Maximum Achievable Diversity for MIMO-OFDM Systems with Arbitrary Spatial Correlation

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Abstract— **In this paper, the maximum achievable diversity order is determined for MIMO-OFDM systems with arbitrary spatial correlation. We show that the maximum achievable diversity order is the rank of the spatial correlation matrix of the channel including all delay paths. We also show that any space-frequency code designed to achieve full diversity in spatially independent MIMO-OFDM systems can be used to achieve full diversity in spatially correlated scenarios. Extensive simulation results are provided to support the theoretical analysis.**

I. INTRODUCTION

Recently there has been much interest in applying multiple transmit and receive antennas in broadband wireless communication systems. In addition to spatial diversity offered by the multiple input multiple output (MIMO) system, broadband channels have inherent frequency diversity due to multipath fading. Orthogonal frequency division multiplexing (OFDM) is usually used in such frequency selective systems to reduce the intersymbol interference [1], [2]. In order to achieve the spatial and frequency diversity offered by the MIMO-OFDM system, space-frequency (SF) coding techniques have been proposed [3]–[8]. However, the code design developed until now assumed a channel model with independent spatial fading at both transmitter and receiver. For the spatially correlated channel model, there is no exact formula for the maximum achievable diversity, and no code design has been proposed that can achieve this diversity.

In [9], the authors studied the effect of spatial correlation on the performance of SF codes. However, their derivations were made for the special case that the spatial correlations at the transmitter and the receiver are independent to each other, which was assumed for analytical tractability. Moreover, the maximum achievable diversity gain was shown only for the special case when the channel spatial correlation matrix was full rank, and it provided an upper bound when the spatial correlation matrix is rank deficient.

In this paper, with an arbitrary spatial correlation model for the MIMO channel, we determine the maximum achievable diversity gain for both full rank and rank deficient spatial correlation channel matrix. We show that the maximum achievable diversity order is the rank of the spatial correlation matrix of the channel including all delay paths. We also show that any SF code designed to achieve full diversity in the independent fading case can be used with a spatially correlated channel to get the maximum achievable diversity.

The rest of the paper is organized as follows. In Section II, we describe the system model used in the paper. In Section III, we drive the performance criteria of the system in order to show the parameters that determine the diversity gain of the system. In Section IV, we derive the exact formula for maximum achievable diversity for a general spatially correlated channel model. Simulation results are shown in Section V. Finally, some conclusions are made in Section VI.

Notations used in the paper: $\mathbf{I_N}$ denotes the $N\times N$ identity matrix, the superscripts T, $\mathcal{H}$, and $*$ represent the transpose, conjugate transpose and elementwise conjugation respectively, and $\otimes$ represents the tensor product. $\mathrm{vec}(\mathbf{C})$ transforms the matrix $\mathbf{C}=[\mathbf{c}_1\ldots\mathbf{c}_M]$, with $\mathbf{c}_i$ denoting its i-th column, into the column vector $\mathrm{vec}\left(\mathbf{C}\right)=\left[\mathbf{c}_1^T\,\ldots\,\mathbf{c}_M^T\right]^T$.

II. SYSTEM MODEL

In this paper, we consider a multipath frequency selective fading MIMO system with M_t transmit antennas and M_r receive antennas. The number of significant delay paths between any antenna pair is L. The channel impulse response from transmit antenna i to receive antenna j can be modeled as

$$h_{ij}\left(\tau\right)=\sum_{l=0}^{L-1}\alpha_{ij}\left(l\right)\delta\left(\tau-\tau_l\right), \tag{1}$$

where τ_l is the delay of the l-th path and $\alpha_{ij}\left(l\right)$ is the complex gain of the l-th path between transmit antenna i and receive antenna j. The $\alpha_{ij}\left(l\right)\sim\mathcal{CN}\left(0,\sigma_l^2\right)$ are modelled as zero mean, circularly symmetric complex Gaussian random variables with variance σ_l^2. The time delay τ_l and the variance σ_l^2 are the same for each transmit receive link [10]. The power of the L paths are normalized such that $\sum_{l=0}^{L-1}\sigma_l^2=1$.

