Single-Block Differential Transmit Scheme for Frequency Selective MIMO-OFDM systems

Thanongsak Himsoon, Weifeng Su, and K. J. Ray Liu
Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742.

***Abstract*— In this paper, we propose a differential encoding and decoding scheme for MIMO-OFDM systems under frequency-selective fading channels. We differentially encode signal within each OFDM symbol period. The scheme does not only reduce encoding and decoding delay, but also relaxes the restriction on channel assumption. The successful differential decoding of the proposed scheme depends on the assumption that fading channels keep constant over two OFDM symbol periods rather than multiple of them as required in previous schemes. We provide the pairwise error probability formulation, and quantify the performance criteria in terms of diversity and coding advantages. Our design criteria reveals that the existing diagonal cyclic codes can be applied to achieve full diversity with high coding gain. Performance simulations in various channel conditions show that our proposed scheme yields superior performance to the previously proposed differential schemes.**

I. INTRODUCTION

Differential space-time (DST) modulation [1]-[4] has been widely accepted as one of many practical alternatives that bypasses multi-channel estimation in frequency non-selective multiple-input multiple-output (MIMO) system. Recently, a technique of incorporating the DST modulation with orthogonal frequency division multiplexing (OFDM) transmission, called differential space-time-frequency (DSTF) MIMO-OFDM [5]-[11], was introduced for wideband systems under frequency-selective fading environments. The DSTF scheme differentially encodes across spatial, temporal, and frequency domains such that both spatial and frequency diversities can be explored. However, a complete transmission of one DSTF codeword expands several OFDM symbol periods which are, in fact, proportional to the number of transmit antennas. In order to perform successful differential decoding, all of the DSTF schemes in [5]-[11] assumed that the fading channels keep constant within several OFDM blocks and slowly change from a duration of several OFDM blocks to another OFDM blocks. Nevertheless, such channel condition is not valid in most practical situations since the channel coefficients would change before two entire DSTF codeword matrices are completely transmitted. The related work on non-coherent space-frequency (SF) coding has been investigated in [12]; however, a set of SF codes was obtained through random search, and the scheme introduced high decoding complexity.

In this paper, we propose a differential encoding and decoding scheme for MIMO-OFDM system which is able to transmit the differentially encoded signal matrix within one OFDM symbol period, regardless of the number of transmit antennas. The scheme allows us to relax the channel fading assumption to vary from a duration of one OFDM block to the next, but remain approximately constant over only two OFDM symbol periods. The pairwise error probability analysis in case of frequency-selective fading channels with arbitrary power delay profiles is also given. We address design criteria of the proposed scheme, and it reveals that the diagonal cyclic codes [3] can be used to achieve the maximum diversity order with high coding gain. The merit of our proposed scheme is shown through computer simulations.

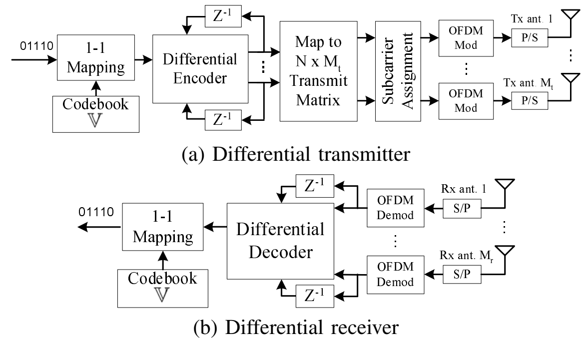

Fig. 1: Description of the differential MIMO-OFDM system.

The rest of the paper is organized as follows. Section II outlines the system description. In Section III, we derive the differential encoding and decoding scheme for MIMO-OFDM systems. The pairwise error probability is analyzed, and the design criteria of the proposed scheme is given in Section IV. We show some simulation results and discussions in Section V. Finally, Section VI concludes the paper.

II. SYSTEM DESCRIPTION

We consider a MIMO wireless communication system equipped with M_t transmit and M_r receive antennas, as shown in Figure 1. Each antenna employs an OFDM modulator with N subcarriers. In each transmit-receive link, the frequency-selective fading channel is assumed to have L independent delay paths with arbitrary power delay profiles, and the base-band equivalent channel is modelled by

$$h_{ij}^k(t) = \sum_{l=0}^{L-1} \alpha_{ij}^k(l)\delta(t-\tau_l), \tag{1}$$

where $\alpha_{ij}^k(l)$ is the path gain coefficient of the l^{th} path between transmit antenna i and receive antenna j at the k^{th} OFDM symbol period, and τ_l represents the l^{th} path delay. The $\alpha_{ij}^k(l)$ is modelled as zero-mean complex Gaussian random variable with variance $E\left|\alpha_{ij}^k(l)\right|^2 = \delta_l^2$. The channel coefficients are assumed to be spatially uncorrelated and the power of the L independent delay paths is normalized such that $\sum_{l=0}^{L-1}\delta_l^2 = 1$.

At the transmitter, an information bit sequence is differentially encoded and mapped onto an $N \times M_t$ transmit matrix $\mathbf{X}^k = \left[\mathbf{x}_1^k \cdots \mathbf{x}_{M_t}^k\right]$. The $N \times 1$ vector $\mathbf{x}_i^k, i = 1, \cdots, M_t$,

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of $\mathbf{X}^k$ has $x_i^k(n)$ as its $\{n,i\}^{th}$ element which represents a differentially encoded symbol to be transmitted over the n^{th} subcarrier by the i^{th} antenna during the k^{th} OFDM block. We assume that $\mathbf{X}^k$ is normalized to satisfy the energy constraint $E\|\mathbf{X}^k\|_F^2 = N$, where $\|\cdot\|_F$ denotes the Frobenius norm [17]. We will explain details of the proposed differential encoding and decoding scheme in Section III. In order to transmit $\mathbf{X}^k$, each of the i^{th} column of $\mathbf{X}^k$ is OFDM modulated using N-point IFFT and augmented by cyclic prefix. The resulting OFDM symbol is transmitted over the i^{th} transmit antenna. Note that all of the M_t OFDM symbols are transmitted simultaneously over different transmit antennas within one OFDM symbol period.

