Femto-macro Co-channel Interference Coordination via Pricing Game

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***Abstract*—Recently, intercell interference coordination in heterogenous networks attracts great attention. This paper presents an analytical framework to evaluate time mute scheme in closed access femto and macro co-existent networks. We use stochastic geometry to model the downlink scenario and derive the coverage probability of indoor macro users and femto users. Considering the selfishness of the owners of femtos, we formulate the two-tier interference coordination as pricing game, and obtain the closed-form of Nash equilibrium (NE). Simulation results demonstrate the influences of different parameters on the coverage probability of macro users achieved at the NE of the pricing game.**

***Index Terms*—Femto, Almost Blank Subframe, Game theory**

I. Introduction

The development of new cellular technologies and topologies are motivated by the rapid increase of mobile data activity. Femtocell [1] is one of the interesting trends of cellular evolution. A big challenge for femtocell deployments is the less predictable and more complicated intercell interference. In [2], the authors show that the overall interference conditions are not exacerbated when the femto base stations (FBSs) are open access and users select the strongest cells. However, when a macro user (MUE) gets close to a closed access FBS, it will see severe interference in the downlink. Due to the extremely poor channel condition, the user cannot connect to any cell and hence is in outage [3]. Different tools have been proposed to counter the coverage problem in OFDMA femtocells including power control [4], time mute [5], frequency partitioning [6], precoding [7], and subband scheduling [8]. An example of the time mute scheme is almost blank subframe (ABS), which has been proposed by 3GPP members to combat co-channel cross-tier interference in heterogeneous networks [9]. The rationale of time mute scheme in femtocell is muting some subframes of femto tier and scheduling the vulnerable macro users in these subframes. Hence, the channel conditions of the macro users are improved in these muting time slots.

To the best of our knowledge, only a few works in the literature study time mute scheme in femtocell. Simulation results of utilizing time mute are illustrated in [10][11]. There has been little work done on the theoretical analysis of time mute scheme. In [5], the authors studied the required number of ABS to guarantee the outage throughput of macro users. Different from [5] where the active time ratio is obtained from altruistic interference mitigation of FBSs. In this paper, we consider the case that both FBS and macro BS (MBS) are selfish, and the stable operating parameters of them are achieved by using game theory.

We first derive the coverage probability of indoor macro user and femto user in closed access femto and macro co-existent networks based on the stochastic geometry [12]. Secondly, we consider the case where a FBS is capable of deciding the interference leakage to the MUE according to the reward from the operator of MBSs. By formulating the two-tier interference coordination problem as pricing game, we obtain the closed-form Nash equilibrium (NE), which reveals the stable working parameter of FBS and the payment of the operator. Simulation results show the effects of different parameters on the coverage probability of MUE achieved at the NE of the pricing game.

The rest of this paper is outlined as follows. Section II introduces the system model based on stochastic geometry. In Section III, we derive the coverage probability of indoor macro user and femto user. In Section IV, the two-tier interference coordination problem is formulated as pricing game and closed-form of NE is given. Simulation results are demonstrated and discussed in Section V. Finally, concluding remarks are given in Section VI.

II. System Model

We consider a two-tier heterogenous network with MBSs and closed access FBSs in tier 1 and 2, respectively. The two tiers share the same physical transmission resources. Assume that each user can only be served by a BS belonging to its accessible tier, and We consider an arbitrary user anywhere in the network, and focus on the scenario where macro users are indoor, since macro users are most vulnerable to the interference from FBSs in this case.

The locations of the BSs in tier i are assumed to be given by a homogeneous Poisson Point Process (PPP)[12] Φ_i on the plane with intensity λ_i (units of BSs per m^2). We assume a single antenna transceiver at both BS and user, and the received power at a user located at a distance of r from a BS b of tier i is given by,

$$y_{i,b} = \frac{h_{i,b}}{r^{\delta_i}}, \quad (1)$$

where $\delta_i > 2$ is the pathloss exponent, $h_{i,b}$ is the attenuation in power due to fading on the link, and the effect of transmit power, antenna gain, etc. $\{h_{i,b}\}$ are independent distribution over all BSs in the two tiers, and for the sake of tractability, we do not model shadow fading, and assume that all links follow Rayleigh distribution, and $h_{i,b}$ obeys exponential distribution with $E[h_{i,b}] = \mu_i$ [2]. Denote $\{\mu_i\} = \{\mu_{1,1}, \mu_{2,0}, \mu_{2,2}\}$ as the set of channel gain, where $\mu_{i,k}$ is associated with the channel gain from a BS in tier i to the observed user, and k corresponds to the number of walls that the signal goes through.