OFDM with N subcarriers is used to overcome the effect of the multipath delay channel. The cyclic prefix is taken to be longer than the channel delay spread in order to guarantee

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transforming the channel to be flat fading over each subcarrier. From (1), the frequency response of the channel at the n-th subcarrier is given by

$$H_{ij}(n) = \sum_{l=0}^{L-1} \alpha_{ij}(l) e^{-j2\pi n \Delta f \tau_l}, \tag{2}$$

where $\Delta f = 1/T$ is the subcarrier frequency separation, and T is the OFDM symbol period.

The MIMO channel is assumed to have spatial correlation at both the transmitter and receiver sides, which will be reflected in their spatial correlation matrix later. The channel gains are assumed to be jointly Gaussian. It is also assumed that path gains for different delays are independent, i.e., different scatterers are assumed to be independent.

Space-frequency (SF) coding is used to achieve the spatial and frequency diversity inherent in the system. The input bit stream is divided into b bit-long segments, forming 2^b-ary source symbols. These symbols are then mapped onto a SF codeword to be transmitted over the M_t transmit antennas. Each space frequency codeword can be expressed as an $M_t \times N$ matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}(0) & \mathbf{c}(1) & \dots & \mathbf{c}(N-1) \end{bmatrix}, \tag{3}$$

where $\mathbf{c}(n) = \begin{bmatrix} c_1(n) & c_2(n) & \dots & c_{M_t}(n) \end{bmatrix}^T$ is an $M_t \times 1$ column vector, representing the channel symbol vector transmitted on the n-th subcarrier. The SF code is assumed to satisfy the energy constraint $E\left[\|\mathbf{C}\|_F^2\right] = NM_t$, where $E[\cdot]$ denotes expectation, and $\|\mathbf{C}\|_F$ is the Frobenius norm of $\mathbf{C}$. The OFDM transmitter applies IFFT to each row of the matrix $\mathbf{C}$, appends a cyclic prefix then transmits the i-th row from the i-th antenna.

At the receiver, after matched filtering, removing the cyclic prefix, and applying FFT, the received signal at the n-th subcarrier at receive antenna j is given by

$$y_j(n) = \sqrt{\frac{E_s}{M_t}} \mathbf{h}_j^T(n)\, \mathbf{c}(n) + v_j(n), \tag{4}$$

where

$$\mathbf{h}_j(n) = \begin{bmatrix} H_{1j}(n) & H_{2j}(n) & \dots & H_{M_tj}(n) \end{bmatrix}^T, \tag{5}$$

and $v_j(n) \sim \mathcal{CN}(0, N_o)$ denotes the additive white complex Gaussian noise, with zero mean and variance N_o, at the n-th subcarrier at receive antenna j. Thus the average SNR at each receive antenna is given by E_s/N_o.

III. Performance Criteria

The received signal (4) can be rewritten in matrix form as

$$\mathbf{y} = \sqrt{\frac{E_s}{M_t}} \mathbf{H} \mathrm{vec}(\mathbf{C}) + \mathbf{v}, \tag{6}$$

where the channel matrix $\mathbf{H}$ is of size $NM_r \times NM_t$ and is formatted as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \dots & \mathbf{H}_{M_r}^T \end{bmatrix}^T, \tag{7}$$

where $\mathbf{H}_j$ represents the channel frequency response to receive antenna j, and is formatted as an $N \times NM_t$ block diagonal matrix as follows

$$\mathbf{H}_j = \mathrm{diag}\left(\mathbf{h}_j^T(0), \mathbf{h}_j^T(1), \dots, \mathbf{h}_j^T(N-1)\right), \tag{8}$$

the received signal vector $\mathbf{y}$ is of size $NM_r \times 1$, and is given by

$$\mathbf{y} = [y_1(0) \dots y_1(N-1)\, y_2(0) \dots y_2(N-1) \dots \\ y_{M_r}(0) \dots y_{M_r}(N-1)]^T, \tag{9}$$

and the noise vector $\mathbf{v}$ has the same form as $\mathbf{y}$.

