Dynamic Distributed Rate Control for Wireless Networks by Optimal Cartel Maintenance Strategy

Zhu Han, Zhu Ji, and K.J. Ray Liu
Electrical and Computer Engineering Department, University of Maryland, College Park

Abstract— **In this paper, we consider wireless networks with multiple distributed users. The users transmit their data packets to a communication node via some multi-access protocol. Then the communication node transmits the packets to a remote destination via a lossy wireless link. The cost for each user's transmission is modelled as a function of its transmission rate. If a user's packet is successfully transmitted to the destination, a profit will be obtained. However, this profit is reduced by the transmission contention among users and the lossy wireless channel. Each user controls its transmission rate to maximize its profit in a distributed manner. If no rule is applied for rate control, each user will select a large transmission rate resulting in having a low performance. In order to improve the system performance, we develop a distributed rate control algorithm using optimal Cartel maintenance strategy. In this strategy, each user prefers to cooperate for rate control, because deviation from cooperation will be punished by other users for a period of time. Since the users want to optimize their performances over time, there is no incentive for them to deviate to gain a benefit while they have to endure more punishment in the future. From the simulation results, we can see that the proposed distributed scheme can enforce the cooperation among users and achieve much better performance than the noncooperative transmission.** [1]

I. Introduction

In wireless networks, there exists competition among users to share the system resources. Efficient resource allocation such as rate control is an important technology to increase the system performance by controlling users' appetite for resources. In some wireless networks with scattered topology, resource allocation scheme must be implemented in a distributed way. Moreover dynamic optimization over time is necessary for the wireless system with fluctuating channels and long term goals. The above aspects challenge the design of a dynamic distributed resource allocation scheme, which becomes an important research topic recently.

Dynamic resource allocation in different wireless networks was discussed in details in [1]. Since individual mobile users do not have the knowledge of other users conditions and cannot cooperate with each other, they act selfishly to maximize their own performances in a distributed fashion. Such a fact motivates us to adopt the game theory [5]. Repeated game theory analyzes the behaviors of users in multiple stages, so it can be applied to analyze the dynamic optimization of the wireless resource allocation. In [2], repeated game theory was applied to routing problems. In [3], multiple access resource allocation was studied using game theory approach. In [4], repeated game was further applied to physical layer problems.

[1]The authors would like to thank Professor Peter Cramton for his teachings and helps

In this paper, we consider a special case of wireless networks. Suppose there are many distributed users in a local area. They share a communication node to communicate with a remote destination. There are costs for the users to transmit their packets and also benefits if their packets are successfully transmitted. The successful probability is affected by two factors: the channel condition from the communication node to the destination and the competition among users to share the communication node. So the problem is how to control each user's transmission rate in a distributed manner such that the overall profit (i.e. benefit minus cost) will be maximized. The above wireless networks fit a variety of practical situations such as wireless sensor networks, where a strong communication node collects sensors' data and transmits them back.

In order to solve the above problem, we are inspired by the micro-economy approach in [6]. We propose a scheme that users agree to cooperate at some transmission rates first. At each time, users will observe the probability of the successful transmission. If the probability is lower than some threshold, it probably means some users deviate from the agreed transmission rates and cause more contention in the communication node. Under this condition, the other users will transmit non-cooperatively at much higher rates for a period of time. Consequently, the probability of successful transmissions drops dramatically. Since users optimize their rates over time, the gain for the deviation will be wiped out by the loss caused by the punishment. Because of this reason, all users have no incentive to deviate from the agreed transmission rates. So the proposed scheme can force the users to cooperate in a distributed manner.

This paper is organized as follows: In Section II, the wireless multiuser system is described and system model is presented. In Section III, we propose the distributed rate control scheme using optimal Cartel maintenance strategy. In Section IV, simulation results are provided. Conclusion is given in Section V.

