

# Fair Multiuser Channel Allocation for OFDMA Networks Using Nash Bargaining Solutions and Coalitions

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**Abstract**—In this paper, a fair scheme to allocate subcarrier, rate, and power for multiuser orthogonal frequency-division multiple-access systems is proposed. The problem is to maximize the overall system rate, under each user's maximal power and minimal rate constraints, while considering the fairness among users. The approach considers a new fairness criterion, which is a generalized proportional fairness based on Nash bargaining solutions and coalitions. First, a two-user algorithm is developed to bargain subcarrier usage between two users. Then a multiuser bargaining algorithm is developed based on optimal coalition pairs among users. The simulation results show that the proposed algorithms not only provide fair resource allocation among users, but also have a comparable overall system rate with the scheme maximizing the total rate without considering fairness. They also have much higher rates than that of the scheme with max-min fairness. Moreover, the proposed iterative fast implementation has the complexity for each iteration of only  $O(K^2 N \log_2 N + K^4)$ , where  $N$  is the number of subcarriers and  $K$  is the number of users.

**Index Terms**—Channel allocation, coalition, cooperative game, Nash bargaining solution, orthogonal frequency-division multiple access (OFDMA).

## I. INTRODUCTION

ORTHOGONAL frequency-division multiple access (OFDMA) is a promising multiple-access technique for high-data-rate transmissions over wireless radio channels. Efficient resource allocation, which involves bit loading, transmission power allocation, and subcarrier assignment, can greatly improve system performance, and so draws great attention in recent research.

The resource-allocation problem for a single user across parallel orthogonal channels is to maximize the total achievable rate subject to a total power constraint, which can be optimally solved by means of the waterfilling method [22]. The rate allocation in each subcarrier is then determined by the corresponding power allocation. The waterfilling solution can also be applied in single-cell multiuser systems with a given set of allocated

subcarriers to each user, since, in that case, resource allocation for each user can be considered independently.

However, if we consider the different users' link qualities and the discrete nature of the subcarrier-assignment problem, it is more difficult to optimally assign the subcarriers to different users in a multiuser environment. By adaptively assigning subcarriers of various frequencies, we can take advantage of channel diversity among users in different locations, which is called multiuser diversity. Such multiuser diversity stems from channel diversity, including independent path loss and fading of users. Most of the existing works focus on improving the system efficiency by exploring multiuser diversity [1]–[7]. In [1], the authors studied the dual problem, namely, to find the optimal subcarrier allocation so as to minimize the total transmitted power and satisfy a minimum rate constraint for each user. The dual problem is further formulated as an integer programming problem, and a suboptimal solution is found by using the continuous relaxation. In [2], a low-complexity suboptimal algorithm is proposed, which decouples the problem into two subproblems, finding the required power and the number of subcarriers for each user, and finding the exact subcarrier and rate allocation. In [3], the discrete subcarrier-allocation problem is relaxed into a constrained optimization problem with continuous variables. The problem is shown to belong to the class of convex programming problems, thus allowing the optimal assignment to be found with numerical methods. In [4], the problem is formulated using a max-min criterion for downlink application. The optimal channel-assignment problem is formulated as a convex optimization problem, and a low-complexity suboptimal algorithm is developed. Real-time subcarrier-allocation schemes are studied in [5] and [6], which only use subcarrier allocation to enhance the performance while fixing modulation levels. The Hungarian method [17] can be used to solve such problems with a high computational complexity of  $O(N^4)$ , where  $N$  is the number of subcarriers. The suboptimal algorithms are developed in [5] and [6] to simplify the Hungarian algorithm and achieve similar performances. In [7], adaptive modulation is applied for an uplink OFDMA system.

Most of the previous approaches study how to efficiently maximize the total transmission rate or minimize the total transmitted power under some constraints. The formulated problem and their solutions are focused on the efficiency issue. But these approaches benefit the users closer to the base station (BS) or with a higher power capability. The fairness issue has been mostly ignored. On the other hand, as for the fairness among users, the max-min criterion has been considered for

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channel allocation in multiuser orthogonal frequency-division multiplexing (OFDM) systems [4]. However, by using this criterion, it is not easy to take into account the notion that users might have different requirements. Moreover, since the max-min approach deals with the worst-case scenario, it penalizes users with better channels and reduces the system efficiency. In addition, most of the existing solutions have high complexities, which prohibit them from practical implementation. Therefore, it is necessary to develop an approach that considers altogether the fairness of resource allocation, system efficiency, and complexity.

In daily life, a market serves as a central gathering point, where people can exchange goods and negotiate transactions, so that people can be satisfied through bargaining. Similarly, in single-cell multiuser OFDMA systems, there is a BS that can serve as a function of the market. The distributed users can negotiate via the BS to cooperate in making the decisions on the subcarrier usage, such that each of them can operate at its optimum and joint agreements are made about their operating points. Such a fact motivates us to apply the game theory [8], [9], [11]–[14] and especially cooperative game theory, which can achieve the crucial notion of fairness and maximize the overall system rate. The concepts of the Nash bargaining solution (NBS) and coalitions are taken into consideration, because they provide a fair operation point in a distributed implementation.

Motivated by the above reasons, we apply the cooperative game theory for resource allocation in OFDMA systems. The goal is to maximize the overall system rate, under the constraints of each user's minimal rate requirement and maximal transmitted power. First, we develop a two-user bargaining algorithm to negotiate the usage of subcarriers. The approach is based on NBS, which maximizes the system performance while keeping the NBS fairness, where the NBS fairness is a generalized proportional fairness. Then we group the users into groups of size two, which is defined as a coalition. Within each coalition, we use a two-user algorithm to improve the performance. In the next iteration, new coalitions are formed, and subcarrier allocation is optimized until no improvement can be obtained. By using the Hungarian method, optimal coalitions are formed, and the number of iterations can be greatly reduced. A significant point for the proposed iterative algorithm is that the complexity for each iteration is only  $O(K^2 N \log_2 N + K^4)$ , where  $K$  is the number of users. From the simulation results, the proposed algorithms allocate resources fairly and efficiently, compared with the other two schemes: maximal rate and max-min fairness. The NBS fairness is demonstrated by the fact that a user's rate is not influenced by the interfering users.

This paper is organized as follows. In Section II, the system model is given. In Section III, basics for the NBS of cooperative game theory are presented. In Section IV, the optimization problem is formulated. A two-user algorithm and a multiuser algorithm are constructed. In Section V, simulations are developed, and in Section VI, conclusions are drawn.

## II. SYSTEM MODEL AND DESCRIPTION

Consider an uplink scenario of a single-cell multiuser OFDMA system. There are, in total,  $K$  users randomly located

within the cell. The users want to share their transmissions among  $N$  different subcarriers. Each subcarrier has a bandwidth of  $W$ . The  $i$ th user's transmission rate is  $R_i$  and is allocated to different subcarriers as  $R_i = \sum_{j=1}^N r_{ij}$ , where  $r_{ij}$  is the  $i$ th user's transmission rate in the  $j$ th subcarrier. Define the rate-allocation matrix  $\mathbf{r}$  as  $[\mathbf{r}]_{ij} = r_{ij}$ . Define the subcarrier-assignment matrix  $[\mathbf{A}]_{ij} = a_{ij}$ , where

$$a_{ij} = \begin{cases} 1, & \text{if } r_{ij} > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

For single-cell multiuser OFDMA, no subcarrier can support the transmissions for more than one user, i.e.,  $\sum_{i=1}^K a_{ij} = 1 \forall j$ .