The receiver is assumed to know exactly the channel, while the transmitter is assumed to have no channel information. The receiver applies a maximum likelihood decoder

$$\hat{\mathbf{C}} = \underset{\mathbf{C}}{\mathrm{argmin}} \|\mathbf{y} - \sqrt{\frac{E_s}{M_t}} \mathbf{H} \mathrm{vec}(\mathbf{C}) \|^2. \tag{10}$$

The pairwise error probability between two codewords $\mathbf{C}$ and $\tilde{\mathbf{C}}$ for a given channel realization can be upper bounded by [11]

$$Pr\left(\mathbf{C} \to \tilde{\mathbf{C}} | \mathbf{H}\right) \le \frac{1}{2} \exp\left(\frac{-E_s}{4M_tN_o} \|\mathbf{\Phi}\|^2\right), \tag{11}$$

where $\mathbf{\Phi}$ is an $NM_r \times 1$ vector given by

$$\mathbf{\Phi} = \mathbf{H}\left[\mathrm{vec}(\mathbf{C}) - \mathrm{vec}\left(\tilde{\mathbf{C}}\right)\right]. \tag{12}$$

Since the channel coefficients are jointly Gaussian, the vector $\mathbf{\Phi}$ for fixed codewords has a Gaussian distribution with zero mean and covariance matrix

$$\mathbf{R}_{\mathbf{\Phi}} = E\left[\mathbf{\Phi}\mathbf{\Phi}^{\mathcal{H}}\right], \tag{13}$$

of size $NM_r \times NM_r$. Since N is usually greater than LM_t, the matrix $\mathbf{R}_{\mathbf{\Phi}}$ can be shown to be rank deficient.

Averaging the pairwise error probability (11) over all channel realizations we get [12]

$$Pr\left(\mathbf{C} \to \tilde{\mathbf{C}}\right) \le \left(\frac{E_s}{4M_tN_o}\right)^{-r(\mathbf{R}_{\mathbf{\Phi}})} \left(\prod_{i=0}^{r(\mathbf{R}_{\mathbf{\Phi}})-1} \lambda_i(\mathbf{R}_{\mathbf{\Phi}})^{-1}\right), \tag{14}$$

where $r(\mathbf{R}_{\mathbf{\Phi}})$ and $\lambda_i(\mathbf{R}_{\mathbf{\Phi}})$ are the rank and i-th eigenvalue respectively of the covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$.

IV. Maximum Achievable Diversity

According to (14), the diversity gain depends on the rank of the covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$ (13). The spatial correlation matrix that we consider here is more general than that in [9], in which it was assumed for analytical tractability that the correlation between the fading of two distinct antenna pairs is the product of the corresponding transmit correlation and receive correlation. In this paper, we consider an arbitrary spatial correlation structure. The covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$ can be further determined in the following theorem.

Theorem 1: The covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$ in (13) can be decomposed as

$$\mathbf{R}_{\mathbf{\Phi}} = \sum_{l=0}^{L-1} \left[I_{M_r} \otimes \mathbf{D}^{\tau_l} \left(\mathbf{C} - \tilde{\mathbf{C}} \right)^T \right] \mathbf{R}_\alpha (l) \left[I_{M_r} \otimes \left(\mathbf{C} - \tilde{\mathbf{C}} \right)^* \mathbf{D}^{*\tau_l} \right], \quad (15)$$

where the matrix

$$\mathbf{D} = \mathrm{diag}\left(1, e^{-j2\pi\Delta f}, \ldots, e^{-j2\pi(N-1)\Delta f}\right), \quad (16)$$

and the matrix $\mathbf{R}_\alpha(l)$ represents the spatial covariance matrix of the l-th delay path, which is specified as

$$\mathbf{R}_\alpha(l) = E\left[\alpha(l)\,\alpha^{\mathcal{H}}(l)\right], \quad (17)$$

in which the vector

$$\alpha(l) = \left[\alpha_{11}(l) \ldots \alpha_{M_t 1}(l) \ldots \alpha_{1 M_r}(l) \ldots \alpha_{M_t M_r}(l)\right]^{\mathcal{T}}, \quad (18)$$

has size $M_r M_t \times 1$.

