PERFORMANCE ANALYSIS FOR MULTI-NODE DECODE-AND-FORWARD RELAYING IN COOPERATIVE WIRELESS NETWORKS

Ahmed K. Sadek, Weifeng Su, and K. J. Ray Liu

Department of Electrical and Computer Engineering, and Institute for Systems Research
University of Maryland, College Park, MD 20742, USA. {aksadek, weifeng, kjrliu}@eng.umd.edu

ABSTRACT

In this paper, we provide symbol-error-rate (SER) performance analysis for a multi-node wireless network employing a decode-and-forward cooperation strategy. An approximate expression for the SER of an N relay network with M'ary phase-shift-keying (M-PSK) signalling is derived at high enough signal-to-noise ratio (SNR). The approximation hinges on ignoring terms in the SER which are of order higher than $(N+1)$ in the SNR. The validity of the derived approximate SER is justified through computer simulations for networks with different number of relays. The simulation results show that the approximation is tight at high SNR and that the cooperation protocol can achieve full diversity order equal to the number of cooperating terminals.

1. INTRODUCTION

Combating fading via spatial diversity, introduced by multiple-input-multiple-output systems, in point-to-point communications has been extensively studied in literature. In wireless networks, e.g., cellular and ad-hoc networks, it might be difficult to have multiple antennas installed at the terminals due to space limitations. To overcome this problem, the concept of cooperative diversity in wireless networks has been recently introduced. In such a strategy, single antenna terminals cooperate together to transmit information to a destination forming a virtual antenna array.

Various cooperative diversity protocols have been proposed in [1] and [2], and outage probability performance has been provided. The concepts of decode-and-forward and amplify-and-forward have been introduced in these works. In decode-and-forward, the relay decodes the received symbol and retransmits it to the destination. While in amplify -and-forward, the relay simply amplifies and forwards the received signal. The authors in [3] and [4] introduced the concept of cooperation diversity for CDMA systems.

In this paper, we consider a decode-and-forward cooperation protocol for a multi-node wireless network. SER performance analysis for the cooperative protocol over an N relay wireless network is provided. The performance analysis of the system reveals that the SER expression contains terms of order $(N+1)$ and higher in the SNR. Based on the performance analysis, we derive an approximate SER at high SNR for a multi-node wireless network with M-PSK signalling. The approximation comes from ignoring terms in the SER expression that are of order higher than $(N+1)$ in the SNR. To verify the validity of the approximation, we perform some computer simulations for networks with different number of nodes employing the cooperative protocol. The results reveal that the approximate SER is tight at high SNR.

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2. PROTOCOL DESCRIPTION AND SYSTEM MODEL

We consider a cooperation strategy that employs a decode-and-forward protocol at the relaying nodes [2]. Fig. 1 illustrates an example for an N relay cooperative diversity network, where the source is denoted by $\mathbf{s}$, the i-th relay is denoted by $\mathbf{r}_i$, $1 \leq i \leq N$, and the destination is denoted by $\mathbf{d}$. The cooperation protocol has $(N+1)$ phases. In Phase 1, the source sends information to the destination, and the information is also received by the N relays in the network. In Phase $(l+1)$, relay $\mathbf{r}_l$ tries to decode the information it received in the previous Phases from the source and relays 1 to $(l-1)$. Then, if relay $\mathbf{r}_l$ decodes correctly, it transmits the information to the destination, and this information is also received by relays $(l+1)$ to N. Finally, the decoder coherently combines all of the received messages.

The link between any two terminals is modeled as a flat fading Rayleigh channel contaminated with additive white Gaussian noise (AWGN). We assume that the receiving terminals know the exact channel state information (CSI) needed for them to apply maximum likelihood decoding (ML). For medium access, we assume the relays are transmitting over orthogonal channels, thus no inter-relay interference is considered in the signal model. The additive noise at all receiving terminals is modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_o$.