II. System Descriptions and Models

Fig. 1 shows the block diagram of the multiuser wireless networks. There are many distributed users and one communication node. Each user can transmit its data packets to the communication node by using the multiple access protocols such as Aloha, CSMA, etc. The communication node has the ability to transmit the data packets to the remote destination via a wireless link. We assume there is a reliable feedback channel. So, the system can be described as multiple users sharing a communication link. Without loss of generality, we assume users are homogenous. (Het-

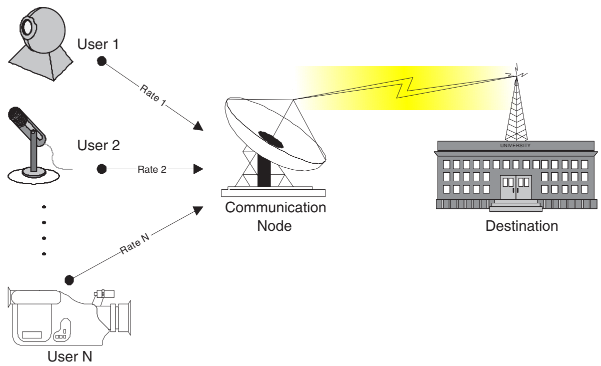

Fig. 1. Wireless Networks Block Diagram

erogeneous case can be extended in a similar way of this paper.) Each user can control its transmission rate. The users need to compete with each other for the communication link which is fluctuating due to the wireless channel. Thus one user's rate can affect the performances of itself, other users, and the whole system. So it is necessary to find a rate control algorithm such that the system can operate at the optimal point. Moreover, it is hard to have communication channels among users. Distributed algorithm is required for rate control.

There are N users in wireless networks. The transmission time for the data packets is divided into time slots. The users transmit their packets by the Poisson distribution. The average transmission rate vector for all users is denoted by $\overrightarrow{\lambda}^t = [\lambda_1^t, ..., \lambda_N^t]'$, where λ_i^t is the rate of user i in period t. Total arriving rate at the communication node is then $\Lambda^t = \sum_{i=1}^N \lambda_i^t$. Each user intends to increase its transmission rate. However, arbitrary increases of the transmission rates will result in a higher probability of collision at the communication node and reduce the system throughput. In addition, the probability of successful transmission at the communication node is also affected by the wireless link quality from the communication node to the destination. The overall probability of successful transmission can be observed by all users and can be expressed as:

$$\hat{P}_t = P(\Lambda^t)\theta_t, \tag{1}$$

where $P : \Re_+ \rightarrow \Re_+$ denotes the system throughput which is a function of Λ^t for the multiple access protocol. θ_t is the channnel error probability, which is an identically and independently distributed sequence of random variables with mean μ, probability density function (PDF) f, and cumulative distribution function (CDF) F. In this paper, for simplicity, we approximate P as a linear and decreasing function in total arriving rate Λ^t as

$$P(\Lambda^t) = a - b\Lambda^t, \tag{2}$$

where a and b are positive constants. This approximation fits the scenarios of the highly-loaded wireless networks, where the attempted transmissions at each time slot is high. Other approximations can also be applied in a similar way.

For each user, the cost function for the transmissions is hypothesized to be homogeneous as

$$C_t = c_0 + c_1\lambda_i^t, \tag{3}$$

where c_0 is the basic cost to maintain the link and c_1 is the cost per transmission rate.

Then, we omit subscript t for simplicity. For each successful transmission, the user has the benefit of c_2. User i has the profit as the benefit minus cost as

$$\pi_i(\overrightarrow{\lambda}) = c_2\hat{P}_t\lambda_i - C = c_2\theta(a - b\Lambda)\lambda_i - c_0 - c_1\lambda_i. \tag{4}$$

Given $\Lambda_{-i} = \Lambda - \lambda_i = \sum_{j\neq i}\lambda_j$, the total arrival rate of the other users, the single time slot expected profit can be further represented as:

$$\pi_i(\overrightarrow{\lambda}) = [A - B(\Lambda_{-i} + \lambda_i)]\lambda_i - c_0, \tag{5}$$

where $A = \mu a - c_1$ and $B = \mu b$. It is assumed that

$$0 < c_1 < \mu a \tag{6}$$

and

$$0 < c_0 < (\mu a - c_1)^2/\mu b(N+1)^2. \tag{7}$$

(6) and $b > 0$ imply that A and B are positive constants.

III. Cartel Maintenance Strategy

In this section, we first present the motivation of our proposed strategy. Then we formulate the problem and construct a distributed algorithm for each user. Finally, the optimal parameters of the algorithm are deduced.

A. Motivations

Since users are located distributively, they act noncooperatively and independently to increase their profits by adapting their rates, i.e.,

$$\arg\max_{\lambda_i} \pi_i. \tag{8}$$

Let $\overrightarrow{s} = [s_1, ..., s_N]'$ denote the optimal rate vector for the above noncooperative optimization. By taking the derivative, we have

$$s_i = s = A/B(N+1),\ \forall i, \tag{9}$$

and

$$\pi_i(\overrightarrow{s}) = [A^2/B(N+1)^2] - c_0,\ \forall i. \tag{10}$$

(7) guarantees that users earn positive profits.