III. Coverage Probability

In this section, we derive coverage probability of macro and femto users. Coverage probability is the probability that a user has a signal-to-interference-plus-noise (SINR) higher than an outage threshold.

A. Indoor Macro User

Denote $r_{i,b}$ as the distance from BS b in tier i to the observed user. Assume that a user is served by the nearest BS b_0 in its accessible tier i. Denote $r = r_{1,b_0}$ as the distance between the observed macro user and its serving macro BS. The SINR of a macro user on a subcarrier is given by,

$$\gamma_{MUE} = \frac{h_{1,b_0} r^{-\delta_1}}{Z_r}, \tag{2}$$

where Z_r is interference plus noise. As can be seen from Fig.1(a), for an indoor macro user, there exists a dominant interferer $\tilde{b}$, i.e., the femto BS located in the same room with the macro user. Assume that the distance between femto BS $\tilde{b}$ and the observed macro user $r_{2,\tilde{b}} = R$. Then,

$$Z_r = h_{2,\tilde{b}} R^{-\delta_2} + \kappa \sum_{b \in \Phi_1 \setminus b_0} h_{1,b} r_{1,b}^{-\delta_1} + \sum_{b \in \Phi_2 \setminus \tilde{b}} h_{2,b} r_{2,b}^{-\delta_2} + \sigma^2. \tag{3}$$

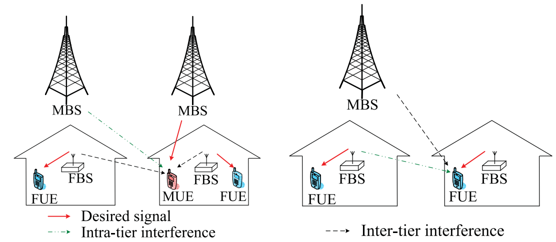
The first term in the right handside of (3) is the interference from FBS $\tilde{b}$. The second term represents the interference received from intra-tier Φ_1, where κ is the interference leakage coefficient of other MBSs. The third term stands for the interference from FBS $b \in \Phi_2 \setminus \tilde{b}$. The σ^2 is the variance of a zero-mean circularly symmetric complex Gaussian noise. With the probability density function (pdf) of r [13],

$$f_r(r) = e^{-\lambda_i \pi r^2} 2\pi\lambda_i r \tag{4}$$

and the assumption that pathloss exponents $\delta_1 = \delta_2 = \delta$, and noise is much smaller than the interference, we can derive the coverage probability of indoor MUE as follows in Theorem 1.

Theorem 1: Assume that $\delta_1 = \delta_2 = \delta$, and noise is much smaller than the interference, i.e. $\sigma^2/Z_r \to 0$, the coverage probability of an indoor macro user on the shared channel in the two-tier heterogenous network model is

$$\mathbb{P}_\delta^{MS} = \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\pi\lambda_1 v \big(1 + \rho(\kappa T, \delta) + \frac{\lambda_2}{\lambda_1} \frac{(\frac{\mu_{2,2}}{\mu_{1,1}} T)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\big)\right)}{1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\delta/2} R^{-\delta}} \mathrm{d}v \tag{5}$$



(a) Indoor macro user (b) Femto user

Fig. 1. Receiving signals of an indoor macro user and a femto user

where T is target SINR and

$$\rho(y, \delta) = y^{2/\delta} \int_{y^{-2/\delta}}^\infty \frac{1}{1 + u^{\delta/2}} \mathrm{d}u. \tag{6}$$