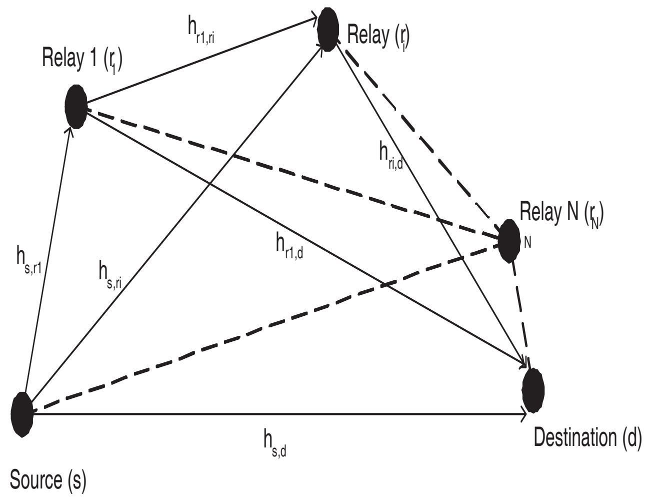

Fig. 1. An example for a multi-node wireless network.

In Phase 1, the source transmits information, and the received signal at the destination and the i-th relay can be modeled respectively as

$$y_{s,d} = \sqrt{P_0} h_{s,d} x + n_{s,d}, \tag{1}$$

$$y_{s,r_i} = \sqrt{P_0} h_{s,r_i} x + n_{s,r_i}, \quad 1 \leq i \leq N, \tag{2}$$

where P_0 is the power transmitted at the source, x is the transmitted symbol, $h_{s,d} \sim CN(0, \sigma_{s,d}^2)$ and $h_{s,r_i} \sim CN(0, \sigma_{s,r_i}^2)$ are the channel fading coefficients between the source and the destination, and i-th relay, respectively. The terms $n_{s,d}$ and n_{s,r_i} denote the AWGN. The statistic used in ML detection at relay 1 is y_{s,r_1} (2).

In Phase 2, if the first relay decodes correctly, it transmits the decoded symbol with power P_1 to the destination and relays 2 to N, otherwise the first relay remains idle. Thus, the received signals in Phase 2 can be modeled as

$$y_{r_1,d} = \sqrt{\hat{P}_1} h_{r_1,d} x + n_{r_1,d}, \tag{3}$$

$$y_{r_1,r_i} = \sqrt{\hat{P}_1} h_{r_1,r_i} x + n_{r_1,r_i}, \quad 2 \leq i \leq N, \tag{4}$$

where $\hat{P}_1 = P_1$ if relay 1 correctly decodes the message, and $\hat{P}_1 = 0$ otherwise. In Phase $(l+1)$, the l-th relay combines the received signals from the previous Phases using a maximal-ratio-combiner (MRC) [5] as follows

$$y_{r_l} = \sqrt{P_0} h_{s,r_l}^* y_{s,r_l} + \sum_{i=1}^{l-1} \sqrt{\hat{P}_i} h_{r_i,r_l}^* y_{r_i,r_l}, \tag{5}$$

where $h_{r_i,r_l} \sim CN(0, \sigma_{r_i,r_l}^2)$ is the channel fading coefficient between the i-th and the l-th relays. In (5), y_{r_i,r_l} denotes the signal received at the l-th relay from the i-th relay, and can be modeled as

$$y_{r_i,r_l} = \sqrt{\hat{P}_i} h_{r_i,r_l} x + n_{r_i,r_l}, \tag{6}$$

where $\hat{P}_i$ is the power transmitted at relay i in Phase $(i+1)$, and $\hat{P}_i = P_i$ if relay i correctly decodes the transmitted symbol, otherwise $\hat{P}_i = 0$. The l-th relay uses y_{r_l} in (5) as the detection statistics. If relay l decodes correctly it transmits with power $\hat{P}_l = P_l$, otherwise it remains idle. Finally, in Phase $(N+1)$, the destination coherently combines all of the received signals using an MRC as follows

$$y_d = \sqrt{P_0} h_{s,d}^* y_{s,d} + \sum_{i=1}^{N} \sqrt{\hat{P}_i} h_{r_i,d}^* y_{r_i,d}. \tag{7}$$

