Blind Adaptive Spatial–Temporal Equalization Algorithms for Wireless Communications Using Antenna Arrays

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***Abstract*—Spatial–temporal equalizer can be used in the wireless communication systems with antenna arrays to improve the performance. This article introduces two blind adaptive algorithms for spatial–temporal equalization. Computer simulation demonstrates that the new algorithms converge faster than fractionally spaced constant-modulus algorithm (FS-CMA).**

***Index Terms*—Adaptive equalizer, blind equalization, spatial–temporal processing.**

I. Problem Formulation

ANTENNA arrays are used for diversity reception so as to improve the quality of wireless communication systems. To make full use of the information contained in each sensor, a spatial–temporal equalizer is used to remove intersymbol interference and mitigate the additive channel noise. The wireless communication systems with antenna arrays can be modeled as *single-input/multiple-output systems* (SIMO) shown as in Fig. 1. Since the channels are time-varying and training sequences are not always available, blind adaptive techniques have to be used in these systems.

In Fig. 1, the input sequence $\{s[n]\}$ is sent through M different linear channels with impulse response $\{h_m[n]\}$ for $m = 1, 2, \cdots, M$. Hence, the channel outputs can be written in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n] \text{ or } \mathcal{X}_K[n] = \mathcal{H}_K \mathcal{S}_K[n] \qquad (1.1)$$

where we have used the definitions:

$$\mathcal{H}_K \triangleq \begin{pmatrix} \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] \end{pmatrix} \qquad (1.2)$$

$$\mathcal{X}_K[n] \triangleq (\mathbf{x}_K[n], \cdots, \mathbf{x}_K[n+N-1]) \qquad (1.3)$$

$$\mathcal{S}_K[n] \triangleq (\mathbf{s}_K[n], \cdots, \mathbf{s}_K[n+N-1]) \qquad (1.4)$$

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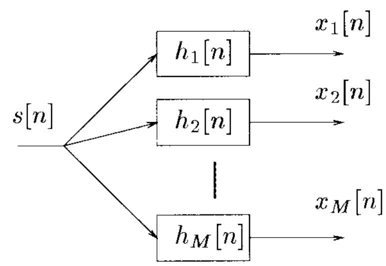

Fig. 1. SIMO systems.

with

$$\mathbf{s}_K[n] \triangleq (s[n-L+1], \cdots, s[n+K-1])^T$$
$$\mathbf{h}[n] \triangleq (h_1[n], \cdots, h_M[n])^T$$
$$\mathbf{x}_K[n] \triangleq (\mathbf{x}^T[n], \cdots, \mathbf{x}^T[n+K-1])^T$$
$$\mathbf{x}^T[n] \triangleq (x_1[n], \cdots, x_M[n]).$$

The integer K in the above equations determines the dimensions of the matrices and the vectors.

In this letter, we will assume that the SIMO channels are of finite impulse response (FIR) with length L, and furthermore, they satisfy the *length-and-zero condition* [2], which makes $\mathcal{H}_K$ for any $K \geq L-1$ to be of full column rank. Hence, from (1.1), there exists a $KM \times (K+L-1)$ matrix $\mathcal{F}$ (not unique) such that

$$\mathbf{s}_K[n] = \mathcal{F}^H \mathbf{x}_K[n], \mathcal{F} \triangleq (\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_{K+L-1}) \qquad (1.5)$$

where $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ are column vectors with KM elements.

The task of blind adaptive equalization of SIMO channels is to find algorithms to adaptively adjust the parameters $\mathbf{f}_k$ such that

$$\mathcal{F}^H \mathcal{H}_K = cI_{K+L-1} \qquad (1.6)$$

for some nonzero constant c.

II. Algorithm Development

From the above definitions, we have

$$\mathbf{f}_k^H \mathbf{x}_K[n] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1] \qquad (2.1)$$

for $k = 1, \cdots, K+L-2$, and every integer n. It has been proved in [1] that if the channel satisfies the length-and-zero condition and the channel input in *the* $(K+1)$*th-order*