At each receive antenna, the receiver performs match filtering, cyclic prefix removing, and OFDM demodulating by N-point FFT. The received signal is a noisy superposition of transmitted symbols from multiple transmit antennas. We model the received signal at the n^{th} subcarrier at the j^{th} receive antenna during the k^{th} OFDM block as

$$y_j^k(n) = \sqrt{\rho}\sum_{i=1}^{M_t} x_i^k(n) H_{ij}^k(n) + w_j^k(n), \tag{2}$$

where ρ is the average signal to noise ratio per receiver, and

$$H_{ij}^k(n) = \sum_{l=0}^{L-1} \alpha_{ij}^k(l) e^{-j2\pi n \Delta f \tau_l} \tag{3}$$

is the subchannel gain. Here, $\Delta f = 1/T_s$ is the inter-subcarrier spacing, and T_s is the OFDM symbol period. The additive noise $w_j^k(n)$ is modelled as independent complex Gaussian random variable with zero mean and unit variance, $\mathcal{CN}(0,1)$. We observe from (2) that the OFDM modem converts a frequency-selective fading channel into a set of parallel flat fading channels. The differential modulation scheme does not require the knowledge of channel state information at either the transmitter or the receiver. However, the subchannel gains are assumed constant over two OFDM symbol periods, i.e. $H_{ij}^k(n) \approx H_{ij}^{k-1}(n)$.

Let $\mathbf{y}_j^k = \left[y_j^k(0), y_j^k(1), \cdots, y_j^k(N-1)\right]^T$ be an $N\times 1$ vector comprising the received signal at the j^{th} receive antenna during the k^{th} OFDM symbol period. We can describe $\mathbf{y}_j^k$ as

$$\mathbf{y}_j^k = \sqrt{\rho}\ \mathcal{D}(\mathbf{X}^k)\mathbf{h}_j^k + \mathbf{w}_j^k, \tag{4}$$

where we denotes $\mathcal{D}(\mathbf{X}^k)$ as an operation on an $N\times M_t$ matrix $\mathbf{X}^k$ that converts each column of $\mathbf{X}^k$ into a diagonal matrix and results in an $N \times NM_t$ matrix, expressed by

$$\mathcal{D}(\mathbf{X}^k) = \mathcal{D}(\left[\mathbf{x}_1^k \cdots \mathbf{x}_{M_t}^k\right]) = \left[diag(\mathbf{x}_1^k)\cdots diag(\mathbf{x}_{M_t}^k)\right]. \tag{5}$$

In (4), the $NM_t \times 1$ channel gain vector $\mathbf{h}_j^k$ is represented by $\mathbf{h}_j^k = \left[(\mathbf{h}_{1j}^k)^T \cdots (\mathbf{h}_{M_t j}^k)^T\right]^T$ in which $\mathbf{h}_{ij}^k = \left[H_{ij}^k(0)\cdots H_{ij}^k(N-1)\right]^T$ and the noise vector has the form $\mathbf{w}_j^k = \left[w_j^k(0)\cdots w_j^k(N-1)\right]^T$. By stacking all M_r received signal vectors together, we obtain the $NM_r\times 1$ received vector

$$\mathbf{y}^k = \sqrt{\rho}\left(\mathbf{I}_{M_r} \otimes\ \mathcal{D}(\mathbf{X}^k)\right)\mathbf{h}^k + \mathbf{w}^k, \tag{6}$$

where $\mathbf{y}^k = \left[(\mathbf{y}_1^k)^T\ (\mathbf{y}_2^k)^T \cdots (\mathbf{y}_{M_r}^k)^T\right]^T$, $\mathbf{h}^k = \left[(\mathbf{h}_1^k)^T\ (\mathbf{h}_2^k)^T \cdots (\mathbf{h}_{M_r}^k)^T\right]^T$, $\mathbf{w}^k = \left[(\mathbf{w}_1^k)^T\ (\mathbf{w}_2^k)^T \cdots (\mathbf{w}_{M_r}^k)^T\right]^T$, and $\otimes$ denotes the tensor product [17].

III. Single-Block Differential Transmit scheme

In what follows, we propose a differential encoding and decoding scheme for MIMO-OFDM systems under frequency-selective fading channels. By taking advantage of the coding strategy in [13], the proposed scheme is able to completely transmit the differentially encoded signal matrix within one OFDM symbol period. This allows us to relax the channel assumption for efficient differential decoding. Specifically, our scheme requires that the fading channels keep constant within only one OFDM block, and slowly change from one OFDM block to the next.

A. Transmit Signal Structure

We will introduce a differential encoding and decoding scheme based on a transmit scheme proposed in [13]. Specifically, for an integer Γ such that $1 \le \Gamma \le L$, a transmit signal matrix $\mathbf{X}^k$ is partitioned into $P = \lfloor N/(\Gamma M_t)\rfloor$ sub-matrices as follows [13]:

$$\mathbf{X}^k = \left[(\mathbf{X}_1^k)^T\ (\mathbf{X}_2^k)^T\ \cdots\ (\mathbf{X}_P^k)^T\ \ (\mathbf{0}_{N-P\Gamma M_t})^T\right]^T, \tag{7}$$

where $\mathbf{0}_{N-P\Gamma M_t}$ denotes an $(N-P\Gamma M_t)\times M_t$ zero padding matrix to be inserted if N cannot be divided by ΓM_t. The $\Gamma M_t \times M_t$ matrix $\mathbf{X}_p^k$, for $p = 1, 2, \ldots, P$, is modelled as

$$\mathbf{X}_p^k = diag(\mathbf{x}_{p,1}^k\ \mathbf{x}_{p,2}^k\ \cdots\ \mathbf{x}_{p,M_t}^k), \tag{8}$$

where $\mathbf{x}_{p,i}^k$, for $i = 1, 2, \ldots, M_t$, is a $\Gamma \times 1$ vector,

$$\mathbf{x}_{p,i}^k = \left[s_{p,(i-1)\Gamma+1}^k\ s_{p,(i-1)\Gamma+2}^k\ \cdots\ s_{p,i\Gamma}^k\right]^T, \tag{9}$$

and all $s_{p,m}^k$, $m = 1, 2, \ldots, \Gamma M_t$, are differentially encoded symbols that will be specified later.