Proof: From (12), the covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$ can be written as

$$\begin{aligned} \mathbf{R}_{\mathbf{\Phi}} &= E\left[\mathbf{H}\mathrm{vec}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)\mathrm{vec}^{\mathcal{H}}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)\mathbf{H}^{\mathcal{H}}\right] \\ &= E\left(\mathbf{H}\mathbf{\Delta}\mathbf{\Delta}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\right), \end{aligned} \quad (19)$$

where $\mathbf{\Delta} = \mathrm{vec}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)$. Using (7), we can write (19) as

$$\mathbf{R}_{\mathbf{\Phi}} = E\left\{ \begin{bmatrix} \mathbf{H}_1\mathbf{\Delta} \\ \vdots \\ \mathbf{H}_{M_r}\mathbf{\Delta} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta}^{\mathcal{H}}\mathbf{H}_1^{\mathcal{H}} & \ldots & \mathbf{\Delta}^{\mathcal{H}}\mathbf{H}_{M_r}^{\mathcal{H}} \end{bmatrix} \right\}. \quad (20)$$

Denote $E\left[\mathbf{H}_j\mathbf{\Delta}\mathbf{\Delta}^{\mathcal{H}}\mathbf{H}_p^{\mathcal{H}}\right]$ as $\mathbf{A}_{jp}$, we can compute the value of this matrix using (8) as follows

$$\mathbf{A}_{jp} = E\left\{ \begin{bmatrix} \mathbf{\Delta}^T(0)\,\mathbf{h}_j(0) \\ \vdots \\ \mathbf{\Delta}^T(N-1)\,\mathbf{h}_j(N-1) \end{bmatrix} \left[\mathbf{h}_p^{\mathcal{H}}(0)\,\mathbf{\Delta}^*(0) \ldots \mathbf{h}_p^{\mathcal{H}}(N-1)\,\mathbf{\Delta}^*(N-1)\right] \right\}, \quad (21)$$

where $\mathbf{\Delta}(n) = (\mathbf{c}(n) - \tilde{\mathbf{c}}(n))$. Using (2), the vector $\mathbf{h}_j$ can be written as $\mathbf{h}_j(n) = \sum_{l=0}^{L-1} e^{-j2\pi n\tau_l\Delta f}\alpha_{\mathbf{j}}(l)$, in which $\alpha_j(l) = \begin{bmatrix}\alpha_{1j}(l) & \ldots & \alpha_{M_t j}(l)\end{bmatrix}^T$. With the assumption that different delay paths are independent, we have

$$E\left[\mathbf{h}_j(n)\,\mathbf{h}_p^{\mathcal{H}}(k)\right] = \sum_{l=0}^{L-1} e^{-j2\pi(n-k)\tau_l\Delta f} E\left[\alpha_j(l)\,\alpha_p^{\mathcal{H}}(l)\right]. \quad (22)$$

From (21) and (22), we have $\mathbf{A}_{jp}$ as follows

$$\sum_{l=0}^{L-1} E\left\{ \begin{bmatrix} \mathbf{\Delta}^T(0)\,\alpha_j(l) \\ \vdots \\ e^{-j2\pi(N-1)\tau_l\Delta f}\mathbf{\Delta}^T(N-1)\,\alpha_j(l) \end{bmatrix} \left[\alpha_p^{\mathcal{H}}(l)\,\mathbf{\Delta}^*(0) \ldots \alpha_p^{\mathcal{H}}(l)\,\mathbf{\Delta}^*(N-1)\,e^{j2\pi(N-1)\tau_l\Delta f}\right] \right\}. \quad (23)$$

Using the fact that $\mathbf{\Delta}(n)$ is the n-th column of the matrix $(\mathbf{C} - \tilde{\mathbf{C}})$ we can further simplify (23) as follows

$$\mathbf{A}_{jp} = \sum_{l=0}^{L-1} D^{\tau_l}(\mathbf{C}-\tilde{\mathbf{C}})^T E\left[\alpha_j(l)\,\alpha_p^{\mathcal{H}}(l)\right](\mathbf{C}-\tilde{\mathbf{C}})^* D^{*\tau_l}. \quad (24)$$

Combining (20) and (24), we get (15).

