networks via

TABLE II
OPERATING PARAMETERS OF ALTRUISTIC FBSS, ($\kappa = 0.1$)

p_c^M	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
P_2 of PC	0.0153	0.0037	0.0014	0.0006	0.0002	0.0001	0.00001	0	0
β_2 of TM	0.8714	0.7316	0.5918	0.4521	0.3123	0.1725	0.0327	0	0

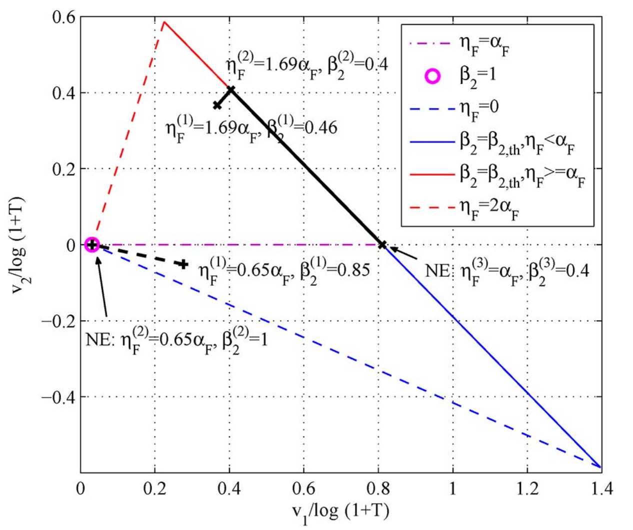

Fig. 8. Dynamic process to achieve the NEs of $\mathcal{G}_T$. The solid and dash lines represent the coordinative and incoordinate cases, respectively. $\alpha_M = 2$, $\alpha_F = 1$, $\beta_{2,th} = 0.4$, $\kappa = 0.2$, $\delta = 3$.

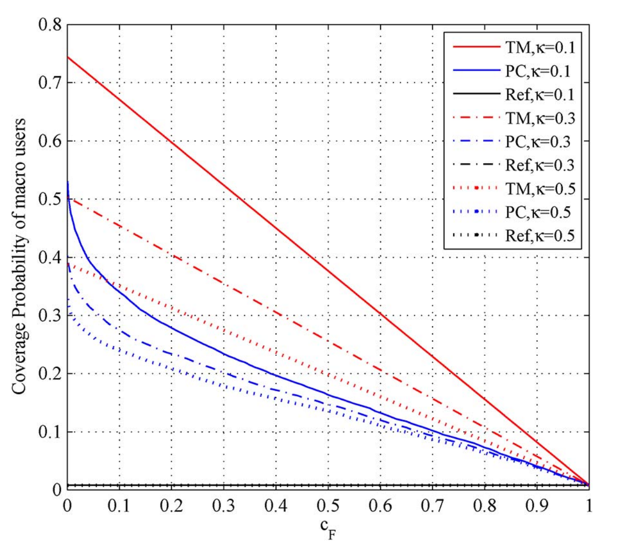

Fig. 9. Coverage probability of MUEs at coordinative NE of the interference coordination pricing games. $T = \{3$ dB, and $L_{ow} = 20$ dB.

game theory. The coverage probability of indoor MUEs and FUEs was derived based on the stochastic model, and the operating parameters of altruistic time mute and power control schemes were obtained. Considering the selfishness of FBSs, we used pricing games to formulate the two-tier interference coordination, and the coordinative NEs were achieved when the operator rewarded and charged a FBS with the same unit price, and the FBS decreased the outage throughput to the threshold. Simulation results showed that the intra-macro tier interference greatly impacted the performance of the two-tier interference coordination schemes, nevertheless, when $T \geq 3$ dB, the time mute scheme outshined power control scheme in dealing with the indoor MUE coverage problem.

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