We now specify information matrices as follows. For each p, $p = 1, 2, \ldots, P$, let $\mathbf{V}_p^k$ denote a $\Gamma M_t \times \Gamma M_t$ unitary information matrix having diagonal form as

$$\mathbf{V}_p^k = diag([v_{p,1}^k\ v_{p,2}^k \cdots\ v_{p,\Gamma M_t}^k]^T), \tag{10}$$

in which $v_{p,m}^k$, $m = 1, 2, \ldots, \Gamma M_t$, is an information symbol to be transmitted over subcarrier $(p-1)\Gamma M_t + m$ during the k^{th} OFDM symbol period. We will independently design the matrix $\mathbf{V}_p^k$ for different p. The set of all possible information matrices constitutes a constellation $\mathbb{V}_p$. In order to support a data rate of $R\ b/s/Hz$, $\mathbb{V}_p$ is designed to have constellation size $\mathcal{L} = |\mathbb{V}_p| = 2^{R\Gamma M_t}$.

B. Differential Encoder and Transmission Matrix

The differential encoding procedure comprises a concatenation of two functional blocks, namely, a differential encoder and a multiplicative mapping matrix.

1) Differential Encoder: Let $\mathbf{S}_p^k$ be a $\Gamma M_t \times \Gamma M_t$ differentially encoded signal matrix to be transmitted during the k^{th} OFDM symbol period. We recursively construct $\mathbf{S}_p^k$ from the fundamental differential transmission equation [2], [3]

$$\mathbf{S}_p^k = \begin{cases} \mathbf{V}_p^k\ \mathbf{S}_p^{k-1}, & k \ge 1 \\ \mathbf{I}_{\Gamma M_t}, & k = 0 \end{cases}, \tag{11}$$

where the differential transmission initially sends $\mathbf{S}_p^0 = \mathbf{I}_{\Gamma M_t}$ to learn the channels. The matrix $\mathbf{S}_p^k$ is also unitary since it results from recursive multiplication of unitary information matrices. Due to the diagonal structure of $\mathbf{V}_p^k$, $\mathbf{S}_p^k$ can be expressed as

$$\mathbf{S}_p^k = diag([s_{p,1}^k, s_{p,2}^k, ..., s_{p,\Gamma M_t}^k]^T), \quad (12)$$

where $s_{p,m}^k$, $m = 1, 2, \ldots, \Gamma M_t$, is the differentially encoded complex symbol to be transmitted at subcarrier $(p-1)\Gamma M_t + m$ during the k^{th} OFDM block.

Note that, depending on how the elements of $\mathbf{S}_p^k$ are transmitted over the M_t transmit antennas, the differential schemes can be different. The DSTF schemes in [5]-[11] transmit the $\mathbf{S}_p^k$ matrix through M_t OFDM modulators over multiple OFDM blocks. This leads to performance degradation when the fading channels do not stay constant over several OFDM blocks. In what follows, we introduce a multiplicative mapping matrix that allows us to transform $\mathbf{S}_p^k$ into the code structure in (8) and completely transmit $\mathbf{S}_p^k$ within one OFDM block. This does not only improve system performance under rapid fading environment, but also reduces encoding and decoding delay.

2) Multiplicative Mapping Matrix: We define the $\Gamma M_t \times M_t$ multiplicative mapping matrix as

$$\mathbf{\Phi}_p = [\boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2 \cdots \ \boldsymbol{\phi}_{M_t}], \quad (13)$$

in which $\boldsymbol{\phi}_i$ is a $\Gamma M_t \times 1$ vector,

$$\boldsymbol{\phi}_i = \mathbf{e}_i \otimes \mathbf{1}_\Gamma, \qquad i = 1, \cdots, M_t, \quad (14)$$

where $\mathbf{e}_i$ is an $M_t \times 1$ unit vector having one at the i^{th} component and the rest are zeroes, and $\mathbf{1}_\Gamma$ denotes a $\Gamma \times 1$ vector of all ones. We post-multiply $\mathbf{S}_p^k$ by $\mathbf{\Phi}_p$, resulting in the $\Gamma M_t \times M_t$ transmit matrix

$$\mathbf{X}_p^k = \mathbf{S}_p^k \mathbf{\Phi}_p. \quad (15)$$

Consequently, the differentially encoded complex symbol $s_{p,m}^k$, as specified in (12), is transmitted at the $\lceil \frac{m}{\Gamma} \rceil$ transmit antenna, where $\lceil \cdot \rceil$ represents the ceiling function.

C. Differential Decoding

According to (6) and (7), the receive signal vector corresponding to the transmitted matrix $\mathbf{X}_p^k$ is given by

$$\mathbf{y}_p^k = \sqrt{\rho} \left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k) \right) \mathbf{h}_p^k + \mathbf{w}_p^k, \quad (16)$$

where $\mathcal{D}(\mathbf{X}_p^k)$ ($\mathcal{D}(\cdot)$ is defined in (5)) is a $\Gamma M_t \times \Gamma M_t M_t$ transmit matrix. The $\Gamma M_t M_t M_r \times 1$ channel vector $\mathbf{h}_p^k = \left[(\mathbf{h}_{p,1}^k)^T \ (\mathbf{h}_{p,2}^k)^T \ \cdots \ (\mathbf{h}_{p,M_r}^k)^T \right]^T$ comprises

$$\mathbf{h}_{p,j}^k = \left[(\mathbf{h}_{p,1j}^k)^T \ (\mathbf{h}_{p,2j}^k)^T \ \cdots \ (\mathbf{h}_{p,M_t j}^k)^T \right]^T, \quad (17)$$

where

$$\mathbf{h}_{p,ij}^k = \left[H_{ij}^k((p-1)\Gamma M_t) \ \cdots \ H_{ij}^k(p\Gamma M_t - 1) \right]^T. \quad (18)$$