The total transmitted power is fixed: $\sum_{i=0}^{N} P_i = P$. We assume that when information is transmitted in any of the above Phases, an ideal cyclic-redundancy-check (CRC) [6] code is applied over the information, such that the receiver can judge whether it correctly decoded the information or not.

3. PERFORMANCE ANALYSIS

In this section, we analyze the SER performance of the cooperation protocol described in Section 2. We provide an analytical approximation for the SER of a multi-node decode-and-forward cooperative wireless network, which reveals the asymptotic performance of the system at high SNR.

3.1. SER Performance Analysis

First, we present some terminologies that will be used throughout the paper. For a given transmission, each relay can be in one of two states: either the relay decoded correctly or not. Define an $n \times 1$ vector S_n to denote the states of the first n relays for a given transmission, where $1 \leq n \leq N$. The m-th entry of the vector S_n denotes the state of the m-th relay as follows

$$S_n(m) = \begin{cases} 1, & \text{if relay } m \text{ correctly decodes,} \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

where $1 \leq m \leq n$. Thus, the decimal value of the binary vector S_N can take on values from 0 to $2^N - 1$. Denote the event that the destination decodes in error by e, and the probability of error at the destination given all the CSI by $P_{e|CSI_d}^d$, where CSI_d denotes the CSI needed at the destination.

Using the above terminology, the probability of error at the destination can be computed as

$$P_{e|CSI_d}^d = \sum_{i=0}^{2^N-1} Pr(e|S_N = B_{i,N}, CSI_d) Pr(S_N = B_{i,N}), \tag{9}$$

where $B_{i,N}$ denotes the $N \times 1$ binary representation of a decimal number i, with $B_{i,N}(1)$ being the most significant bit, and $Pr(S_N(j) = B_{i,N}(j))$ denotes the probability that relay j is in state $B_{i,N}(j)$. The joint probability $Pr(S_N)$ can be written as

$$P(S_N) = P(S_N(1))P(S_N(2)|S_N(1))\cdots P(S_N(N)|S_{N-1}). \tag{10}$$

Denote the probability that relay n decodes in error given all CSI by $P^n_{e|CSI_n}$, where CSI_n is the CSI needed for relay n to perform ML detection. This probability can be computed as

$$P^n_{e|CSI_n} = \sum_{i=0}^{2^{n-1}-1} Pr(S_N(n) = 0|S_{n-1} = B_{i,n-1}, CSI_n) \times Pr(S_{n-1} = B_{i,n-1}). \tag{11}$$

Thus the probability that relay n is in state $B_{i,N}(n)$ can be computed as

$$Pr(S_N(n) = B_{i,N}(n)) = \begin{cases} P^n_{e|CSI_n} & \text{if } B_{i,N}(n) = 0, \\ 1 - P^n_{e|CSI_n} & \text{if } B_{i,N}(n) = 1. \end{cases} \tag{12}$$

For a general number of relays N, it is difficult to write down all of the terms in (9) in order to find the average SER over all channel realizations. Thus, we will try to simplify the expression in (9) at high SNR by ignoring some terms. More specifically, we will apply the following two approximations:

(i) Let $1 - P^n_{e|CSI_n} \simeq 1$.

(ii) Any expression of order higher than $(N + 1)$ in the SNR is neglected.