On the other hand, if there exists a centralized control, users can cooperate to maximize the system overall profit. Then, the optimization goal is

$$\arg\max_{\overrightarrow{\lambda}} \sum_{i=1}^N \pi_i. \tag{11}$$

Denote the rate vector which maximizes expected joint overall profit by $\overrightarrow{r} = [r_1, ..., r_N]'$. Given Λ_{-i}, we have the solutions as

$$r_i = r = A/2BN,\ \forall i \tag{12}$$

and

$$\pi_i(\overrightarrow{r}) = (A^2/4BN) - c_0,\ \forall i. \tag{13}$$

From (9), (10), (12) and (13), we can see that, as long as there is more than one user, single user's expected profits with centralized control will be higher than those noncooperative results. However there exist two problems for centralized control. First, the network topology is distributed and it is hard to implement the centralized control. Second, it can be shown that if any user deviates from (12) and transmits at a higher rate, it can get a greater benefit in (13). So instead of cooperating, users will deviate because of their greediness. At a result, the competition for the communication node becomes intense and P will decrease. Consequently, each user's profit will drop to the noncooperative value in (9). So ***our goal*** is to construct a distributed scheme such that users have to cooperate and have no incentive to deviate.

One possible solution comes from game theory literature [6]. In staged repeated games, the player's overall payoff is weighted average payoffs over time. At each stage, the "noncooperative" game can be played, which means that the players' choices are based only on their perceived self-interest. On the other hand, players can also play "cooperative" games, which allows the cooperation among users to achieve better payoffs. If some consequences of the player's actions can be observed at the end of each stage, it becomes possible for players to adjust their strategy, which can lead to better equilibrium outcomes that do not arise when the game is played only once. So we want to construct the scheme based on the following underlying rationale: At the beginning, all players agree to operate in the cooperative way. If any player deviates, in the next stage, other players will observe the deviation and play in the noncooperative way instead. So this deviating player will get less because of the punishment due to the other players. Since each player tries to optimize the payoff over time, no player will have the incentive to deviate from the cooperative stage. So using the threaten of punishment from other users, system forces all users to act cooperatively in a distributed manner.

For the system shown in Fig. 1, each user can play either non-cooperatively or cooperatively. Each user tries to optimize its rate such that the overall profit over time can be optimized. At each time slot, each user can observe the transmission status such as the successful transmission probability in the communication node. The above facts motivate us to apply the repeated game approach.

B. Problem Formulation and Algorithm

In the wireless networks, each user's goal is to maximize a discounted expected payoff over time slots. We define the discount factor as β. For most applications of wireless network, β is close to 1. The optimization problem can be represented as

$$\max_{\lambda_i^t} \sum_{t=0}^{\infty} \beta^t \pi_i(\lambda_i^t, \Lambda_{-i}^t),\ \forall i. \tag{14}$$

TABLE I

CARTEL MAINTENANCE ALGORITHM

Initialization:
$t = 0$ is a cooperative period;
Strategy:
If t is a cooperative period and $\hat{P}_t \geq P^*$, then $t+1$ is a cooperative period;
If t is a cooperative period and $\hat{P}_t < P^*$, then $t+1, ..., t+T-1$ are noncooperative periods with $\lambda_i = s, \forall i$, and $t+T$ is a cooperative period.

Intuitively, every user wants to maximize its own expected profit by increasing its transmission rate. It will result in too many collisions among users which limit each user's profit. Thus, we need to introduce certain mechanism, namely, the game rule, to force the users act cooperatively to achieve better profits and being robust to the cheating phenomenon.

In [6], Porter developed a Cartel trigger-price strategy for dynamical industry model, where a company deters others from deviating from collusive output levels by threatening to produce at noncooperative quantities for a period of duration whenever the market price falls below some trigger price. Based on the similar idea, we develop the Cartel maintenance algorithm in Table I.

At the beginning, all users are in a cooperative period with rate $\overrightarrow{\lambda}$. Then they will monitor the overall probability of successful transmission in (1). If the probability is higher than some threshold P^*, it means that all users transmit at the cooperative operating rate probably. On the other hand, if the probability drops lower than the threshold, it means that some users may cheat. Then the other users will play punishment by transmitting non-cooperatively according to (9) for a period of T. Then they will come back to play the cooperative period again. Since users are afraid of future punishment, they are inclined to play cooperatively.