Similarly, the $\Gamma M_t M_r \times 1$ receive signal vector is given by $\mathbf{y}_p^k = \left[(\mathbf{y}_{p,1}^k)^T \ (\mathbf{y}_{p,2}^k)^T \ \cdots \ (\mathbf{y}_{p,M_r}^k)^T \right]^T$, where $\mathbf{y}_{p,j}^k = \left[y_j^k((p-1)\Gamma M_t) \ \cdots \ y_j^k(p\Gamma M_t - 1) \right]^T$. The noise vector $\mathbf{w}_p^k$ is in the same form as $\mathbf{y}_p^k$ with $y_j^k(n)$ replaced by $w_j^k(n)$.

To perform differential decoding, two consecutive received signal vectors, i.e. $\mathbf{y}_p^k$ and $\mathbf{y}_p^{k-1}$ in (16), are required to recover the information matrix at each OFDM symbol period. Since the two consecutive received signal vectors are related through the differentially encoded signal matrix $\mathbf{S}_p^k$ (see (11)), we will introduce the equivalent expression of $\left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k) \right) \mathbf{h}_p^k$ in terms of $\mathbf{S}_p^k$ for subsequent differential decoding.

From (13) and (15), we can express $\mathcal{D}(\mathbf{X}_p^k)$ as

$$\mathcal{D}(\mathbf{X}_p^k) = \left[diag(\mathbf{S}_p^k \boldsymbol{\phi}_1) \ \cdots \ diag(\mathbf{S}_p^k \boldsymbol{\phi}_{M_t}) \right]. \quad (19)$$

According to (17) and (19), we have

$$\mathcal{D}(\mathbf{X}_p^k)\mathbf{h}_{p,j}^k = \sum_{i=1}^{M_t} diag(\mathbf{S}_p^k \boldsymbol{\phi}_i) \mathbf{h}_{p,ij}^k, \quad (20)$$

which can be re-written using Hadamard product [17] as

$$\begin{aligned} \mathcal{D}(\mathbf{X}_p^k)\mathbf{h}_{p,j}^k &= \sum_{i=1}^{M_t} (\mathbf{S}_p^k \boldsymbol{\phi}_i) \circ \mathbf{h}_{p,ij}^k \\ &= \mathbf{S}_p^k \sum_{i=1}^{M_t} \boldsymbol{\phi}_i \circ \mathbf{h}_{p,ij}^k \triangleq \mathbf{S}_p^k \ \tilde{\mathbf{h}}_{p,j}^k, \end{aligned} \quad (21)$$

where $\tilde{\mathbf{h}}_{p,j}^k$ in the last equality is explicitly defined. By substituting (18) into (21), we can express $\tilde{\mathbf{h}}_{p,j}^k$ as

$$\tilde{\mathbf{h}}_{p,j}^k = \left[(\tilde{\mathbf{h}}_{p,1j}^k)^T \ (\tilde{\mathbf{h}}_{p,2j}^k)^T \cdots (\tilde{\mathbf{h}}_{p,M_t j}^k)^T \right]^T \quad (22)$$

in which

$$\tilde{\mathbf{h}}_{p,ij}^k = [H_{ij}^k(n_{p,i}^0) \ H_{ij}^k(n_{p,i}^1) \ \cdots \ H_{ij}^k(n_{p,i}^{\Gamma-1})]^T, \quad (23)$$

where

$$n_{p,i}^\gamma = (i-1)\Gamma + (p-1)\Gamma M_t + \gamma \quad (24)$$

for $\gamma = 0, 1, \ldots, \Gamma - 1$. Denoting $\tilde{\mathbf{h}}_p^k = \left[(\tilde{\mathbf{h}}_{p,1}^k)^T \ (\tilde{\mathbf{h}}_{p,2}^k)^T \cdots (\tilde{\mathbf{h}}_{p,M_r}^k)^T \right]^T$ as a $\Gamma M_t M_r \times 1$ channel gain vector and using (21), we obtain an equivalent expression

$$\left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k) \right) \mathbf{h}_p^k = \left(\mathbf{I}_{M_r} \otimes \mathbf{S}_p^k \right) \tilde{\mathbf{h}}_p^k. \quad (25)$$

For notation convenience, let us define $\boldsymbol{\mathcal{S}}_p^k \triangleq \left(\mathbf{I}_{M_r} \otimes \mathbf{S}_p^k \right)$ and $\boldsymbol{\mathcal{V}}_p^k \triangleq \left(\mathbf{I}_{M_r} \otimes \mathbf{V}_p^k \right)$ such that

$$\boldsymbol{\mathcal{S}}_p^k = \left(\mathbf{I}_{M_r} \otimes \mathbf{V}_p^k \right) \boldsymbol{\mathcal{S}}_p^{k-1} = \boldsymbol{\mathcal{V}}_p^k \boldsymbol{\mathcal{S}}_p^{k-1}. \quad (26)$$

Accordingly, using (25) and (26), we can rewrite the two consecutive receive signal vectors in (16) as

$$\mathbf{y}_p^{k-1} = \sqrt{\rho} \boldsymbol{\mathcal{S}}_p^{k-1} \tilde{\mathbf{h}}_p^{k-1} + \mathbf{w}_p^{k-1}, \quad (27)$$

$$\mathbf{y}_p^k = \sqrt{\rho} \boldsymbol{\mathcal{S}}_p^k \tilde{\mathbf{h}}_p^k + \mathbf{w}_p^k. \quad (28)$$