Due to page limitation, we show the proof in the supplementary information [14]. If we assume that the interference from FBSs $b \in \Phi_2 \setminus \tilde{b}$ can be neglected, then the coverage probability can be approximately written as:

$$\mathbb{P}_{\delta,\text{apx}}^{MS} = \pi\lambda_1 \int_0^\infty \frac{\exp\left(-\pi\lambda_1 v \big(1 + \rho(\kappa T, \delta)\big)\right)}{1 + \frac{\mu_{2,0}}{\mu_{1,1}} T v^{\delta/2} R^{-\delta}} \mathrm{d}v. \tag{7}$$

Assume that there is no interference leakage from FBSs on the muting subframes, and the coverage probability of indoor macro users on the dedicated channel is

$$\mathbb{P}_\delta^{MD} = \frac{1}{\big(1 + \rho(\kappa T, \delta)\big)}. \tag{8}$$

B. Femto User

For a femto user (FUE), its serving BS is the femto BS b_0 inside the same room. Since the area of a room is rather small compared to the observed area, for simplicity, we assume that the distance between any user (indoor MUE/FUE) and its nearest FBS is the same, i.e., $r_{2,b_0} = R$. Then, the SINR of the femto user on a subcarrier is,

$$\gamma_{FUE} = \frac{h_{2,b_0} R^{-\delta_2}}{Z_R}, \tag{9}$$

where Z_R is interference plus noise,

$$Z_R = \sum_{b \in \Phi_2 \setminus b_0} h_{2,b} r_{2,b}^{-\delta_2} + \sum_{b \in \Phi_1} h_{1,b} r_{1,b}^{-\delta_1} + \sigma^2. \tag{10}$$

Theorem 2: Assume that $\delta_1 = \delta_2 = \delta$, and $\sigma^2/Z_r \to 0$, the coverage probability of a femto user on the shared channel in the two-tier heterogenous network model is

$$\mathbb{P}_\delta^{FS} = \exp\left(-\pi R^2 \big(\lambda_2 \frac{(\mu_{2,2}/\mu_{2,0} T)^{2/\delta}}{\mathrm{sinc}(2/\delta_2)} + \lambda_1 \frac{(\mu_{1,1}/\mu_{2,0} T)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\big)\right). \tag{11}$$

The proof of Theorem 2 is similar to Theorem 1, and we omit the proof due to page limitation. If we assume that the interference from FBS $b \in \Phi_2 \setminus b_0$ can be neglected, the coverage probability can be further simplified as:

$$\mathbb{P}_{\delta,\text{apx}}^{FS} = \exp\big(-\pi R^2 \lambda_1 \frac{(\mu_{1,1}/\mu_{2,0} T)^{2/\delta}}{\mathrm{sinc}(2/\delta)}\big). \tag{12}$$

IV. TIME MUTE VIA PRICING GAME

We assume that there is only one dominant interfering FBS for each macro user, which will be proved to be true in Section V. Based on this assumption, each FBS sets individual operating parameters depending on the existence of nearby macro users. we assume that a FBS can determine the interference leakage to the macro users, and the operator of MBSs pay for the rate loss of a FUE. Since the FBS and MBSs share channels, they generally belong to the same operator. Each month, for example, the operator charges a FUE for using the core network. The fee is the least if the a FBS is in altruistic mode, and the highest if a FBS is noncooperative. For simplicity, we assume that $\delta_1 = \delta_2 = \delta$, and utilize the approximated expressions of coverage probability of both macro and femto users in this section.

Denote $\beta_2 (0 \leq \beta_2 \leq 1)$ as the active time ratio of a FBS. We assume that a FBS adjusts active time ratio β_2 to control interference to indoor MUEs, and the operator pays for the rate loss of a FUE with a unit price η_F. Let player 1 be the operator, and player 2 be the owner of a FBS. Given the player set $\Omega = \{1, 2\}$, we define the utility of player i as

$$v_1(\eta_F, \beta_2) = \alpha_M \log(1+T)\mathbb{P}_\delta^M - \eta_F(1-\beta_2)C_{out}^{FS}, \quad (13)$$