The probability of successful transmission in the communication node is determined by two factors: the users' rates and the wireless channel link condition. It is possible that all users act cooperatively but the probability is still under the threshold because of the bad channel. Under this situation, the users will play the non-cooperative period, because they cannot tell if the low successful transmission probability is caused by the deviations or the bad channels. This is a penalty for the distributed implementation.

C. Derivation of Optimal Parameters

The remaining problem is how to find the optimal values of cooperative rate $\overrightarrow{\lambda}$, threshold P^* and punishment duration T, which we will show in this subsection.

In cooperative periods, the expected discounted profit of user i is given by

$$V_i(\overrightarrow{\lambda}) = \pi_i(\overrightarrow{\lambda}) + P_r\{\hat{P}_t \geq P^*\}\beta V_i(\overrightarrow{\lambda}) +$$

 0-7803-8794-5/04/$20.00

$$P_r\{\hat{P}_t < P^*\} \left(\sum_{\tau=1}^{T-1} \beta^\tau \pi_i(s) + \beta^T V_i(\overrightarrow{\lambda}) \right), \quad (15)$$

where the first term on right hand side (RHS) is the current expected value, the second term on RHS is the expected value of next period if cooperation, and the third term on RHS is the expected value of next period if noncooperation. (15) can be rewritten as:

$$V_i(\overrightarrow{\lambda}) = \frac{\pi_i(s)}{1-\beta} + \frac{\pi(\overrightarrow{\lambda}) - \pi(s)}{1-\beta+(\beta-\beta^T)F(P^*/P)}, \quad (16)$$

where F is CDF of random variable θ. In order to find the optimal transmission rate vector, threshold, and punishment duration, the following derivatives are set to zeros:

$$\frac{\partial V_i}{\partial \overrightarrow{\lambda}} = 0, \ \frac{\partial V_i}{\partial P^*} = 0, \text{ and } \frac{\partial V_i}{\partial T} = 0. \quad (17)$$

Since the system is homogenous, the optimal transmission rate is the same for all users. Heterogeneous case can be analyzed in a similar way. The detailed deductions is similar in [6]. The optimal λ^*, P^*, and T^* can be determined by the following steps:

$$\begin{aligned} \lambda^* &= \frac{A}{2BN}\left(\frac{N+\eta^*+(N+1)(a/A)}{N+1+\eta^*}\right), \text{ if } \eta^* > \eta^0, \\ &= s, \qquad \text{otherwise}; \end{aligned} \quad (18)$$

where

$$\eta^0 = \frac{(N+1)[(N+1)(a/A)-N]}{(N-1)}$$

and

$$\eta^* = \frac{f(P^*/P(\lambda^*))}{F(P^*/P(\lambda^*))} \frac{P^*}{P(\lambda^*)}.$$

P^* is determined by:

$$\frac{f(P^*/P(\lambda^*))}{F(P^*/P(\lambda^*))} \frac{P^*}{P(\lambda^*)} - \frac{f'(P^*/P(\lambda^*))}{f(P^*/P(\lambda^*))} \frac{P^*}{P(\lambda^*)} = 1, \quad (19)$$

where f and f' is the PDF and its derivative of θ, respectively. The optimal P^* and λ^* are calculated by (18) and (19) iteratively. Obviously we have $\lambda^* \in (r, S]$, and $\lambda^* \to r$ when $\eta^* \to \infty$. After the values converge, we can calculate the optimal punishment period as:

$$T^* = \frac{1}{\ln \beta} \ln\{\beta - \frac{(1-\beta)[A-(N+1)B\lambda^*]}{f(\theta^*)(b\theta^*/P(\lambda^*))\Delta - F(\theta^*)[A-(N+1)B\lambda^*]}\} \quad (20)$$

where $\theta^* = P^*/P(\lambda^*)$ and $\Delta = \pi_i(\lambda^*) - \pi_i(s)$.