We relate the equivalent terms of (27) and (28) through (26), and assume that the channel coefficients are almost constant over two consecutive OFDM blocks, i.e. $\tilde{\mathbf{h}}_p^k \approx \tilde{\mathbf{h}}_p^{k-1} \approx \tilde{\mathbf{h}}_p$, we obtain $\mathbf{y}_p^k = \boldsymbol{\mathcal{V}}_p^k \mathbf{y}_p^{k-1} + \tilde{\mathbf{w}}_p^k$, where $\tilde{\mathbf{w}}_p^k \triangleq \mathbf{w}_p^k - \boldsymbol{\mathcal{V}}_p^k \ \mathbf{w}_p^{k-1}$ has twice variance as that of $\mathbf{w}_p^k$. Without acquiring channel state information, the detector follows the decision rule [3]

$$\hat{\boldsymbol{\mathcal{V}}}_p^k = \arg \min_{\boldsymbol{\mathcal{V}}_p^k \in \mathbb{V}_p} \| \mathbf{y}_p^k - \boldsymbol{\mathcal{V}}_p^k \ \mathbf{y}_p^{k-1} \|_F^2. \quad (29)$$

It is worth to mention that the detector is able to differentially decode within two OFDM symbol periods regardless of the number of transmit antennas. Therefore, our proposed scheme significantly reduces the decoding delay compared to the DSTF schemes. Note also that the proposed differential scheme includes the differential scheme in [14] for single antenna OFDM system as a special case.

IV. Pairwise Error Probability and Design Criteria of the Proposed Differential Scheme

The PEP analysis of the differential scheme for MIMO-OFDM systems have been considered in [6]-[11]. In this paper, we provide an alternative PEP formulation based on the results in [15] which showed the asymptotic PEP for differential detections. The PEP upper bound in [15] is not only asymptotically tight, but also provides a simple interpretation of the performance in terms of the eigenvalues of signal and correlation matrices.

Suppose that $\boldsymbol{\mathcal{V}}_p^k$ and $\hat{\boldsymbol{\mathcal{V}}}_p^k$ are two different information matrices. With the assumption of slow fading channels, the average PEP is upper bounded by [15]

$$P\left(\boldsymbol{\mathcal{V}}_p^k \to \hat{\boldsymbol{\mathcal{V}}}_p^k\right) \leq \begin{pmatrix} 2\nu - 1 \\ \nu \end{pmatrix} \left(\prod_{m=1}^{\nu} \beta_{p,m}\right)^{-1} \left(\frac{\rho}{2}\right)^{-\nu}, \quad (30)$$

where ν is the rank and $\beta_{p,m}$'s are the non-zero eigenvalues of the matrix

$$\boldsymbol{\Psi}_p \triangleq \boldsymbol{\mathcal{S}}_p^{k-1} \boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p} (\boldsymbol{\mathcal{S}}_p^{k-1})^H (\boldsymbol{\mathcal{V}}_p^k - \hat{\boldsymbol{\mathcal{V}}}_p^k)^H (\boldsymbol{\mathcal{V}}_p^k - \hat{\boldsymbol{\mathcal{V}}}_p^k), \quad (31)$$

in which $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p} = E[\tilde{\mathbf{h}}_p \tilde{\mathbf{h}}_p^H]$ denotes the correlation matrix of channel vector $\tilde{\mathbf{h}}_p$. Note that the PEP upper bound in (30) is a function of $\rho/2$, which corresponds to the 3-dB performance loss when compared to its coherent counterpart.

We will reformulate the PEP upper bound in (30) for the case of spatially uncorrelated MIMO channels such that we can obtain design criteria for our proposed scheme. To simplify the expression for matrix $\boldsymbol{\Psi}_p$ in (31), we evaluate the channel correlation matrix $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p}$ as follows. First, we re-write the frequency response in (3) as

$$H_{ij}^k(n) \quad = \quad \boldsymbol{\omega}^T(n)\ \mathbf{a}_{ij}^k, \quad (32)$$

where $\mathbf{a}_{ij}^k \triangleq [\alpha_{ij}^k(0), \cdots, \alpha_{ij}^k(L-1)]^T \in \mathcal{C}^{L\times 1}$, $\boldsymbol{\omega}(n) \triangleq [\omega^{n\tau_0}, \ldots, \omega^{n\tau_{L-1}}]^T \in \mathcal{C}^{L\times 1}$, and $\omega \triangleq e^{-j2\pi\Delta f}$. According to (32), we can represent $\tilde{\mathbf{h}}_{p,ij}^k$ in (23) as

$$\tilde{\mathbf{h}}_{p,ij}^k = \boldsymbol{\Omega}_{p,i}\ \mathbf{a}_{ij}^k, \quad (33)$$

where $\boldsymbol{\Omega}_{p,i} = \left[\boldsymbol{\omega}(n_{p,i}^0)\ \boldsymbol{\omega}(n_{p,i}^1)\ \cdots\ \boldsymbol{\omega}(n_{p,i}^{\Gamma-1})\right]^T \in \mathcal{C}^{\Gamma\times L}$ and $n_{p,i}^{\gamma}$ is defined in (24). Substituting (33) into (22), we have

$$\tilde{\mathbf{h}}_{p,j}^k = \boldsymbol{\Omega}_p\ \mathbf{a}_j^k, \quad (34)$$

where $\boldsymbol{\Omega}_p = diag\left(\boldsymbol{\Omega}_{p,1}, \cdots, \boldsymbol{\Omega}_{p,M_t}\right) \in \mathcal{C}^{\Gamma M_t \times L M_t}$, and $\mathbf{a}_j^k = \left[(\mathbf{a}_{1j}^k)^T \cdots (\mathbf{a}_{M_t j}^k)^T\right]^T \in \mathcal{C}^{LM_t\times 1}$. Based on (34) and the assumption that each transmit-receive link has the same power delay profile, we can calculate the correlation matrix of channel vector $\tilde{\mathbf{h}}_{p,j}^k$ as