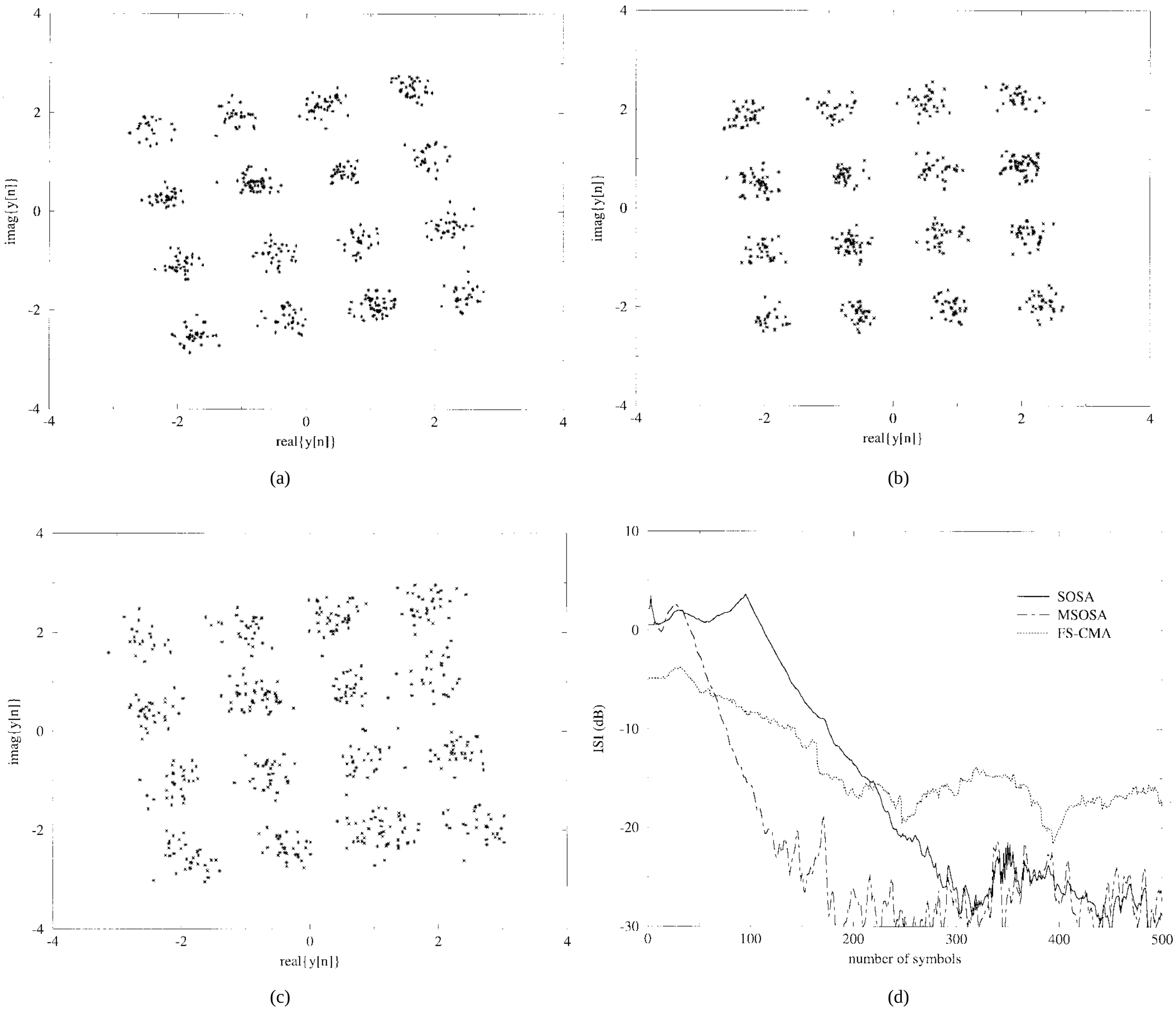

Fig. 2. Comparison of the SOSA, the MSOSA, and the CMA when $SNR = 20$ dB, 500 symbol eye patterns of (a) the SOSA, (b) the MSOSA, and (c) the orthogonal FS-CMA after 500 iterations, and (d) the convergence of the ISI for the three algorithms, respectively.

persistently exciting [3] then the $\mathbf{f}_k$'s satisfying (2.1) will satisfy (1.6). This is the property only SIMO systems have, by means of which we are able to design second-order statistics based blind adaptive spatial–temporal equalization algorithm.

If the channel noise is considered, $\mathbf{f}_k$ for $k = 1, \cdots, K + L - 1$ can be estimated by minimizing the cost function:

$$\mathcal{C} = \sum_{k=1}^{K+L-2} E|\mathbf{f}_k^H \mathbf{x}_K[n] - \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1]|^2 \tag{2.2}$$

subject to

$$\sum_{k=1}^{K+L-1} |\mathbf{f}_k^H \mathbf{x}_K[n]|^2 = c_o$$

where c_o is a nonnegative constant. The constraint is added here to prevent from the trivial solution of $\mathbf{f}_k = \mathbf{0}$ for all $k = 1, \cdots, K + L - 1$.