$$v_2(\eta_F, \beta_2) = -\alpha_F C_{out}^{FS} + \alpha_F \beta_2 C_{out}^{FS} + \eta_F(1-\beta_2)C_{out}^{FS}, \quad (14)$$

where $\mathbb{P}_\delta^M = (1-\beta_2)\mathbb{P}_\delta^{MD} + \beta_2 \mathbb{P}_{\delta,\text{apx}}^{MS}$ is the average coverage probability of MUE. $C_{out}^{FS} = \mathbb{P}_{\delta,\text{apx}}^{FS} \log(1+T)$ is the outage throughput of FUE. The utility of player 1 consists of two terms, where $\alpha_M \log(1+T)\mathbb{P}_\delta^M$ is fee that operator charges MUEs for providing the service, and $\eta_F(1-\beta_2)C_{out}^{FS}$ is the cost that operator pays for the help of the FBS. The utility of player 2 is composed of three terms, where $\alpha_F C_{out}^{FS}$ is the initial fee the owner of FBS pays to the operator, $\alpha_F \beta_2 C_{out}^{FS}$ is the benefit from transmission, and $\eta_F(1-\beta_2)C_{out}^{FS}$ is the reward from operator.

Assume that player 2 maintains a minimum outage throughput c_F, regardless of the reward of player 1. Given the BSs density of two tiers $\{\lambda_1, \lambda_2\}$, the set of channel gain $\{\mu_{1,1}, \mu_{2,0}, \mu_{2,2}\}$, target SINR T, the two-tier interference coordination can be formulated as a strategic game $\mathcal{G}_T$:
Player 1:

$$\max_{\eta_F} \quad \alpha_M \log(1+T)\mathbb{P}_\delta^M - \eta_F(1-\beta_2)C_{out}^{FS}$$
$$\text{s.t.} \quad \eta_F > 0 \quad (15)$$

Player 2:

$$\max_{\beta_2} \quad -\alpha_F C_{out}^{FS} + \alpha_F \beta_2 C_{out}^{FS} + \eta_F(1-\beta_2)C_{out}^{FS}$$
$$\text{s.t.} \begin{cases} \beta_2 C_{out}^{FS} \geq c_F \\ 0 < \beta_2 \leq 1 \end{cases}$$

The strategy of player 1 and player 2 are η_F and β_2 respectively. The NEs of $\mathcal{G}_T$ are shown in Theorem 3.

Theorem 3: The Nash equilibria of $\mathcal{G}_T$ are

$$(\eta_F, \beta_2) = \begin{cases} (\alpha_F, c_F/C_{out}^{FS}), \\ (\eta_F < \alpha_F, 1). \end{cases} \quad (16)$$

Proof: Denote $\beta_{2,th} = c_F/C_{out}^{FS}$. Assume that the strategies of the players are initialized to $(\eta_F^{(1)}, \beta_2^{(1)} > \beta_{2,th})$.

If $\eta_F^{(1)} > \alpha_F$, $v_2(\eta_F, \beta_2)$ is a decreasing function of P_2. Player 2 chooses $\beta_2 = \beta_{2,th}$, leading to $(\eta_F^{(2)}, \beta_2^{(2)}) = (\eta_F^{(1)}, \beta_{2,th})$. Then $v_1(\eta_F, \beta_2)$ becomes a decreasing function of η_F. Player 1 changes his strategy $\eta_F^{(3)} = \alpha_F$, leading to $(\eta_F^{(3)}, P_2^{(3)}) = (\alpha_F, \beta_{2,th})$. After that the two players will not deviate, and $(\alpha_F, \beta_{2,th})$ is a NE of game $\mathcal{G}_T$.