Adaptive modulation provides each user with the ability to match each subcarrier's transmission rate  $r_{ij}$ , according to its channel condition.  $M$ -ary quadrature amplitude modulation (MQAM) is a modulation method with a high spectrum efficiency, which is adopted in our system without loss of generality. In [16], the bit-error rate (BER) of MQAM as a function of rate and signal-to-noise ratio (SNR) is approximated by

$$\text{BER}_{ij} \approx c_1 e^{-c_2 \frac{\Gamma_{ij}}{2^{r_{ij}} - 1}} \quad (2)$$

where  $c_1 \approx 0.2$ ,  $c_2 \approx 1.5$ , and  $\Gamma_{ij}$  is the  $i$ th user's SNR at the  $j$ th subcarrier, given by

$$\Gamma_{ij} = \frac{P_{ij} G_{ij}}{\sigma^2} \quad (3)$$

where  $G_{ij}$  is the subcarrier channel gain, and  $P_{ij}$  is the transmitted power for the  $i$ th user in the  $j$ th subcarrier. The thermal noise power for each subcarrier is assumed to be the same, and equal to  $\sigma^2$ . Define power-allocation matrix  $[\mathbf{P}]_{ij} = P_{ij}$ . From (2), without loss of generality, we assume a fixed and the same BER for all users in all subcarriers. Then we have

$$r_{ij} = W \log_2 \left( 1 + \frac{P_{ij} G_{ij} c_3}{\sigma^2} \right) \quad (4)$$

where  $c_3 = c_2 / \ln(c_1 / \text{BER})$  with  $\text{BER} = \text{BER}_{ij} \forall i, j$ .

We assume the slow-fading channel such that the channel is stable within each OFDM frame. The channel conditions of different subcarriers for each user are assumed perfectly estimated. There exist reliable feedback channels from BS to mobile users without any delay. Moreover, for a practical system, the OFDM frequency offset between the mobile user and the BS is around several tenths of Hertz. The intercarrier interference caused by the frequency offset may cause some error-floor increase. However, this is not the bottleneck limiting the system performance, and this offset can be fed back to the mobile for adjustment. In [15], a guard subcarrier is put at the edge of each subcarrier such that multiple-access interference can be minimized, and a synchronized algorithm is applicable for each subcarrier. So, in this paper, we assume mobiles and the BS are synchronized.

In Fig. 1, an illustrative three-user example is given for the system setup. The number of subcarriers for communication is eight. Each subcarrier is occupied by one user. According to the channel conditions, a user selects an adaptive modulation level and adjusts its rate for this subcarrier. The conflicts are that some subcarrier is good for more than one user, and the problem is who this subcarrier should be assigned to. So our goal is to

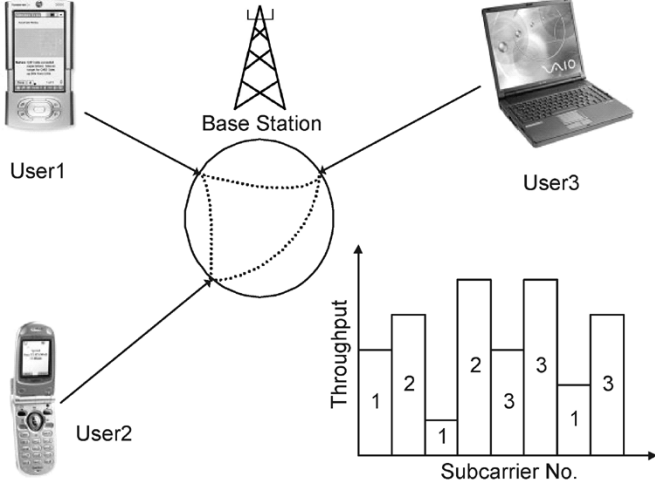


Fig. 1. System model.

assign the subcarriers by negotiating with other users via the BS, so that each user can obtain its minimal rate while the system overall performance is optimized. In the following sections, we will discuss in detail how to implement the negotiation process.

### III. BASICS FOR NASH BARGAINING SOLUTION

In this section, we will briefly review the basic concepts and theorems for NBS. Then, we will give an overview on how to apply these ideas to OFDMA resource allocation.

The bargaining problem of cooperative game theory can be described as follows [8], [9], [11]. Let  $\mathbf{K} = \{1, 2, \dots, K\}$  be the set of players. Let  $\mathbf{S}$  be a closed and convex subset of  $\mathbb{R}^K$  to represent the set of feasible payoff allocations that the players can get if they all work together. Let  $R_{\min}^i$  be the minimal payoff that the  $i$ th player would expect; otherwise, he will not cooperate. Suppose  $\{R_i \in \mathbf{S} | R_i \geq R_{\min}^i \forall i \in \mathbf{K}\}$  is a nonempty bounded set. Define  $\mathbf{R}_{\min} = (R_{\min}^1, \dots, R_{\min}^K)$ , then the pair  $(\mathbf{S}, \mathbf{R}_{\min})$  is called a  $K$ -person bargaining problem.

Within the feasible set  $\mathbf{S}$ , we define the notion of Pareto optimal as a selection criterion for the bargaining solutions.

**Definition 1:** The point  $(R_1, \dots, R_K)$  is said to be **Pareto optimal**, if and only if there is no other allocation  $R'_i$  such that  $R'_i \geq R_i \forall i$ , and  $R'_i > R_i, \exists i$ , i.e., there exists no other allocation that leads to superior performance for some users without inferior performance for some other users.

There might be an infinite number of Pareto optimal points. We need further criteria to select a bargaining result. A possible criterion is the fairness. One commonly used fairness criterion is max-min [4], where the performance of the user with the worst channel conditions is maximized. This criterion penalizes the users with good channels, and as a result, generates inferior overall system performance. In this paper, we use the criterion of fairness NBS. The intuitive idea is that after the minimal requirements are satisfied for all users, the rest of the resources are allocated proportionally to users according to their conditions. We will discuss the *proportional fairness* concept, which is a special case of NBS fairness, in the next section, and show the fair results in the simulation section. There exist many kinds of cooperative game solutions [11]. Among them, NBS provides

a unique and fair Pareto optimal operation point under the following conditions. NBS is briefly explained as follows.

**Definition 2:**  $\bar{\mathbf{r}}$  is said to be an **NBS** in  $\mathbf{S}$  for  $\mathbf{R}_{\min}$ , i.e.,  $\bar{\mathbf{r}} = \phi(\mathbf{S}, \mathbf{R}_{\min})$ , if the following axioms are satisfied.

- 1) *Individual Rationality:*  $\bar{R}_i = \sum_{j=1}^N \bar{r}_{ij} \geq R_{\min}^i \forall i$ .
- 2) *Feasibility:*  $\bar{\mathbf{r}} \in \mathbf{S}$ .
- 3) *Pareto Optimality:* For every  $\hat{\mathbf{r}} \in \mathbf{S}$ , if  $\sum_{j=1}^N \hat{r}_{ij} \geq \sum_{j=1}^N \bar{r}_{ij} \forall i$ , then  $\sum_{j=1}^N \hat{r}_{ij} = \sum_{j=1}^N \bar{r}_{ij} \forall i$ .
- 4) *Independence of Irrelevant Alternatives:* If  $\bar{\mathbf{r}} \in \mathbf{S}' \subset \mathbf{S}$ ,  $\bar{\mathbf{r}} = \phi(\mathbf{S}, \mathbf{R}_{\min})$ , then  $\bar{\mathbf{r}} = \phi(\mathbf{S}', \mathbf{R}_{\min})$ .
- 5) *Independence of Linear Transformations:* For any linear scale transformation  $\psi$ ,  $\psi(\phi(\mathbf{S}, \mathbf{R}_{\min})) = \phi(\psi(\mathbf{S}), \psi(\mathbf{R}_{\min}))$ .
- 6) *Symmetry:* If  $\mathbf{S}$  is invariant under all exchanges of agents,  $\phi_j(\mathbf{S}, \mathbf{R}_{\min}) = \phi_{j'}(\mathbf{S}, \mathbf{R}_{\min}) \forall j, j'$ .