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}} = E[\tilde{\mathbf{h}}_{p,j}^k (\tilde{\mathbf{h}}^k)_{p,j}^H] = \boldsymbol{\Omega}_p\ \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H, \quad (35)$$

where $\boldsymbol{\Lambda}_{\delta^2} = diag(\delta_0^2, \ldots, \delta_{L-1}^2)$ represents an $L\times L$ diagonal matrix of power delay profile. Observe from (35) that $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}}$ is the same for all j's. Denote $\boldsymbol{\Sigma} \triangleq \boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}}$, then we have

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p} = \mathbf{I}_{M_r} \otimes \boldsymbol{\Sigma}. \quad (36)$$

Applying the property of tensor product $(\mathbf{A}_1 \otimes \mathbf{B}_1)(\mathbf{A}_2 \otimes \mathbf{B}_2)(\mathbf{A}_3 \otimes \mathbf{B}_3) = (\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3 \otimes \mathbf{B}_1\mathbf{B}_2\mathbf{B}_3)$ to (31), we obtain

$$\boldsymbol{\Psi}_p = \mathbf{I}_{M_r} \otimes \boldsymbol{\Theta}_p, \quad (37)$$

in which

$$\boldsymbol{\Theta}_p = \mathbf{S}_p^{k-1} \boldsymbol{\Sigma} (\mathbf{S}_p^{k-1})^H \boldsymbol{\Delta}, \quad (38)$$

and $\boldsymbol{\Delta} = \left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)^H \left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)$. Hence, by (37), the PEP in (30) can be expressed as

$$P\left(\mathbf{V}_p^k \to \hat{\mathbf{V}}_p^k\right) \leq \begin{pmatrix} 2rM_r - 1 \\ rM_r \end{pmatrix} \left(\prod_{m=1}^{r} \lambda_{p,m}\right)^{-M_r} \left(\frac{\rho}{2}\right)^{-rM_r} \quad (39)$$

where r is the rank of $\boldsymbol{\Theta}_p$ and $\lambda_{p,m}$'s are the non-zero eigenvalues of $\boldsymbol{\Theta}_p$.

The PEP upper bound in (39) suggests two design criteria

1) Rank criterion: For any $\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k$, design a constellation set of unitary matrices $\mathbb{V}_p$ such that the minimum rank of $\boldsymbol{\Theta}_p$ is maximized.

2) Product criterion: For any $\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k$, design a constellation set of unitary matrices $\mathbb{V}_p$ such that the minimum value of the product $\prod_{m=1}^{r} \lambda_{p,m}$ is maximized.

To quantify the maximum achievable diversity order, we substituting (35) into (38), and re-express $\boldsymbol{\Theta}_p$ as

$$\boldsymbol{\Theta}_p \quad = \quad \mathbf{S}_p^{k-1} \boldsymbol{\Omega}_p \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H \left(\mathbf{S}_p^{k-1}\right)^H \boldsymbol{\Delta}. \quad (40)$$

Observe from (40) that $\mathbf{S}_p^{k-1}$ and $\mathbf{V}_p^k$ are of size $\Gamma M_t \times \Gamma M_t$, the correlation matrix $\boldsymbol{\Omega}_p$ is of size $\Gamma M_t \times L M_t$, and $\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}$ is an $LM_t \times LM_t$ diagonal matrix. Since $\Gamma \leq L$, the rank of $\boldsymbol{\Theta}_p$ is at most ΓM_t. Therefore, the maximum achievable diversity gain is

$$G_d^{max} = M_r \max \left(\min_{\forall\ \mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} rank\left(\boldsymbol{\Theta}_p\right) \right) = \Gamma M_t M_r. \quad (41)$$

When the maximum diversity order is achieved, the maximum product criterion is determined by the normalized coding advantage or the so-called diversity product [3], [13]

$$\zeta = \frac{1}{2} \min_{\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} \left| \prod_{m=1}^{\Gamma M_t} \lambda_{p,m} \right|^{\frac{1}{2\Gamma M_t}}, \quad (42)$$

where a larger ζ results in better performance.

In this case, we can evaluate the product of the non-zero eigenvalues of the matrix $\boldsymbol{\Theta}_p$ as

$$\begin{aligned} \prod_{m=1}^{\Gamma M_t} \lambda_{p,m} &= det\left(\mathbf{S}_p^{k-1} \boldsymbol{\Omega}_p \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H \left(\mathbf{S}_p^{k-1}\right)^H\right) det\left(\boldsymbol{\Delta}\right) \\ &= \prod_{i=1}^{M_t} det\left(\boldsymbol{\Omega}_{p,i} \boldsymbol{\Lambda}_{\delta^2} \boldsymbol{\Omega}_{p,i}^H\right) \prod_{m=1}^{\Gamma M_t} \left|v_{p,m}^k - \hat{v}_{p,m}^k\right|^2 \quad (43) \end{aligned}$$

where $\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k = diag(v_{p,1}^k - \hat{v}_{p,1}^k, \ldots, v_{p,\Gamma M_t}^k - \hat{v}_{p,\Gamma M_t}^k)$. In the second equality, we apply the identity $det\left(\mathbf{AB}\right) = det\left(\mathbf{BA}\right)$ and the unitary property of matrix $\mathbf{S}_p^{k-1}$. Substitute (43) into (42), resulting in

$$\zeta = \left| \prod_{i=1}^{M_t} det\left(\boldsymbol{\Omega}_{p,i} \boldsymbol{\Lambda}_{\delta^2} \boldsymbol{\Omega}_{p,i}^H\right) \right|^{\frac{1}{2\mathcal{M}}} \frac{1}{2} \min_{\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} \prod_{m=1}^{\mathcal{M}} \left| v_{p,m}^k - \hat{v}_{p,m}^k \right|^{\frac{1}{\mathcal{M}}} \quad (44)$$

in which $\mathcal{M} = \Gamma M_t$.

Observe from the equation in (44) that ζ can be maximized by designing the two terms on the right hand side separately.