If $\eta_F^{(1)} < \alpha_F$, $v_2(\eta_F, \beta_2)$ becomes an increasing function of β_2. Player 2 chooses the strategy $\beta_2^{(2)} = 1$, leading to $(\eta_F^{(2)}, \beta_2^{(2)}) = (\eta_F^{(1)}, 1)$. After that the two players will not deviate, and $(\eta_F < \alpha_F, 1)$ is also a NE of game $\mathcal{G}_T$. ■

From Theorem 3, we know that there are two NEs of game $\mathcal{G}_T$. If a MUE detects a strong interfering FBS, the operator will highly reward the FBS, and the two players achieve the NE $(\alpha_F, c_F/C_{out}^{FS})$. This NE represents that the operator rewards and charges FBS with the same unit price $\eta_F = \alpha_F$, and the FBS deceases the outage throughput to the threshold c_F. To guarantee that the operator benefits from FBS coordination, i.e., $v_1(\alpha_F, c_F/C_{out}^{FS}) > v_1(\eta_F < \alpha_F, 1)$, operator should design α_M and α_F satisfying the constraint $\alpha_M(\mathbb{P}_\delta^{MD} - \mathbb{P}_{\delta,\text{apx}}^{MS}) > \alpha_F \mathbb{P}_\delta^{FS}$.

For a FBS that has not been identified as dominant interferer by a macro user, the reward price is low, leading to the NE $(\eta_F < \alpha_F, 1)$.

V. SIMULATION RESULTS

In this section, the aforementioned theorems are verified through Monte Carlo simulations. In the simulation, the observed user is located at the central of the observed area, and the femto BS inside the same room is located at northeast of it with a distance of R. The number of macro/femto BSs are random variables generated by poisson distribution, and the locations of the macro/femto BSs are uniformly distributed in the observed area. The main parameters of simulations are set according to Table I.

In Fig.2, we verify the coverage probability of indoor MUE on shared channel. Since the coverage probability is the complementary CDF of SINR, here, we use CDF to indirectly prove it. The simulation results are collected from 10000 random realizations of BS locations and channels. We simulate different wall penetration $L_{ow} = \{5 \text{ dB}, 20 \text{ dB}\}$ and normalized transmit power of FBSs $P_2 = \{10^{-3}, 1\}$, with $\delta = 3$, $\kappa = 0.3$. In Fig.2, we can see that both the exact and approximate curves fit well with the simulation results. This implies that the interference from FBSs in $\Phi_2 \setminus \tilde{b}$ can be neglected. The reason is that the signal from these FBSs undergoes two walls before it reaches the observed MUE, and thus is very weak.

In Fig.3, we demonstrate the dynamic processes to achieve the NEs of $\mathcal{G}_T$. Assume that T is a constant, $v_i/\log(1+T)$

TABLE I
PARAMETERS SETUP.

Parameter	Value
Observation area	10×10 km^2
Room area	10×10 m^2
Distance between a user and FBS in the same room	$R = 7.07$ m
BSs Density	4/km^2 (MBSs), 100/km^2(FBSs)
Max Tx Power	46 dBm (MBSs), 20 dBm (FBSs)
Antennas	1Tx, 1Rx (both MBSs and FBSs)
Antenna gains	14 dB (MBSs), 5 dB (FBSs)
Bandwidth	10 MHz (600 subcarriers)
Pathloss exponent	δ =3
Pathloss	$L = 15.3 + 10\delta \log_{10}(r)$, r in m
Wall penetration loss	L_{ow} = 5dB, 20 dB
Target SINR	$T = 3$ dB
Intra-tier 1 leakage	$\kappa = 0.1 \sim 0.5$

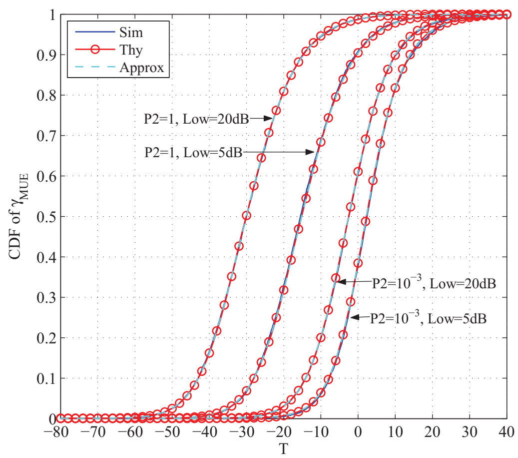


Fig. 2. CDF of γ_{MUE} with regard to wall penetration and transmit power of FBS, $\delta = 3$, and $\kappa = 0.3$.