Axioms 4–6 are called axioms of fairness. The irrelevant alternative axiom asserts that eliminating the feasible solutions that would not have been chosen should not affect the NBS solution. Axiom 5 asserts that the bargaining solution is scale-invariant. Symmetry axiom asserts that if the feasible ranges for all users are completely symmetric, then all users have the same solution.

The following theorem shows that there is exactly one NBS that satisfies the above axioms [11].

**Theorem 1: Existence and Uniqueness of NBS:** There is a unique solution function  $\phi(\mathbf{S}, \mathbf{R}_{\min})$  that satisfies all six axioms in *Definition 1*, and this solution satisfies [11]

$$\phi(\mathbf{S}, \mathbf{R}_{\min}) \in \arg \max_{\bar{\mathbf{r}} \in \mathbf{S}, \bar{R}_i \geq R_{\min}^i \forall i} \prod_{i=1}^K (\bar{R}_i - R_{\min}^i). \quad (5)$$

As discussed above, the cooperative game in the multiuser OFDMA system can be defined as follows. Each user has  $R_i$  as its objective function, where  $R_i$  is bounded above and has a nonempty, closed, and convex support. The goal is to maximize all  $R_i$  simultaneously.  $\mathbf{R}_{\min}$  represents the minimal performance, and is called the initial agreement point. Define  $\mathbf{S}$  as the feasible set of rate-allocation matrix  $\mathbf{r}$  that satisfies  $R_i \geq R_{\min}^i \forall i$ . The problem, then, is to find a simple way to choose the operating point in  $\mathbf{S}$  for all users, such that this point is optimal and fair.

### IV. COOPERATIVE GAME APPROACHES

#### A. Problem Formulation

Since the channel conditions for a specific subcarrier may be good for more than one user, there is a competition among users for their transmissions over the subcarriers with large  $G_{ij}$ . Moreover, the maximal transmitted power for each user is bounded by the maximal transmitted power  $P_{\max}$ , and each user has a minimal rate requirement  $R_{\min}^i$  if it is admitted to the system. In this paper, the optimization goal is to determine different users' transmission function  $\mathbf{A}$  and  $\mathbf{P}$  for the different subcarriers, such that the cost function can be maximized, i.e.,

$$\max_{\mathbf{A}, \mathbf{P}} U, \quad \text{subject to} \begin{cases} \sum_{i=1}^K a_{ij} = 1 & \forall j \\ R_i \geq R_{\min}^i & \forall i \\ \sum_{j=1}^N P_{ij} \leq P_{\max} & \forall i \end{cases} \quad (6)$$

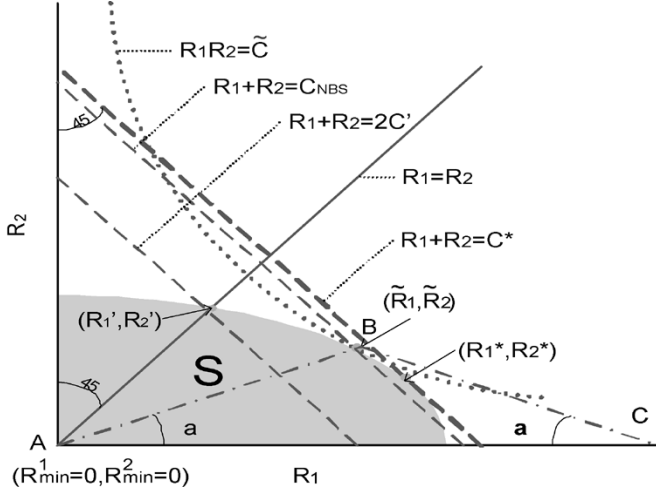


Fig. 2. Two-user illustrative example.

where  $U$  can have three definitions in terms of the objectives

$$\text{Maximal Rate : } U = \sum_{i=1}^K R_i \quad (7)$$

$$\text{Max-min Fairness : } U = \min R_i \quad (8)$$

$$\text{NBS : } U = \prod_{i=1}^K (R_i - R_{\min}^i). \quad (9)$$

For maximal rate optimization, the overall system rate is maximized. For max-min fairness optimization, the worst-case situation is optimized with strict fairness. In this paper, we proposed the NBSs for the following two reasons. First, it will be shown later that this form can ensure fairness of allocation in the sense that NBS fairness is a generalized proportional fairness. Second, cooperative game theories prove that there exists a unique and efficient solution under the six axioms. The difficulty in solving (6) by traditional methods lies in the fact that the problem itself is a constrained combinatorial problem, and the constraints are nonlinear. Thus, the complexities of the traditional schemes are high, especially with a large number of users. Moreover, distributed algorithms are desired for uplink OFDMA systems, while centralized schemes are dominant in the literature. In addition, most of the existing work does not discuss the issue of fairness. We will use the bargaining concept to develop simple and distributed algorithms with limited signaling that can achieve an efficient and fair resource allocation in the rest of this section.

Fig. 2 illustrates a two-user example where  $\mathbf{R}_{\min}$  is assumed to be zero. Shaded area  $\mathbf{S}$  is the feasible range for  $\mathbf{R}_1$  and  $\mathbf{R}_2$ . For the NBS cost function, the optimal point is  $B$  at  $(\hat{R}_1, \hat{R}_2)$  with  $R_1 R_2 = \hat{C}$ , where  $\hat{C}$  is the largest constant for the feasible set  $\mathbf{S}$ . The physical meaning of this is that “after the users are assigned with the minimal rate, the remaining resources are divided between users in a ratio equal to the rate at which the utility can be transferred” [11]. The geometrical interpretation is that an isosceles triangle  $ABC$  can be drawn with  $(\hat{R}_1, \hat{R}_2)$  as the apex, such that its one side is tangent to the set  $\mathbf{S}$ , and the other side passes  $(R_{\min}^1, R_{\min}^2)$ , i.e., the origin. Since line  $BC$  is also tangent to curve  $R_1 R_2 = \hat{C}$ , the ratio that two rates

can be exchanged within the set  $\mathbf{S}$  is equal to the ratio of the two rates. The maximal-rate approach has the optimal point at  $R_1^* + R_2^* = C^*$ , which is the point within feasible set  $\mathbf{S}$  where the sum  $C^*$  of  $R_1$  and  $R_2$  is maximized. Compared with the maximal-rate approach, the overall rate of the NBS solution is  $C_{\text{NBS}}$ , which is slightly smaller than  $C^*$ . So, the NBS solution has a small overall rate loss, but keeps the fairness. The max-min approach considers the worst-case scenario and has the optimal point with  $R_1' = R_2' = C'$ , where  $C'$  is the largest constant for feasible set  $\mathbf{S}$ . The overall rate for the max-min approach is  $2C'$ . Compared with the max-min algorithm, the NBS solution has a much higher overall rate, i.e.,  $2C' \ll C_{\text{NBS}}$ .

In addition, we will show in the following definition and theorem that proportional fairness [10], which is widely used in wired networks, is a special case of the fairness provided by NBS.