The first term depends only on the power delay profile, and it can be maximized by the use of proper subcarrier selection method, e.g., an optimum permutation strategy proposed in [13]. The interested reader is referred to [13] for more detail treatment of the optimum permutation. In this paper, however, we resort to random permutation strategy to enable fair performance comparison between the proposed scheme and others, e.g. [8]-[11]. The second term relies on the code structure. Here, we adopt the diagonal cyclic group code [3], which is well systematically designed and applicable for MIMO systems with any number of transmit antennas and transmission rates.

In particular, for a specific integer $\mathcal{M}$ and transmission rate R such that $\mathcal{L} = 2^{R\mathcal{M}}$. We denote a set of parameters used to fully specify the signal constellation $\mathbb{V}_p$ as $G_{\mathcal{M},\mathcal{L}} = (\mathcal{M}, \mathcal{L}, [u_1, u_2, ..., u_{\mathcal{M}}])$, where $u_1, u_2, \ldots, u_{\mathcal{M}}$ are chosen from a set of integer number $\mathcal{I}_L = \{0, 1, ..., \mathcal{L}-1\}$ that satisfies [3]

$$[u_1, \ldots, u_{\mathcal{M}}] = \arg \max_{\{u_m \in \mathcal{I}_L\}} \left(\min_{l \in \mathcal{I}_L} \left| \prod_{m=1}^{\mathcal{M}} \sin\left(\pi u_m l / \mathcal{L}\right) \right|^{\frac{1}{\mathcal{M}}} \right).$$

Some of the sets of optimum parameters, $u_1, u_2, \ldots, u_{\mathcal{M}}$, obtained from exhaustive computer search are shown in [3]. Based on $G_{\mathcal{M},\mathcal{L}}$, the constellation $\mathbb{V}_p$ are constructed from

$$\mathbf{V}_{p,l} = diag\left(e^{j\theta_{\mathcal{L}} u_1 l}, e^{j\theta_{\mathcal{L}} u_2 l}, \ldots, e^{j\theta_{\mathcal{L}} u_{\mathcal{M}} l}\right), \qquad (45)$$

for $l = 0, 1, ..., \mathcal{L}-1$, and $\theta_{\mathcal{L}} = 2\pi/\mathcal{L}$.

V. Simulation Results

In all simulations, each OFDM modulator utilized $N = 128$ subcarriers with the total bandwidth of 1 MHz. The corresponding OFDM symbol period was $T_s = 1/\Delta f = 128\mu s$. We added a guard interval of $20\mu s$ against intersymbol interference due to channel multipath delay spread. We considered a simple two-ray and a more realistic six-ray typical urban (TU) power delay profiles. Each delay path of the two-ray power delay profile had equal power with a delay of $20\mu s$ between the two paths. The TU channel description was the same as those shown in ([18], Table 2.2). The fading channels were assumed constant within each OFDM block and slow varying from one OFDM block to another according to the Jakes' fading model [18] with f_D representing the maximum Doppler frequency in Hz.

We simulated the performance under different mobile environments by varying the normalized Doppler frequencies, namely, $f_D T_s$ = 0.0025, 0.005, 0.01, and 0.025 which correspond to a mobile speed of 6, 13, 26, and 65 m/s, respectively. The performance curves are demonstrated in terms of averaged bit error rate (BER) versus averaged signal energy per bit (E_b/N_o) in dB. We compare the performance of our proposed differential scheme to that of an existing DSTF scheme in [11] with the same rate R. The random permutation strategy, in which the n^{th} subcarrier is moved to the $\tilde{n}^{th}$ subcarrier, follows the Takeshita-Constello method as [16]:

$$\tilde{n} = mod\left(\frac{n(n+1)}{2}, N\right) + 1, \qquad n = 1, 2, ..., N. \qquad (46)$$

We considered a system with M_t = 2, M_r = 1, and $\Gamma = 2$. Figures 2 depicts the simulation results for the two-ray power delay profile with $R = 1.5$ $b/s/Hz$ (omitting the cyclic-prefix and guard interval) and using $G_{4,64} = (4, 64, [1, 17, 45, 53])$. It is apparent that the performances of our proposed scheme (showed by solid lines) are superior to that of previously proposed scheme (showed by dashed lines) in every normalized Doppler frequency. For instance, in case of fading channels with $f_D T_s = 0.0025$ and 0.005, our proposed scheme yields almost the same performance of $BER \approx 5 \times 10^{-5}$ at E_b/N_0 of 24 dB, which outperform those of previous scheme that achieved $BER = 1.5 \times 10^{-4}$. When fading rate increases from 0.005 to 0.01, the performances of our proposed scheme and the previous scheme degrade to $BER = 1.22 \times 10^{-4}$ and 4.5×10^{-4}, respectively, at E_b/N_0 = 24 dB. Observe that the performances of the previous scheme degrades faster than that of our proposed scheme. For a more rapid fading at $f_D T_s = 0.025$, the performance of the previous scheme degrades even faster from $BER = 1.5 \times 10^{-4}$ to 6.81×10^{-3} and nearly reach error floor, while the performance of our propose scheme degrades from $BER \approx 5 \times 10^{-5}$ to 5.2×10^{-4}. This confirms our expectation that by coding within only one OFDM block, our propose scheme is robust to the effect of rapid channel variation. In contrast, the DSTF scheme relies on constant channel over several OFDM blocks, thereby more susceptible to rapid fading condition. Note that in all figures, we provide simulation results for coherent detections of our scheme for $f_D T_s = 0.0025$. The 3 dB performance loss due to differential detection can be observed.

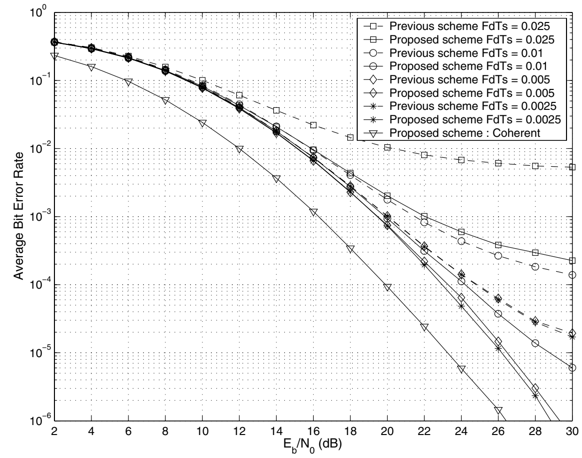

Fig. 2: Two-ray power delay profile, M_t = 2, M_r = 1, $R = 1.5$ $b/s/Hz$.