We note that $Pr(e|S_N, CSI_d)$ is of order in the SNR equal to the number of relays that decoded correctly plus one, as this is equal to the number of signal copies received at the destination from the source and the relays that decoded correctly. For the term $Pr(S_N = B_{i,N})$, written explicitly in (10), $P(S_N(k) = 1|S_{k-1})$ is taken to be equal to 1 as assumed in approximation (i). If the k-th relay decoded in error, then the probability $P(S_N(k) = 0|S_{k-1})$ has an order in the SNR equal to the number of previous relays, 1 to $(k - 1)$, that decoded correctly plus one- the one comes from the signal copy received from the destination. Accordingly, the only terms in (9) that have an order $(N + 1)$ in the SNR are those corresponding to the states $B_{2^k-1,N}$, and k runs from 0 to N. This is because if, for example, the l-th relay decoded correctly it sends a copy of the signal, so if a later relay decoded in error, then the SER contributed by this relay is at least of order 2 in the SNR- a copy from the source and another from the l-th relay- which will result in the final SER expression at the destination of order higher than $(N + 1)$.

Thus the only terms in (10) that will be taken into account are those of the form $Pr(S_N(k) = 0|S_{k-1} = B_{0,k-1})$, which for M-PSK modulation can be computed as

$$Pr(S_N(k) = 0|S_{k-1} = B_{0,k-1}) = \Psi_{Psk}(\frac{P_0 \mid h_{s,r_k} \mid^2}{\mathcal{N}_o}), \tag{13}$$

where $\Psi_{Psk}(\gamma) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp(-\frac{b_{psk}\gamma}{\sin^2(\theta)})d\theta$ [7]. The probability of error at the destination, for a given state, can be computed as

$$Pr(e|S_N = B_{2^j-1,N}, CSI_d) = \Psi_{Psk}(\frac{P_0 \mid h_{s,d} \mid^2 + \sum_{i=N-j+1}^{N} P_i \mid h_{r_i,d} \mid^2}{\mathcal{N}_o}). \tag{14}$$

From (13) and (14), we get an approximate expression for the unconditional SER in (9), as follows

$$P^d_{e|CSI_d} = \sum_{j=1}^{N+1} \Psi_{Psk}(\frac{P_0 \mid h_{s,d} \mid^2 + \sum_{i=j}^{N} P_i \mid h_{r_i,d} \mid^2}{\mathcal{N}_o}) \times \prod_{k=1}^{j-1} \Psi_{Psk}(\frac{P_0 \mid h_{s,r_k} \mid^2}{\mathcal{N}_o}). \tag{15}$$

Averaging the above probability of error over all channel realizations, we can determine the SER of the system in the following Theorem.

Theorem 1: At high enough SNR, when all the channel links between different terminals are available, i.e. they have nonzero variance, the SER of an N relay decode-and-forward cooperative diversity network employing M-PSK modulation can be approximated by

$$SER \simeq \frac{\mathcal{N}_o^{N+1}}{b_{psk}^{N+1}\sigma^2_{s,d}} \sum_{j=1}^{N+1} \frac{g(N - j + 2)g^{j-1}(1)}{P_0^j \prod_{i=j}^{N} P_i \sigma^2_{r_i,d} \prod_{k=1}^{j-1} \sigma^2_{s,r_k}}, \tag{16}$$

where $b_{psk} = \sin^2(\pi/M)$, and $g(x)$ is defined as

$$g(x) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^{2x}(\theta)d\theta. \tag{17}$$

In Theorem 1, note that all terms in the SER expression are of order $(N + 1)$ in the SNR. This shows that the decode-and-forward cooperation strategy described in Section 2 can achieve full diversity order equal to the number of cooperating terminals. Also, note that the quality of the direct channel $\sigma^2_{s,d}$ is a common factor in the SER. Thus the optimum power allocation over the network which minimizes the SER in (16) does not depend on the direct link between the source and the destination.