*Definition 3:* We say the rate distribution is proportionally fair when any change in the distribution of rates results in the sum of the proportional changes of the utilities being nonpositive, i.e.,

$$\sum_i \frac{R_i - \tilde{R}_i}{\tilde{R}_i} \leq 0 \quad \forall R_i \in \mathbf{S} \quad (10)$$

where  $\tilde{R}_i$  and  $R_i$  are the proportionally fair rate distribution and any other feasible rate distribution for the  $i$ th user, respectively.

*Theorem 2:* When  $R_{\min}^i = 0 \forall i$ , the NBS fairness is the same as the proportional fairness.

*Proof:* Since the function of  $\ln$  is concave and monotonic, when  $R_{\min}^i = 0 \forall i$ , the NBS in (5) is equivalent to

$$\max_{\mathbf{r} \in \mathbf{S}} \sum_{i=1}^K \ln(R_i). \quad (11)$$

Define  $\hat{U}_i = \ln(R_i)$ . The gradient of  $\hat{U}_i$  at the NBS point  $\tilde{R}_i$  is  $(\partial \hat{U}_i / \partial R_i)|_{\tilde{R}_i}$ . Since the NBS point optimizes (11), for any point deviating from the NBS point, the following optimality condition holds:

$$\sum_i \frac{\partial \hat{U}_i}{\partial R_i} \Big|_{\tilde{R}_i} (R_i - \tilde{R}_i) = \sum_i \frac{R_i - \tilde{R}_i}{\tilde{R}_i} \leq 0. \quad (12)$$

The above equation means for all feasible  $R_i \in \mathbf{S} \forall i$  that is different from NBS point  $\tilde{R}_i$ , the overall change of benefits is negative, according to the gradients. Moreover, the above equation is the same as the proportional fairness definition in (10). So the proportional fairness is a special case of the NBS fairness when  $R_{\min}^i = 0 \forall i$ . Since the minimal-rate requirement is necessary in practice, we apply NBS fairness in this paper.  $\square$

Next, we want to demonstrate that there exists a unique and optimal solution in (6) when the feasible set satisfying the constraints is not empty. We show the uniqueness and optimality in two steps. First, we prove the uniqueness and optimality with fixed channel-assignment matrix  $\mathbf{A}$ . Then, we prove that the probability that there exists more than one optimal point equalling zero for a different channel-assignment matrix  $\mathbf{A}$ .

First, under the fixed channel-assignment matrix  $\mathbf{A}$ , each user tries to maximize its own rate under the power constraint independently, because any subcarrier is not shared by more than one

TABLE I  
TWO-USER ALGORITHM

<b>1. Initialization:</b> Initialize the subcarrier assignment with the minimal rate requirements. For <i>Maximal Rate</i> , $\varrho_1 = \varrho_2 = 1$ ; For <i>NBS</i> , calculate $\varrho_1$ and $\varrho_2$ .
<b>2. Sort the subcarriers:</b> Arrange the index from the largest to smallest $\frac{\varrho_1}{g_{1j}} - \frac{\varrho_2}{g_{2j}}$ .
<b>3. For <math>j=1, \dots, N-1</math></b> User 1 occupies and water-fills subcarrier 1 to $j$ ; User 2 occupies and water-fills subcarrier $j+1$ to $N$ . Calculate $U$ . <b>End</b>
<b>4. Choose the two-band partition (the corresponding <math>j</math>) that generates the largest <math>U</math> satisfying the constraints.</b> Calculate $\mathbf{A}$ , $\mathbf{P}$ , $R_1$ , and $R_2$ .
<b>5. Update channel assignment</b> - <i>Maximal Rate</i> : Return - <i>NBS</i> : If $U$ can not be increased by updating $\varrho_1$ and $\varrho_2$ , the iteration ends; otherwise, update $\varrho_1 = 1/(R_1 - R_{min}^1)$ , $\varrho_2 = 1/(R_2 - R_{min}^2)$ ; go to Step 2.

user. This is similar to the single-user case. We assume the feasible set that satisfies the constraints in (6) is not empty. Within the feasible set, each user can get its minimal rate requirement  $R_{min}^i$  by allocating its power to the assigned channel set. For all three cost functions in (7)–(9), the problem in (6) is reduced to the following problem:

$$\max_{\mathbf{P}} U_i = R_i \quad \text{subject to} \quad \sum_{j=1}^N P_{ij} a_{ij} \leq P_{max} \quad \forall i. \quad (13)$$

Obviously, the above problem is a waterfilling problem [22] and has a unique optimal solution. Define

$$I_{ij} = \frac{\sigma^2}{c_3 G_{ij}}. \quad (14)$$

The unique optimal solution is

$$P_{ij} = (\mu_i - I_{ij})^+ \text{ and } r_{ij} = W \log_2(1 + \frac{P_{ij} a_{ij}}{I_{ij}}) \quad (15)$$

where  $y^+ = \max(y, 0)$ . Here,  $\mu_i$  is the water level and can be solved by a bisection search of

$$\sum_{j=1}^N P_{ij} a_{ij} = P_{max}. \quad (16)$$

We have proved the optimality and uniqueness with fixed channel assignment. The channel assignment is a combinatorial problem with a finite number of combinations. For example, the total number of combinations for the system with  $K$  users and  $N$  subcarriers is  $K^N$ . So, we can obtain the optimal solution by solving the following problem:

$$\arg \max_{\mathbf{A}} U \quad (17)$$

where  $U$  is obtained by solving (13) with respect to each  $\mathbf{A}$ .

The above problem can be solved by means of a full search to get the optimal channel assignment and power allocation. Among all the implementations of  $\mathbf{A}$ , we select the one that generates the largest  $U$ . Because the optimization goal  $U$ , the

channel gains, and the rates are continuous random values, there is zero probability of having two channel-assignment matrices that generate the same value of optimization goal. So, with probability one, there exists a unique channel-assignment matrix that generates the optimal solution in (6).

### B. Bargaining Algorithm for Two-User Case

In this subsection, we consider the case when  $K = 2$ , and we will develop a fast two-user bargaining algorithm. Similar to bargaining in a real market, the intuitive idea to solve the two-user problem is to allow two users to negotiate and exchange their subcarriers, such that mutual benefits can be obtained. The difficulty is to determine how to optimally exchange subcarriers, which is a complex integer-programming problem. An interesting low-complexity algorithm was given in [3]. The idea is to sort the order of subcarriers first, and then to use a simple two-band partition for the subcarrier assignment. When SNR is high, the two-band partition for two-user subcarrier assignment is near-optimal for the optimization goal of maximizing the weighted sum of both users' rates.

We propose a fast algorithm between two users for the optimization goals by exchanging their subcarriers, as shown in Table I. First, all subcarriers are initially assigned. Then, two users' subcarriers are sorted, and a two-band partition algorithm is applied for them to negotiate the subcarriers. For the maximal rate optimization goal, only one iteration is necessary. For the NBS optimization goal, an intermediate parameter needs to be updated for every iteration. From the simulations, the iterations between Step 2 and Step 5 are converged within two to three rounds. The algorithm has the complexity of  $O(N^2)$  for each iteration, and can be further improved by using a binary search algorithm with a complexity of only  $O(N \log N)$  for each iteration. It is worth mentioning that all the iterations in Table I happen within the BS, so there is no need for signaling between users and BSs.

*Proposition 1:* The algorithm in Table I is near-optimal for both the problem of maximal rate and NBS goals in (6) with the number of users equal to two, when the SNR of each subcarrier

for all users in (3) is much greater than one and there exists a feasible solution.