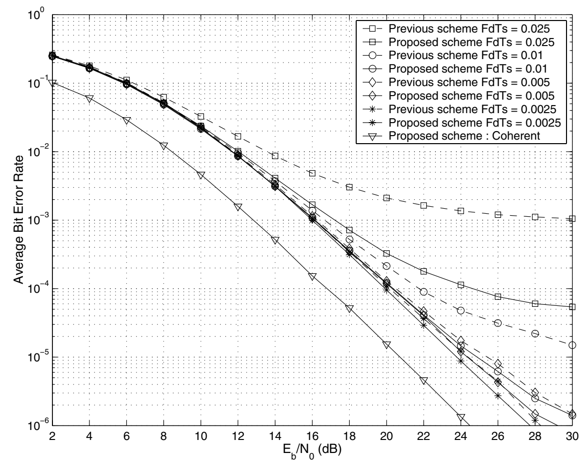

Fig. 3: TU power delay profile, M_t = 2, M_r = 1, and $R = 1$ $b/s/Hz$.

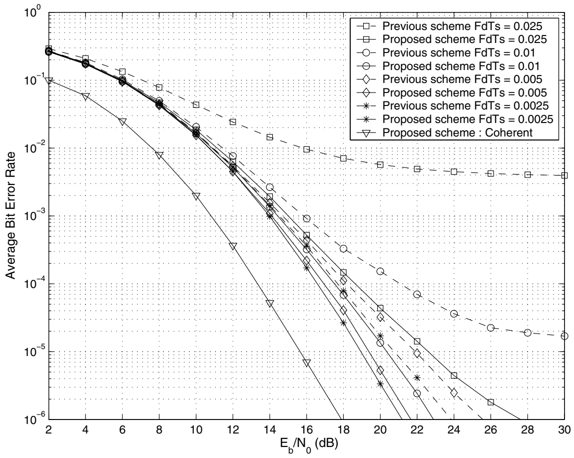

Fig. 4: Two-ray power delay profile, M_t = 3, M_r = 1, and $R \approx 1\ b/s/Hz$.

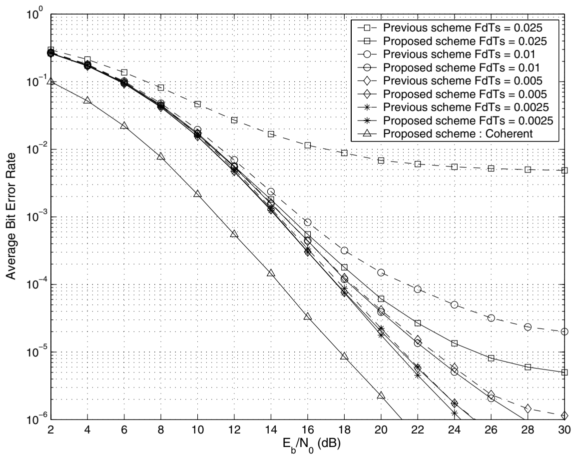

Fig. 5: TU power delay profile, M_t = 3, M_r = 1, and $R \approx 1\ b/s/Hz$.

The performance under the TU power delay profile is shown in Figures 3 for $M_t = 2$, $M_r = 1$, $\Gamma = 2$, and $R = 1\ b/s/Hz$ in which $G_{4,16} = (4, 16, [1, 3, 5, 7])$ is used. Observing that under slow fade rates, i.e., $f_D T_s = 0.0025$ and 0.005, our scheme yields slightly better performances than those in previous scheme at E_b/N_0 of $22\ dB$. Significant performance difference can be observed when $f_D T_s = 0.01$. In this case, our proposed scheme achieves an average BER of 4.13×10^{-5} at E_b/N_0 = $22\ dB$, whereas the previous scheme has a BER of 9.0×10^{-5}. When $f_D T_s$ increases from 0.01 to 0.025, the BER of the previous scheme severely degrades to 1.75×10^{-3} at E_b/N_0 of $22\ dB$, while the BER of our proposed scheme slightly degrades to 1.92×10^{-4}.

The superior performance of our proposed scheme over the previous scheme can be obviously seen in case of $M_t = 3$ and $M_r = 1$. For $\Gamma = 2$ and $R \approx 1\ b/s/Hz$ (due to zero padding insertion), we generated the signal constellation by $G_{6,64} = (6, 64, [1, 9, 15, 17, 23, 25])$. Figures 4 and 5 show performances under the two-ray and the TU power delay profiles, respectively. Similar to the case of two transmit antennas, our scheme yields better performances and more robust to channel fading conditions than those of the previous scheme. In case of fast fading, e.g. $f_D T_s = 0.025$, the performance degradation is significant and high error floor can be observed in the previous scheme. In contrast, the performance of our proposed scheme slightly degrades with an acceptable error floor.

VI. Conclusions

We proposed in this paper a differential scheme for MIMO-OFDM systems that can differentially encode signal within one OFDM block. The scheme allows us to relax the channel assumption to keep constant during each OFDM block and slowly change from a duration of one OFDM block to another, rather than multiple OFDM blocks as assumed in the previously existing works. We formulated the pairwise error probability and design criteria, and showed that our scheme achieves maximum diversity order with high coding gain by utilizing an existing diagonal cyclic codes. Comparing to the previous scheme, the proposed scheme is not only robust to rapid channel variation, but also reduces encoding and decoding delay. Simulation results showed that our proposed scheme yields better performance than those previously proposed in all of the fading conditions and different power delay profiles.

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