From Theorem 1, if a SF code has been designed to achieve full diversity order in the case of no spatial correlation at the transmitter and the receiver, then the rank of the covariance matrix $\mathbf{R}_{\mathbf{\Phi}}$ which determines the diversity order can be calculated in the following theorem.

Theorem 2: The maximum achievable diversity order of the SF-coded MIMO-OFDM system with arbitrary spatial correlation channel is given by

$$r(\mathbf{R}_{\mathbf{\Phi}}) = \sum_{l=0}^{L-1} r(\mathbf{R}_\alpha(l)). \quad (25)$$

Proof: In order to compute the rank of the matrix $\mathbf{R}_{\mathbf{\Phi}}$ in (15), we will rewrite it in the following format

$$\mathbf{R}_{\mathbf{\Phi}} = \mathbf{F}\mathbf{R}_\alpha\mathbf{F}^{\mathcal{H}}, \quad (26)$$

where the matrix $\mathbf{F}$ is of size $NM_r \times LM_tM_r$, and is given by

$$\mathbf{F} = \left[I_{M_r} \otimes \mathbf{D}^{\tau_0}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)^T \ldots I_{M_r} \otimes \mathbf{D}^{\tau_{L-1}}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)^T \right], \quad (27)$$

and $\mathbf{R}_\alpha$ is an $LM_tM_r \times LM_tM_r$ block diagonal matrix given by

$$\mathbf{R}_\alpha = \mathrm{diag}\begin{pmatrix}\mathbf{R}_\alpha(0) & \ldots & \mathbf{R}_\alpha(L-1)\end{pmatrix}. \quad (28)$$

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If the SF code can achieve full diversity gain in the case of independent fading channel, i.e., no spatial correlation at the transmitter and the receiver, then the matrix $\mathbf{F}$ can be shown to be of rank LM_tM_r [8]. We can put (26) after row and column reordering in the form

$$\begin{aligned} \tilde{\mathbf{R}}_{\mathbf{\Phi}} &= \begin{bmatrix} \mathbf{F_1} \\ \mathbf{F_2} \end{bmatrix} \mathbf{R}_\alpha \begin{bmatrix} \mathbf{F}_\mathbf{1}^\mathcal{H} & \mathbf{F}_\mathbf{2}^\mathcal{H} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{F_1}\mathbf{R}_\alpha\mathbf{F}_\mathbf{1}^\mathcal{H} & \mathbf{F_1}\mathbf{R}_\alpha\mathbf{F}_\mathbf{2}^\mathcal{H} \\ \mathbf{F_2}\mathbf{R}_\alpha\mathbf{F}_\mathbf{1}^\mathcal{H} & \mathbf{F_2}\mathbf{R}_\alpha\mathbf{F}_\mathbf{2}^\mathcal{H} \end{bmatrix}, \end{aligned} \quad (29)$$

where $\mathbf{F_1}$ is of size $LM_tM_r \times LM_tM_r$ and is full rank. The matrix $\mathbf{F_2}$ takes the rest of the matrix. According to a theorem [14] on the relation between the singular values of a matrix and any square submatrix obtained by deleting equal number of rows and columns of the original matrix, we get

$$\lambda_i \left(\mathbf{R}_{\mathbf{\Phi}}\right) \geq \lambda_i \left(\mathbf{F_1}\mathbf{R}_\alpha\mathbf{F}_\mathbf{1}^\mathcal{H}\right), \quad (30)$$

where $\lambda_i(\cdot)$ denotes the i-th eigenvalue, and they are ordered in decreasing order. The eigenvalues of $\mathbf{F_1}\mathbf{R}_\alpha\mathbf{F}_\mathbf{1}^\mathcal{H}$ are given by