*Proof:* In [3], the authors proved that if at the optimal subcarrier partition, the SNR is large in every subcarrier for all users, and if the subcarriers are sorted according to the users' subcarrier channel gain, then the optimal subcarrier partition that maximizes  $\alpha_1 R_1 + \alpha_2 R_2$  consists of two contiguous frequency bands with each user occupying one band. Here,  $R_1$  and  $R_2$  are the two users' rates, and  $\alpha_1$  and  $\alpha_2$  are the relative priorities for both users. For the maximal rate optimization goal, the theorem is proved by letting  $\alpha_1 = \alpha_2 = 1$  [3].

For NBS, the optimization goal is  $U = (R_1 - R_{\min}^1)(R_2 - R_{\min}^2)$ , which contains a term of  $R_1 R_2$ . Similar to the approach in [3], we relax the channel-assignment matrix  $\mathbf{A}$  to continuous values with  $0 \leq a_{ij} \leq 1 \forall i, j$ . We write the Lagrangian function of (6) as a function of  $a_{ij}$  and  $P_{ij}$

$$\begin{aligned}
 L = & \left( \sum_{j=1}^N a_{1j} W \log_2 \left( 1 + \frac{P_{1j} G_{1j} c_3}{a_{1j} \sigma^2} \right) - R_{\min}^1 \right) \\
 & \times \left( \sum_{j=1}^N a_{2j} W \log_2 \left( 1 + \frac{P_{2j} G_{2j} c_3}{a_{2j} \sigma^2} \right) - R_{\min}^2 \right) \\
 & + \sum_{j=1}^N \lambda_j \left( \sum_{i=1}^2 a_{ij} - 1 \right) + \sum_{i=1}^2 \kappa_i \left( \sum_{j=1}^N P_{ij} - P_{\max} \right) \\
 & - \sum_{i=1}^2 \sum_{j=1}^N \nu_{ij}^1 P_{ij} - \sum_{i=1}^2 \sum_{j=1}^N \nu_{ij}^2 a_{ij} \quad (18)
 \end{aligned}$$

where  $\lambda_j$ ,  $\kappa_i$ ,  $\nu_{ij}^1$ , and  $\nu_{ij}^2$  are Lagrangian multipliers. By using the Karush–Kuhn–Tucker (KKT) condition [23], we take the derivative of (18) with respect to  $a_{ij}$ , and have

$$\begin{aligned}
 & \log_2 \left( 1 + \frac{P_{1j} G_{1j} c_3}{a_{1j} \sigma^2} \right) - \frac{\frac{P_{1j} G_{1j} c_3}{a_{1j} \sigma^2}}{1 + \frac{P_{1j} G_{1j} c_3}{a_{1j} \sigma^2}} \\
 & \frac{\left( \sum_{j=1}^N a_{1j} W \log_2 \left( 1 + \frac{P_{1j} G_{1j} c_3}{a_{1j} \sigma^2} \right) - R_{\min}^1 \right)}{\log_2 \left( 1 + \frac{P_{2j} G_{2j} c_3}{a_{2j} \sigma^2} \right) - \frac{\frac{P_{2j} G_{2j} c_3}{a_{2j} \sigma^2}}{1 + \frac{P_{2j} G_{2j} c_3}{a_{2j} \sigma^2}}} \\
 & = \frac{\left( \sum_{j=1}^N a_{2j} W \log_2 \left( 1 + \frac{P_{2j} G_{2j} c_3}{a_{2j} \sigma^2} \right) - R_{\min}^2 \right)}{\quad} \quad (19)
 \end{aligned}$$

From (15), we have waterfilling results for discrete  $a_{ij}$ . Define the positive weight factor as shown in (20) at the bottom of the page, where  $\epsilon$  is a small positive number, and  $\varepsilon$  is a small positive value, to ensure the large weight for the user whose rate is less

than  $R_{\min}^i + \epsilon$ . We put (14), (15), and (20) into (19); at high SNR, we have

$$\varrho_1 \left( \log_2 \left( \frac{\mu_1}{I_{1j}} \right) + \frac{I_{1j}}{\mu_1} - 1 \right) = \varrho_2 \left( \log_2 \left( \frac{\mu_2}{I_{2j}} \right) + \frac{I_{2j}}{\mu_2} - 1 \right). \quad (21)$$

If a subcarrier is used by user 1, i.e.,  $a_{1j} = 1$  and  $a_{2j} = 0$ , the left-hand side (LHS) should be strictly greater than the right-hand side (RHS). At high SNR, the fraction  $I_{ij}/\mu_i$  on either side of (21) can be approximated by zero. Let  $g_{ij} = 1/I_{ij}$ . Take the difference between the LHS and the RHS of (19), and define function  $f$  as

$$f \left( \frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}} \right) \approx \log_2 \left( \frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}} \right) + \log_2 \left( \frac{\mu_1^{\varrho_1}}{\mu_2^{\varrho_2}} \right) + \varrho_2 - \varrho_1. \quad (22)$$

We are able to decide whether a subcarrier is used by user 1 or user 2 by checking whether the function is greater than zero or less than zero. We arrange the index of subcarriers to make  $g_{1j}^{\varrho_1}/g_{2j}^{\varrho_2}$  be decreased in  $j$ . With fixed  $\varrho_1$  and  $\varrho_2$ ,  $f(g_{1j}^{\varrho_1}/g_{2j}^{\varrho_2})$  is a monotony function of  $j$ . Then (22) is similar to the weighted maximization in [3], and the optimum partition is a two-band solution.

The LHS and RHS of (21) illustrate the marginal benefits of extra bandwidth for user 1 and user 2 on subcarrier  $j$ , respectively. Within each iteration,  $\varrho_i$  is fixed. Then the algorithm achieves the boundary point of the feasible region [3]. Then, in the next iteration, the new  $\varrho_i$  is updated. Remember that  $\tilde{R}_i$  is the NBS solution. If  $R_1 > \tilde{R}_1$  and  $R_2 < \tilde{R}_2$ , from (20),  $\varrho_1$  is small and  $\varrho_2$  is large. Consequently, the marginal benefit of user 1 will be reduced, and he/she will have a disadvantage for channel allocation in the next iteration, and vice versa. This is one explanation why the proposed scheme converges to the NBS solution. The iterative algorithm converges when (19) is held. It is worth mentioning that the proposed two-user algorithm might not converge toward the NBS solution, because of the nonlinear and combinatorial nature of the formulated problem.  $\square$

### C. Multiple-User Algorithm Using Coalitions

For the case where the number of users is larger than two, most work in the literature concentrates on solving the OFDMA resource-allocation problem for all users together in a centralized way [1]–[7]. Because the problem itself is combinatorial and nonlinear, the computational complexity is very high with respect to the number of subcarriers by the existing methods [1]–[7]. In this paper, we propose a two-step iterative scheme. First, users are grouped into pairs, which are called coalitions. Then for each coalition, the algorithm in Table I is applied for two users to negotiate and improve their performances by exchanging subcarriers. Further, the users are regrouped and renegotiate again and again until convergence. By using this scheme,

$$\varrho_i = \begin{cases} \frac{1}{\varepsilon}, & \sum_{j=1}^N a_{ij} W \log_2 \left( 1 + \frac{P_{ij} G_{ij} c_3}{a_{ij} \sigma^2} \right) - R_{\min}^i \geq R_{\min}^i + \epsilon \\ \text{otherwise} & \end{cases} \quad (20)$$

TABLE II  
MULTIUSER ALGORITHM

<b>1. Initialize the channel assignment:</b> Assign all subcarriers to users.
<b>2. Coalition Grouping:</b> If the number of users is even, the users are grouped into coalitions; otherwise, a dummy user is created to make the total number of users even. No user can exchange its resource with this dummy user. - <i>Random Method</i> : Randomly form 2-user coalition. - <i>Hungarian Method</i> : Form user coalitions by the Hungarian algorithm.
<b>3. Bargain within Each Coalition:</b> Negotiate between two users in all coalitions to exchange the subcarriers using the two-user algorithm in Table I.
<b>4. Repeat:</b> Repeat Step 2 and Step 3, until no further improvement can be achieved.

the computational cost can be greatly reduced. First, we give the strict definition of “coalition” as follows.