IV. Simulation Results

We assume there are $N = 10$ users in the networks. The basic cost is $c_0 = 0.001$, profit per success transmission is $c_2 = 1$, and β is a number very close to 1. We assume the slotted nonpersistent CSMA as the multi-access protocol in the communication node. Without loss of generality, we assume unit service rate. The probability of successful transmission is represented as [7], [8]

$$P = \frac{\alpha \Lambda e^{-\alpha\Lambda}}{1-e^{-\alpha\Lambda}+\alpha}, \quad (21)$$

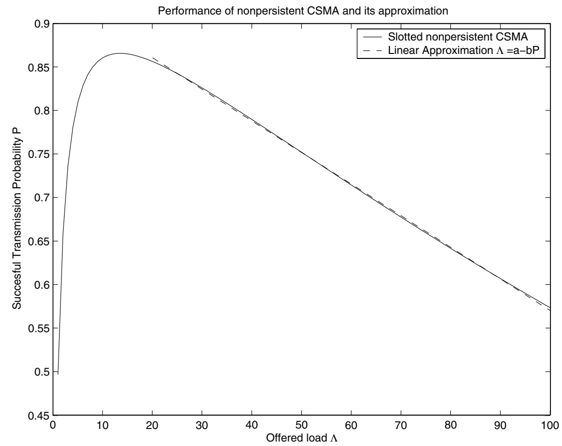

Fig. 2. Linear Approximation

where Λ is the offered load and α is one-way normalized propagation delay which is assume equal to 0.01. In Fig. 2, we show the linear approximation of P using function $P \approx a - b\Lambda$. The linear approximation is shown to be accurate when $\Lambda > 20$, i.e., when the system is overloaded. Here $a = 0.9331$ and $b = -0.0036$.

The distribution of θ is approximated by binomial distribution, which gives the probability of packet loss over Bernoulli trails. The binomial distribution can be approximated by normal distribution. Here we fix the mean of normal distribution as unit and vary the variance to show the performances. Any probability larger than 1 or less than 0 will be truncated. Other probability distributions of packet errors can be applied in a similar way.

In Fig. 3, we show the overall profit as a function of cost per transmission rate c_1 for variance equal to 0.015. Obviously, the overall profit will decrease when increasing c_1, which is because of the increases of the cost. If users play with centrally controlled rate r, the overall profit is much greater than the profit if the users play non-cooperatively with rate s. The proposed scheme has the performance in between, which is because the users prefer to send the packets at the rate λ^* rather than s so as to avoid future punishment. When c_1 is a small number, the cost for transmission is low and the users would like to transmit at higher rate. If users transmit non-cooperatively, the competitions for the communication nodes will be high. As a result, the overall profit is much less than that if they play cooperatively. For the proposed scheme, because users are afraid of others' punishment, they show a behavior of cooperation. So the overall profit is close to the centralized control result.

We vary the variance of the distribution of θ. The channel condition becomes worse when the variance increases. In Fig. 4, we show the effects of channels on the overall profit. In Fig. 5, we show the probability of successful transmission vs. the channel variance. We also show the threshold P^*. When the variance is small, the proposed scheme has the similar overall profit to that of centralized control. On the other hand, if the channel becomes worse, the performance of the proposed scheme drops to the