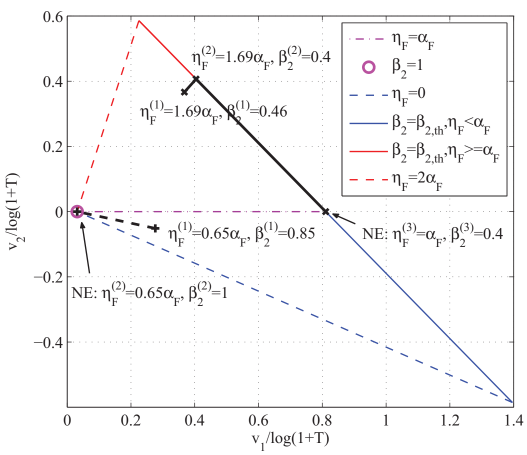


Fig. 3. Dynamic process to achieve the NEs of $\mathcal{G}_T$. The solid and dash lines represent the coordinate and incoordinate cases respectively. $\alpha_M = 2$, $\alpha_F = 1$, $\beta_{2,th} = 0.4$, $\kappa = 0.2$, $\delta = 3$.

is illustrated for simplicity. The unit price for MUE and FUE are $\alpha_M = 2$, $\alpha_F = 1$ respectively. $\beta_{2,th} = c_F / C_{out}^{FS} = 0.4$, and $\kappa = 0.2$, $\delta = 3$. The region surrounded by blue lines

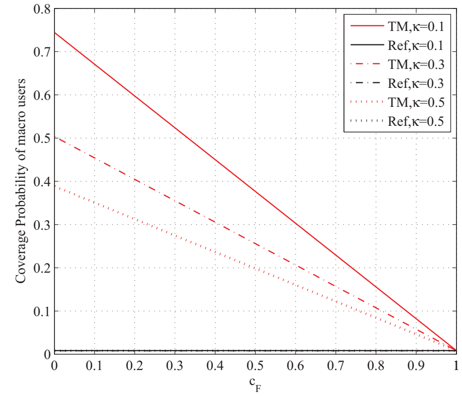


Fig. 4. Coverage probability of MUEs at NE of the interference coordination price game. T = 3dB, and L_{ow} = 20dB.

stands for the incoordinate case, while the area encompassed by red lines represents the coordinate case. The definitions of the boundaries are displayed in the legend. The solid and dash lines represent the processes to achieve the NEs in the coordinate and incoordinate cases respectively. The utilities of player 2 at the two NEs are both zero, but with different meanings. If the NE $(\eta_F, \beta_2) = (\eta_F < \alpha_F, 1)$ is obtained, the outage throughput of femto user is C_{out}^{FS}, and the owner of the FBS has to pay $\alpha_F C_{out}^{FS}$ for using the core network; If the NE $(\eta_F, \beta_2) = (\alpha_F, \beta_{2,th})$ is achieved, the outage throughput of femto user is $\beta_{2,th} C_{out}^{FS}$, and the operator of MBSs charges the owner of the FBS $\alpha_F \beta_{2,th} C_{out}^{FS}$.

In Fig.4, we evaluate the coverage probability of MUEs at NE of the interference coordination price game with target SINR T = 3dB, and L_{ow} = 20dB. From Fig.4, we can see that as the outage throughput threshold of FBSs decreases, the coverage probability of MUEs grows, and the relationship between these two terms are linearly dependent. The increase of intra-tier 1 leakage limits $\mathbb{P}_\delta^{MD}$, and thus reduces the growing speed of coverage probability of MUEs.

VI. CONCLUSION

In this paper, we considered the interference coordination in closed access femto and macro co-existent networks via game theory. The coverage probability of indoor MUEs and FUEs were derived based on the stochastic model, and the operating parameters of altruistic time mute scheme were obtained. Considering the selfishness of FBSs, we used pricing games to formulate the two-tier interference coordination, and obtained the closed-form expression for NE. Simulation results showed the linear relationship between the outage throughput of FUEs and the coverage probability of MUEs achieved at the NE of the pricing game.

VII. ACKNOWLEDGMENT

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