*Definition 4:* For a  $K$ -person game, any nonempty subset of the set of players is called a **coalition**.

The question now is how to group users into coalitions with size two. A straightforward algorithm is to form the coalition randomly and let the users bargain arbitrarily. We call this algorithm the random method, which can be described by the steps in Table II. During the initialization, the goal is to assign all subcarriers to users and try to satisfy the minimal rate and maximal power constraints. We develop a fast algorithm. Starting from the user with the best channel conditions, if the user has a rate larger than or equal to  $R_{\min}^i$ , it is removed from the assignment list. After every user has enough rate, the rest of the subcarriers are greedily assigned to the users according to their channel gains. Note that there is no need for the initial assignment to satisfy all constraints. The constraints can be satisfied during the iterations of negotiations.

We quantify the convergence speed by the round of negotiations. The convergence speed of the random method becomes slow with the number of users increasing. This is because the negotiations within arbitrarily grouped coalitions are less effective, and most negotiations turn out to be the same as, or have little improvement over, the performance of the channel allocation before the negotiations. So, the optimal cooperation grouping among subsets of the users should be taken into consideration. In order to speed up convergence, each user needs to carefully select who it should negotiate with.

Each user’s channel gains are varying over different subcarriers. A user may be preferred by many users to form coalitions with, while only a two-user coalition is allowed. Thus, the problem to decide the coalition pairs can be stated as an assignment problem [17]: “a special structured linear programming which is concerned with optimally assigning individuals to activities, assuming that each individual has an associated value describing its suitability to execute that specific activity.”

Now, we formulate the assignment problem in detail. Define the benefit for the  $i$ th user to negotiate with the  $j$ th user as  $b_{ij}$ . Obviously,  $b_{ii} = 0 \forall i$ . For the other cases, from (6), each element of the cost table  $\mathbf{b}$  can be expressed as

$$b_{ij} = \max \left( U(\tilde{R}_i, \tilde{R}_j) - U(\hat{R}_i, \hat{R}_j), 0 \right) \quad (23)$$

where  $\tilde{R}_i$  and  $\tilde{R}_j$  are the rates if the negotiation happens, and  $\hat{R}_i$  and  $\hat{R}_j$  are the original rates, respectively. Obviously,  $\mathbf{b}$  is also symmetric. The proposed two-user algorithm in the previous section can calculate each  $b_{ij} \forall i, j$ . The total complexity is  $O(K^2 N \log_2 N)$ .

Define a  $K \times K$  assignment table  $\mathbf{X}$ . Each component represents whether or not there is a coalition between two users

$$X_{ij} = \begin{cases} 1, & \text{if user } i \text{ negotiates with user } j \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

Obviously, matrix  $\mathbf{X}$  is symmetric,  $\sum_{i=1}^K X_{ij} = 1 \forall j$ , and  $\sum_{j=1}^K X_{ij} = 1 \forall i$ .

So the assignment problem is how to select the pairs of negotiations, such that the overall benefit can be maximized, which is stated as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \sum_{i=1}^K \sum_{j=1}^K X_{ij} b_{ij} \\ \text{s.t.} \quad & \begin{cases} \sum_{i=1}^K X_{ij} = 1, & j = 1, \dots, K \quad \forall i \\ \sum_{j=1}^K X_{ij} = 1, & i = 1, \dots, K \quad \forall j \\ X_{ij} \in \{0, 1\} & \forall i, j. \end{cases} \end{aligned} \quad (25)$$

One of the solutions for (25) is the Hungarian method [17], which can always find the optimal coalition pairs. The Hungarian method has the minimization optimization goal, so we change the maximization problem in (25) into a minimization problem by defining  $B_{ij} = -b_{ij} + \max(b_{ij})$ . The Hungarian algorithm is briefly explained in Table III.

In each round, the optimal coalition pairs  $\mathbf{A}$  are determined by the Hungarian method, and then the users are set to bargain together using the two-user algorithm in Table I. The whole algorithm stops when no bargaining can further improve the performance, i.e.,  $\mathbf{b}$  is equal to a zero matrix. Based on the above explanations, we develop the multiuser resource allocation in the multiuser OFDMA systems in Table II.

In each iteration, the optimization function  $U$  is nondecreasing in Steps 2 and 3, and the optimal solution is upper bounded. Consequently, the proposed multiuser algorithm is convergent. However, because the proposed problem in (6) is nonlinear and non-convex, and also because of the combinatorial nature of the formulated problem, there might be some local optima that the pro-

TABLE III  
HUNGARIAN METHOD

- 
1. Subtract the entries of each row of  $\mathbf{B}$  by the row minimum, so that each row has at least one zero and all entries are positive or zero.

---

  2. Subtract the entries of each column by the column minimum, so that each row and each column has at least one zero.

---

  3. Select rows and columns across which lines are drawn, in such a way that all the zeros are covered and that no more lines have been drawn than necessary.

---

  4. A test for optimality.
    - (i) If the number of the lines is  $K$ , choose a combination  $\mathbf{A}$  from the modified cost matrix in such a way that the sum is zero.
    - (ii) If the number of the lines is less than  $K$ , go to Step 5.

---

  5. Find the smallest element which is not covered by any of the lines. Then subtract it from each entry which is not covered by the lines and add it to each entry which is covered by a vertical and a horizontal line. Go back to 3.
- 

posed scheme may fall into, even though the Hungarian method can find optimal  $\mathbf{X}$ . From the simulation results, we will show that the problem of local optima is not severe.

The complexity of the Hungarian method is  $O(K^4)$ , so the overall complexity for each iteration of the proposed scheme is  $O(K^2 N \log_2 N + K^4)$ . Since the number of users is much less than the number of subcarriers, the complexity of the proposed algorithm is much lower than the schemes that apply the Hungarian method directly to the subcarrier domain. For example, for IEEE 802.11a, there are 48 subcarriers. For the schemes mentioned above, the complexity is  $N^4 = 5308416$ . When  $K = 4$ , the proposed scheme has the complexity of  $K^2 N \log_2 N + K^4 = 4545$ . Suppose the number of iterations is 10; the complexity is only 0.86% of the complexity of  $O(N^4)$ . Moreover, as shown in the simulation, the convergence is mostly obtained within four to six rounds.

When we apply the algorithm in Table III to the system shown in Fig. 1, each mobile unit tries to negotiate with other mobile units to exchange resources via the BS, which serves as a mediator. The whole system is similar to the market in the real world. People (mobile units) gather in the market place (BS) to exchange their goods (resources such as subcarriers). Since the channel responses for each user over different subcarriers are known in the BS, the bargaining process is performed within the BS without costing bandwidth for signaling between the users and the BS. The random method can be implemented in a distributed manner with limited signaling to form the coalition pairs, while the Hungarian method needs some limited centralized control within the BS to determine the optimal coalition pairs.