Special Cases: i) For a single relay network, i.e. $N = 1$, the SER expression in (16) can be written as

$$SER \simeq \frac{\mathcal{N}_o^2}{b_{psk}^2 \sigma_{s,d}^2} \left[\frac{g(2)}{P_0 P_1 \sigma_{r_1,d}^2} + \frac{g(1)^2}{P_0^2 \sigma_{s,r_1}^2} \right]. \quad (18)$$

ii) For a two relays network, i.e. $N = 2$, the SER expression in (16) can be written as

$$SER \simeq \frac{\mathcal{N}_o^3}{b_{psk}^3 \sigma_{s,d}^2} \left[\frac{g(3)}{P_0 P_1 P_2 \sigma_{r_1,d}^2 \sigma_{r_2,d}^2} + \frac{g(2)g(1)}{P_0^2 P_2 \sigma_{s,r_1}^2 \sigma_{r_2,d}^2} + \frac{g(1)^3}{P_0^3 \sigma_{s,r_1}^2 \sigma_{s,r_2}^2} \right]. \quad (19)$$

3.2. Simulation Results

In this subsection, we compare the approximate SER expression in (16) with simulated SER for wireless networks with $N = 1$, 2 and 3 relays. We include the results of direct transmission, which has diversity order one, as a benchmark. In the simulation setup, we consider QPSK signalling, all channels to be of unit variance, i.e. $\sigma_{s,d}^2 = \sigma_{s,r_i}^2 = \sigma_{r_i,d}^2 = \sigma_{r_i,r_j}^2 = 1$, for $1 \leq i, j \leq N$, and the noise variance is taken to be $\mathcal{N}_o = 1$. For fair comparison, we assume a fixed transmission power P. The power is distributed equally among the source and all of the relays. Fig. 2 demonstrates the SER versus $P/\mathcal{N}_o$. The analytical SER in (16) is plotted in dashed line, while the simulated SER is plotted in solid line. The results show that, at high enough SNR, the approximate SER is almost equal to the simulated SER. Also, as depicted in Fig. 2, the more the relays that cooperate together, the steeper the slope of the SER curve. This shows that the diversity order increases with increasing the number of relays, which agrees with the theoretical results.

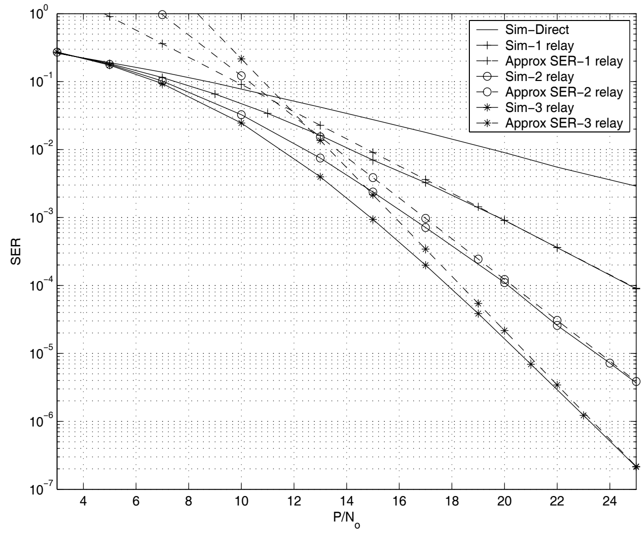

Fig. 2. Comparison between the approx. SER in (16) (dashed line), and the simulated SER (solid line) for different number of relays.

4. CONCLUSION

In this work, we analyzed the performance for a multi-node wireless network employing a decode-and-forward cooperation strategy. An approximate expression of the SER for the cooperative diversity network with N relays employing M-PSK signalling was derived. The analysis shows that the decode-and-forward protocol can achieve full diversity order equal to $(N + 1)$, which is the number of cooperating terminals. Simulations for networks with different number of relays reveal that the derived approximate SER is tight at high enough SNR. Simulation curves also show that the diversity order achieved by the system increases with increasing the number of cooperating relays. The theoretical results reveal that the optimum power allocation over the network will not depend on the direct link between the source and the destination.

5. REFERENCES

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