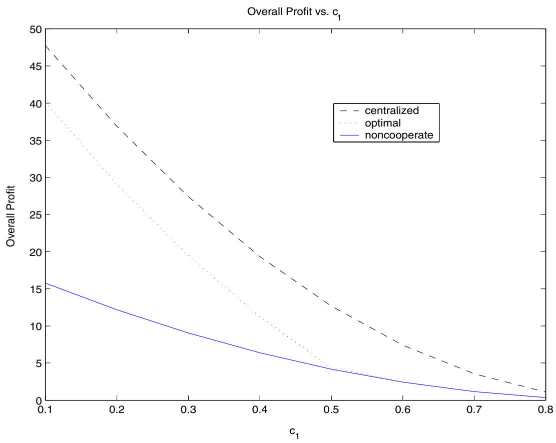

Fig. 3. Overall Profit vs. Cost per Transmission

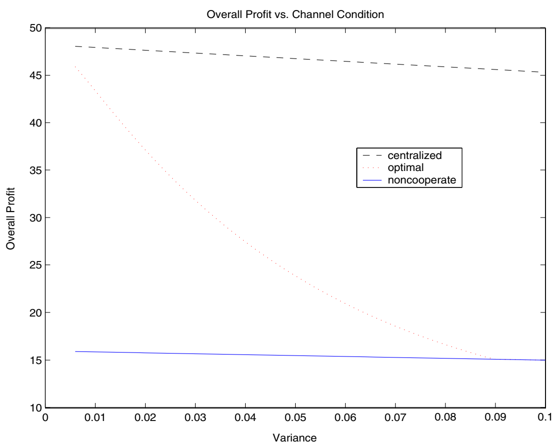

Fig. 4. Overall Profit vs. Channel Condition

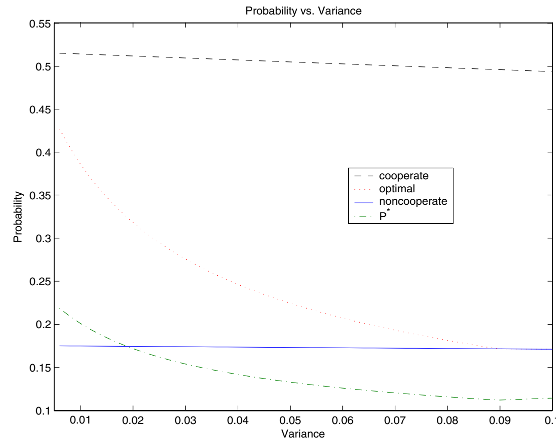

Fig. 5. Probabilities vs. Channel Condition

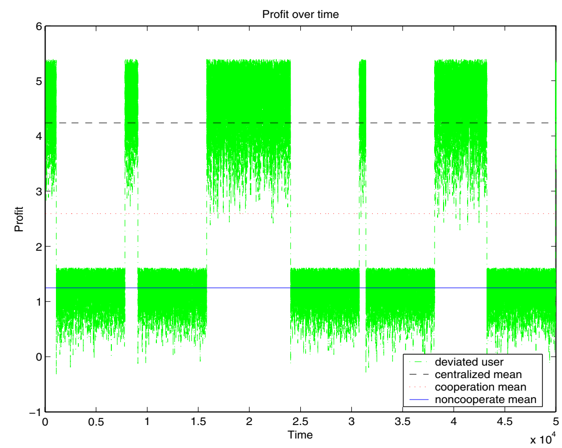

Fig. 6. Punishment for Deviation

noncooperative case. Under this condition, the users cannot distinguish if the probability of successful transmission drops because of the other user's deviation or the channel deterioration. Consequently, they will prefer to transmit more and play non-cooperatively.

In Fig. 6, we show how the scheme punishes the cheating user. Here channel variance is 0.015 and $c_1 = 0.1$. We assume one user deviates from the optimal λ^* and transmit at the higher rate s, while others transmit at λ^*. We show the profit of this deviating user over time. For comparison, we also show the average profit when the user transmit at r, λ^*, and s. We can see that at first the user does get more profit by deviating from λ^*. However this deviation is soon detected by others and punishment phase is performed by other users. So the gain is eliminated over time. This shows the reason why the proposed scheme can enforce the cooperation among users by threatening punishment.

V. Conclusions

In this paper, we develop a distributed rate control scheme using optimal Cartel maintenance strategy. Users will cooperate at some agreed transmission rate first. Then they monitor the success transmission probability of the communication node. If the probability is less than some threshold, the users believe some user has deviated from the agreed rate. Then they will transmit non-cooperatively at a higher rate for a period time to punish the deviating user. So the users show cooperation because they are afraid of punishment if they deviate. We deduce the optimal transmission rate, punishment rate, and punishment time for the proposed scheme. The simulation results show the overall system profit can be greatly improved with the proposed distributed scheme, while the cheating users are punished. Further research can be done for the more sophisticate and realistic wireless communication environments.

References

[1] J. Zander and S.L. Kim, *Radio resource managment for wireless networks*, Artech House Publishers, 2001.

[2] R. J. La and V. Anantharam, "Optimal Routing Control: Repeated Game Approach", *IEEE Transactions on Automatic Control*, pp 437-450, March 2002.

[3] A. B. MacKenzie and S. B. Wicker, "Stability of Multipacket Slotted Aloha with Selfish Users and Perfect Information," *IEEE INFOCOM Conference*, 2003.

[4] A. B. MacKenzie and S. B. Wicker, "A Repeated Game Approach to Distributed Power Control in CDMA Wireless Data Networks," *in revision for IEEE Transactions on Wireless Communications.*

[5] D. Fudenberg and J. Tirole, *Game Theory*, The MIT Press, Cambridge, 1991.

[6] R. H. Porter, "Optimal Cartel Trigger Price Strategies," *Journal of Economic Theory*, vol. 29, pp. 313-318, Apr. 1983.

[7] L. Kleinrock, *Queueing Systems, Volumn II: Computer Applications*, John Wiley and Sons, 1976.

[8] D. Bertsekas and R. Gallager, *Data Networks*. Englewood Cliffs, N. J.: Prentice-Hall, 1992.