## V. SIMULATION RESULTS

In order to evaluate the performance of the proposed schemes, we consider two-user and multiple-user simulation setups. Three different optimization goals (maximal rate, max-min, and NBS) are compared.

First, a two-user OFDMA system is taken into consideration. We simulate the OFDMA system with 128 subcarriers over the 3.2-MHz band. To make the tones orthogonal to each other, the symbol duration is 40  $\mu\text{s}$ . An additional 10  $\mu\text{s}$  guard interval is used to avoid intersymbol interference due to channel delay

spread. This results in a total block length of 50  $\mu\text{s}$  and a block rate of 20 k. The maximal power is  $P_{\max} = 50$  mW, and the desired BER is  $10^{-2}$  (without channel coding). The thermal noise level is  $\sigma^2 = 10^{-11}$  W. The propagation loss factor is three. The distance between user 1 and the BS is fixed at  $D_1 = 100$  m, while  $D_2$  varies from 10 to 200 m.  $R_{\min}^i = 100$  Kb/s  $\forall i$ . Doppler frequency is 100 Hz.

To evaluate the performances, we have tested  $10^5$  sets of frequency-selective fading channels, which is simulated using a four-ray Rayleigh model [24] with the exponential power profile and 100 ns root mean square (RMS) delay spread. Thus, the impulse response of the model can be represented as follows:

$$g(t) = \sum_{l=1}^{L_p} A_l \alpha_l(t) \delta(t - \tau_l) \quad (26)$$

where  $L_p = 4$ ,  $A_l$ , and  $\tau_l$  are the amplitude and time delay for the  $l$ th ray, respectively,  $\alpha_l(t)$  is the channel gain of a flat Rayleigh fading channel, which can be simulated using the Jakes model [25]. Note that the simulated power of each ray is decreasing exponentially according to its delay, and the total power of all rays is normalized as one. The RMS delay spread is the square root of the second central moment of the power delay profile, which is defined as

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \quad (27)$$

where

$$\overline{\tau^2} = \frac{\sum_{l=1}^L A_l^2 \tau_l^2}{\sum_{l=1}^L A_l^2} \quad \text{and} \quad \bar{\tau} = \frac{\sum_{l=1}^L A_l^2 \tau_l}{\sum_{l=1}^L A_l^2}. \quad (28)$$

In Fig. 3, the rates of both users for the NBS, maximal rate, and max-min schemes are shown versus  $D_2$ . For the maximal-rate scheme, the user closer to the BS has a higher rate, and the rate difference is very large when  $D_1$  and  $D_2$  are different. For the max-min scheme, both users have the same rate, which is reduced when  $D_2$  is increasing. This is because the system has to accommodate the user with the worst channel condition. For the NBS scheme, user 1's rate is almost the same, regardless of



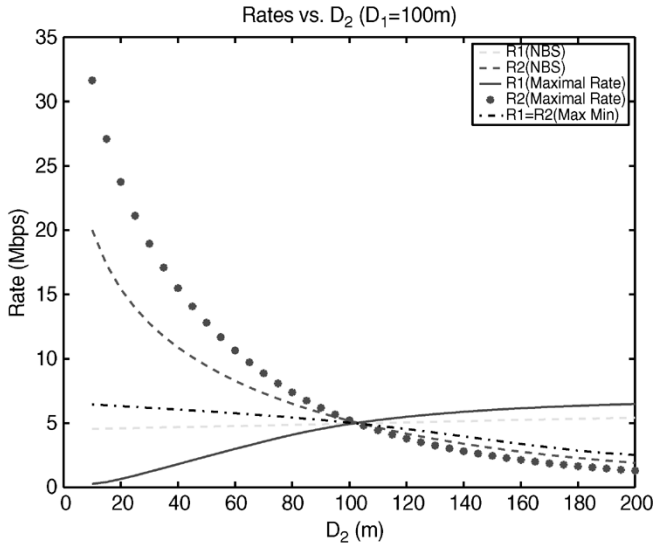
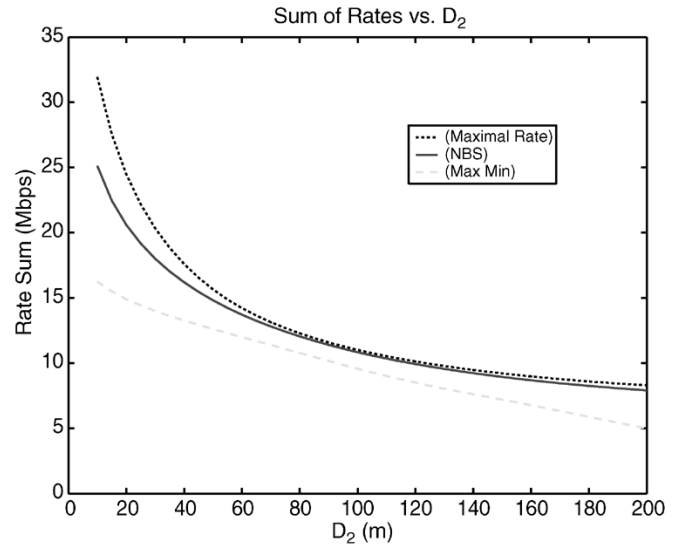
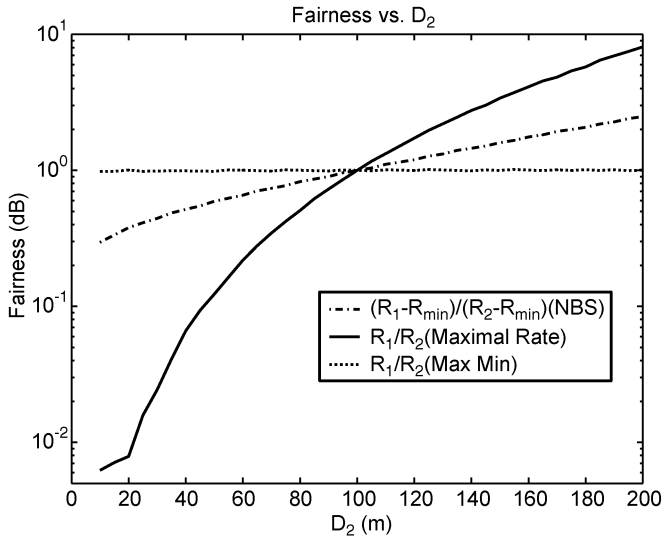
Fig. 3. Each user's rate (Mb/s) versus  $D_2$ .Fig. 5. Overall rate (Mb/s)  $R_1 + R_2$ .

Fig. 4. Fairness for three schemes.

$D_2$ , and user 2's rate is reduced when  $D_2$  is increasing. This shows that the NBS algorithm is fair in the sense that the user's rate is determined only by its channel condition, and not by other interfering users' conditions.

In addition, the ratio of two users' rates is shown in Fig. 4. For the max-min scheme, the ratio is always equal to 1, which is strictly fair but inefficient. For the maximal-rate scheme, the ratio changes greatly for different  $D_2$ , which is very unfair. The user with the better channel condition dominates the resource allocation, while the other user has to starve. The channel gain is mainly determined by the distance and the propagation loss factor. For the proposed NBS scheme, the ratio of  $(R_1 - R_{\min}^1)$  over  $(R_2 - R_{\min}^2)$  changes almost linearly with  $D_2$  in log scale, which shows the NBS fairness. After each user is assigned with the minimal rate requirement, the rest of the resources are allocated to users proportionally according to their channel conditions.<sup>1</sup>

<sup>1</sup>The bumpy part of the max-rate scheme curve when  $D_2$  is small is due to the minimal-rate constraint.