From (2.2), a direct calculation yields that

$$\begin{aligned}\mathcal{C} = \sum_{k=1}^{K+L-2} \{&\mathbf{f}_k^H R_x[0]\mathbf{f}_k - \mathbf{f}_k^H R_x[1]\mathbf{f}_{k+1} \\ &- \mathbf{f}_{k+1}^H R_x^H[1]\mathbf{f}_k + \mathbf{f}_{k+1}^H R_x[0]\mathbf{f}_{k+1}\}\end{aligned} \tag{2.3}$$

where we have used the definition and identity:

$$R_x[m] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n-m]\}, R_x[-m] = R_x^H[m].$$

Hence,

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}_k} = \begin{cases} R_x[0]\mathbf{f}_1 - R_x[1]\mathbf{f}_2, \quad \text{if } k = 1 \\ 2R_x[0]\mathbf{f}_k - R_x[1]\mathbf{f}_{k+1} - R_x^H[1]\mathbf{f}_{k-1}, \\ \qquad\qquad \text{if } k = 2, \cdots, K + L - 2 \\ R_x[0]\mathbf{f}_{K+L-1} - R_x^H[1]\mathbf{f}_{K+L-2}, \\ \qquad\qquad \text{if } k = K + L - 1. \end{cases} \tag{2.4}$$

Let

$$\mathbf{f} \triangleq \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_{K+L-1} \end{pmatrix} \tag{2.5}$$

then, (2.4) can be written as

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}} = R\mathbf{f} \tag{2.6}$$

where R is a $KM(K+L-1) \times KM(K+L-1)$ matrix defined as

$$R \triangleq \begin{pmatrix} R_x[0] & -R_x[1] & \cdots & \mathbf{0} \\ -R_x^H[1] & 2R_x[0] & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & R_x[0] \end{pmatrix}.$$

Using the gradient-based approach, we can obtain an adaptive algorithm to estimate $\mathbf{f}$ as following:

$$\hat{\mathbf{f}}^{(n+1)} = \mathbf{f}^{(n)} - \mu R \mathbf{f}^{(n)} \tag{2.7}$$

$$p^{(n+1)} = \left(\sum_{k=1}^{K+L-1} (\hat{\mathbf{f}}_k^{(n+1)})^H R_x^{(n+1)}[0] \hat{\mathbf{f}}_k^{(n+1)} \right)^{1/2} \tag{2.8}$$

$$\mathbf{f}^{(n+1)} = \frac{c_o}{p^{(n+1)}} \hat{\mathbf{f}}^{(n+1)} \tag{2.9}$$

and $R_x[m]$ in R can be estimated using

$$R_x^{(n+1)}[m] = \lambda R_x^{(n)}[m] + (1-\lambda)\mathbf{x}_K[n]\mathbf{x}_K^H[n-m] \tag{2.10}$$

where μ is a stepsize and $\lambda \in [0,1]$ is a forgetting factor.

The blind adaptive algorithm for SIMO channel equalization defined by (2.7)–(2.10) is called *second-order statistics based algorithm*, or SOSA.

If we examine the identities (1.4) and (1.5) carefully, we will see that

$$\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] = \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2] \tag{2.11}$$

for $k_1, k_2 = 1, 2, \cdots, K+L-1$ and all integer n. Hence, we can modify the cost function (2.2) as

$$\tilde{\mathcal{C}} = \sum_{k_1,k_2=1}^{K+L-1} E|\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] - \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2]|^2. \tag{2.12}$$

From this cost function, we are able to derive a *modified second-order statistics based algorithm* (MSOSA) similar to (2.7)–(2.10), except that R is substituted by

$$\tilde{R} = \begin{pmatrix} (K+L-2)R_x[0] & \cdots & -R_x[K+L-2] \\ \vdots & \ddots & \vdots \\ -R_x^H[K+L-2] & \cdots & (K+L-2)R_x[0] \end{pmatrix}.$$

The dimension parameter K in the above two algorithms is usually between $L-1$ to $L+1$ to get good tradeoff between the performance and the complexity.

Since the MSOSA exploits more information about the structure of $\mathbf{s}_K[n]$, as comfirmed by our computer simulations, it is more robust than the SOSA. However, the MSOSA requires a little bit more computation than the SOSA.

III. Computer Simulation

A Monte Carlo simulation example has been conducted to demonstrate the performance of the new algorithms used in 16-QAM digital communication systems. In our simulation, the channel input sequence $\{s[n]\}$ is i.i.d. and randomly distributed over $\{\pm 1 \pm \jmath\}$, $\{\pm 3 \pm \jmath\}$, $\{\pm 1 \pm 3\jmath\}$, and $\{\pm 3 \pm 3\jmath\}$. It is sent through two time-invariant linear channels with impulse responses $\{0.6662 - \jmath 0.8427, 1.6323 - \jmath 0.2503, -0.6617 - \jmath 0.4102\}$ and $\{0.4607 + \jmath 0.5789, 0.5855 - \jmath 2.6912, 1.3273 - \jmath 0.4184\}$, respectively. The complex white Gaussian noise, with zero-mean and variance making SNR $= 20$ dB, is added at each channel output. The stepsize μ and the forgetting factor λ are chosen to optimize the performance of each equalization algorithm. Fig. 2(a)–(c) shows the eye patterns of the SOSA, the MSOSA, and the orthogonal FS-CMA, respectively, after 500 iterations, and Fig. 2(d) illustrates the convergence of ISI of the three algorithms with respect to the number of iterations. From Fig. 2, the performance of the SOSA and the MSOSA is much better than that of the orthogonal FS-CMA with the MSOSA being the best of the three algorithms.

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