$$\lambda_i \left(\mathbf{F_1}\mathbf{R}_\alpha\mathbf{F}_\mathbf{1}^\mathcal{H}\right) = \theta_i \lambda_i \left(\mathbf{R}_\alpha\right), \quad (31)$$

where θ_i is a nonnegative real number such that $\lambda_1\left(\mathbf{F_1}\mathbf{F}_\mathbf{1}^\mathcal{H}\right) \leq \theta_i \leq \lambda_{LM_tM_r}\left(\mathbf{F_1}\mathbf{F}_\mathbf{1}^\mathcal{H}\right)$ which follows by Ostrowski's theorem [15]. Thus the rank of the matrix $\mathbf{R}_{\mathbf{\Phi}}$ is given by

$$r\left(\mathbf{R}_{\mathbf{\Phi}}\right) = r\left(\mathbf{R}_\alpha\right) = \sum_{l=0}^{L-1} r\left(\mathbf{R}_\alpha(l)\right), \quad (32)$$

where the second equality is due to the diagonal structure of $\mathbf{R}_\alpha$ in (28).

Theorem 2 asserts that if the SF code can achieve full diversity gain in case of independent fading channel with no spatial correlation at both sides, then it will achieve the maximum achievable diversity offered by the channel. The difference between our results and the results in [9] are as follows:

- In [9], the maximum achievable diversity gain was shown only when both the SF code and the channel spatial correlation were full rank, and it provided an upper bound when the spatial correlation matrix is rank deficient. The derivations were also made for the special case when the spatial correlation can be decomposed into the product of the spatial correlation at the transmitter with that at the receiver.
- In this paper, with the general spatial correlation model for the MIMO channel (17), we determined in Theorem 2 the maximum achievable diversity for both full rank and rank deficient channel spatial correlation matrix.
- We also showed that any SF code designed to achieve full diversity in the independent fading case can be used with a spatially correlated channel to get the maximum achievable diversity.

If the spatial correlation is only found at the transmitter side, and all receive antennas have the same fading statistics, then (17) will become block diagonal of the form $\mathbf{R}_\alpha(l) = \mathrm{diag}\begin{pmatrix}\mathbf{R}_l & \ldots & \mathbf{R}_l\end{pmatrix}$, where the matrix $\mathbf{R}_l$ is of size $M_t \times M_t$. Accordingly, the diversity order (25) can be shown to be given by

$$r\left(\mathbf{R}_{\mathbf{\Phi}}\right) = M_r \sum_{l=0}^{L-1} r\left(\mathbf{R}_l\right). \quad (33)$$

V. Simulation Results

To study the effect of spatial correlation on the performance of a SF code we considered two scenarios, the first with two transmit and one receive antennas and the second with two transmit and two receive antennas. In both cases, we considered a 2-ray, equal-power delay profile, with a delay of $20\mu s$ between the two rays. The MIMO-OFDM system has $N = 128$ subcarriers and the total BW is $1MHz$. We chose the full-diversity SF-code from [8] to conduct our simulations. The 2×2 Alamouti's structure [13] was used with repetition two times to guarantee full diversity in the case of independent fading channel. QPSK modulation was used.

To generate the spatially correlated channel coefficients we used the following model

$$\alpha_l = \mathbf{A}_l \tilde{\alpha}_l, \quad (34)$$

where $l \in \{0, \ldots, L-1\}$, $\tilde{\alpha}_l$ is $M_rM_t \times 1$ vector with i.i.d entries chosen from a complex Gaussian distribution with zero mean and σ_l^2 variance, and the matrix $\mathbf{A}_l$ contains the correlation coefficients. It is clear that the rank of $\mathbf{R}_\alpha(l)$ is equal to that of $\mathbf{A}_l$. For brevity, we will give two examples for the matrix $\mathbf{A}_l$ used in the simulations.