In Fig. 5, we show the overall rate of two users for three schemes versus  $D_2$ . Because the max-min algorithm is for the worst-case scenario, it has the worst performance, especially when the two users have the very different channel conditions, because the user with worse channel conditions limits the usage of the system resources. The NBS scheme has the performance between the maximal-rate scheme and max-min scheme, while the maximal-rate scheme is extremely unfair. Moreover, the performance loss of the NBS scheme to that of the maximal-rate scheme is small. As we mentioned before, the NBS scheme maintains the fairness in a way where one user's performance is unchanged from the other user's channel conditions. The proposed algorithm is a good tradeoff between the fairness and the overall system performance.

We set up the simulations with more users to test the proposed algorithms. All the users are randomly located within the cell of radius 200 m. One BS is located in the middle of the cell. Each user has the minimal rate  $R_{\min}^i = 25$  kb/s. The other settings are the same as those of the two-user case simulations.

In Fig. 6, we show the sum of all users' rates versus the number of users in the system for three schemes. We can see that all three schemes have better performances when the number of users increases. This is because of multiuser diversity, provided by the independent varying channels across the different users. The performance improvement satiates gradually. The NBS scheme has a similar performance to that of the maximal-rate scheme, and has a much better performance than that of the max-min scheme. The performance gap between the maximal-rate scheme and the NBS scheme reduces when the number of users is large. This is because more bargaining pair choices are available to increase the system performance.

In Fig. 7, we show the histogram of the number of rounds that is necessary for convergence of the random method and the Hungarian method with eight users. The Hungarian method converges in about one to six rounds, while the random method may converge very slowly. The average number of rounds for convergence of the random method is 4.25 times that of the Hungarian method. By using the Hungarian method, the best negotiation

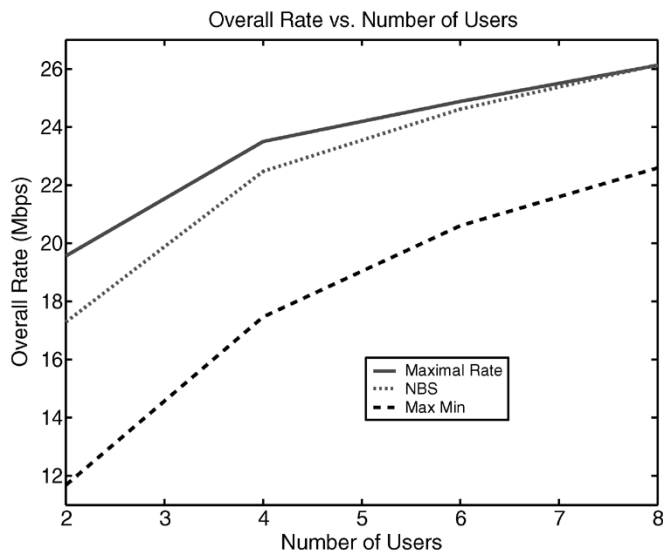


Fig. 6. Overall rate (Mb/s) versus number of users.

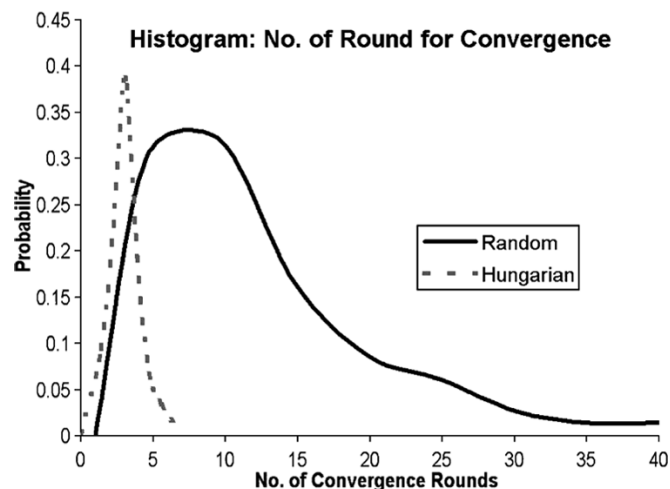


Fig. 7. Histogram for convergence.

pairs can be found. Consequently, the convergence rate is much quicker, and the computation cost is reduced.

In Fig. 8, we show the probability density function of the ratio of  $\prod_{i=1}^K (R_i - R_{\min}^i)$  of the Hungarian method over that of the random method with eight users. If the ratio is larger than one, the Hungarian method converges to a better solution than the random method. From the curve, the Hungarian method converges to a better solution most of the time. This is because the random algorithm finds an arbitrary path for convergence and may fall into different local optima. Notice that for most of the time, the ratio is a small number, so the problem of local optima is not severe. On the other hand, there is a small probability (shown as the shaded area) that the random algorithm has better performance than the Hungarian method. This is because not all six Nash axioms would be satisfied, and the two-user algorithm is suboptimal under low-SNR conditions. Therefore, by using the Hungarian method to find the optimal coalition, we can achieve a better and faster NBS solution for the multiuser situation. Note that the disadvantage of the Hungarian method is that it needs a limited central control in the BS.

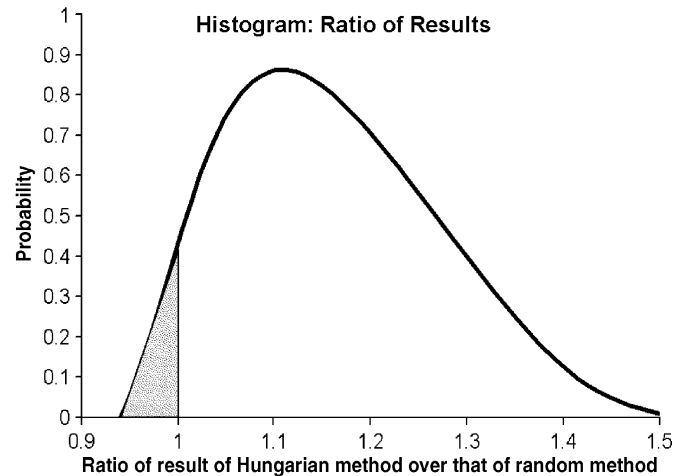


Fig. 8. Histogram for product ratio.

## VI. CONCLUSIONS

In this paper, we use cooperative game theory, including NBS and coalitions, to develop a fair algorithm for adaptive subcarrier, rate, and power allocation in multiuser OFDMA systems. The optimization problem takes consideration of fairness and the practical implementation constraints. The proposed algorithm consists of two steps. First, a Hungarian method is constructed to determine optimal bargaining pairs among users. Then a fast two-user bargaining algorithm is developed for two users to exchange their subcarriers. The above two steps are taken iteratively for users to negotiate the optimal resource allocation. The proposed fast implementation has the low complexity of  $O(K^2 N \log_2 N + K^4)$  for each iteration, which is much lower than that of the existing schemes.

From the simulation results, the proposed algorithm shows a similar overall rate to that of the maximal-rate scheme, and much better performance than that of the max-min scheme. The NBS fairness is demonstrated by the fact that a user's rate is not determined by the interfering users. The proposed algorithm provides a near-optimal fast solution, and finds a good tradeoff between the overall rate and fairness. The significance of the proposed algorithm is the bargaining and NBS fairness that result in the fair individual performance and good overall system performance.

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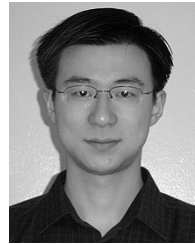
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