A. Simulations with 2Tx and 1Rx antennas

We compared three different cases for this scenario: i) the independent fading case in which $\mathbf{A}_l = I_2$, thus the maximum achievable diversity is 4; ii) the spatial correlation case with $\mathbf{A}_l$ being full rank, thus the maximum achievable diversity is also 4; and iii) the spatial correlation case with $\mathbf{A}_l$ has rank = 1, thus the maximum achievable diversity is 2, an example for this case

$$\mathbf{A}_0 = \begin{bmatrix} -0.07997 - j0.91094 & -0.00934 + j0.40459 \\ -0.65107 + j0.64212 & 0.31776 - j0.25062 \end{bmatrix}. \quad (35)$$

Fig. 1 depicts the performance of the SF code for these three cases. The figure shows that the performance curves for both the independent fading case and spatial correlation with full rank case have the same slope, thus they achieve equal diversity gain. Also, both of these curves has steeper slope than that of the rank deficient case.

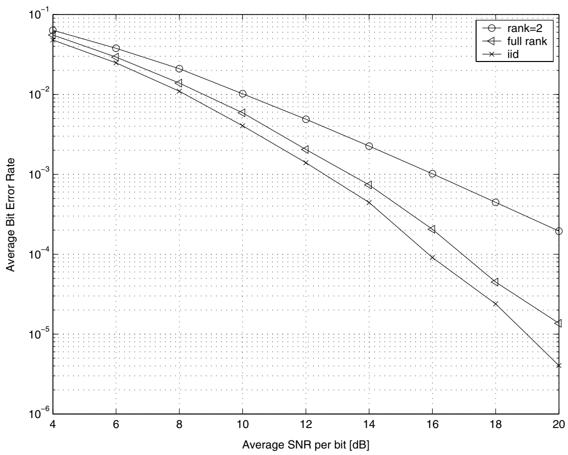

Fig. 1. Bit error rate performance of SF code for 2TX, 1Rx antennas under different spatial correlation conditions.

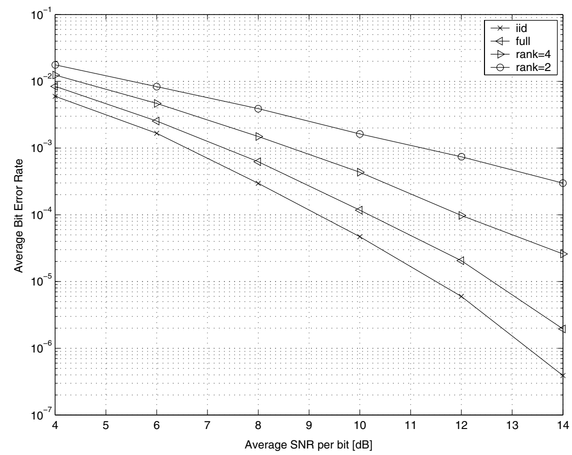

Fig. 2. Bit error rate performance of SF code for 2TX, 2Rx antennas under different spatial correlation conditions.

B. Simulations with 2Tx and 2Rx antennas

In this scenario, we compared four different cases: i) the independent fading case in which $\mathbf{A}_l = I_4$, thus the maximum achievable diversity is 8; ii) the spatial correlation case with $\mathbf{A}_l$ being full rank, thus the maximum achievable diversity is 8; iii) the spatial correlation case with $\mathbf{A}_l$ has rank $= 2$, thus the maximum achievable diversity is 4; and iv) the spatial correlation case with $\mathbf{A}_l$ has rank $= 1$, thus the maximum achievable diversity is 2, an example of $\mathbf{A}_0$ for this case is

$$\begin{bmatrix} -0.41 - j0.25 & 0.55 - j0.47 & 0.17 - j0.34 & -0.27 + j0.06 \\ 0.43 - j0.23 & 0.13 + j0.72 & 0.21 + j0.33 & 0.09 - j0.27 \\ 0.37 + j0.31 & -0.62 + j0.39 & -0.23 + j0.31 & 0.28 - j0.02 \\ -0.28 - j0.39 & 0.69 - j0.22 & 0.30 - j0.25 & -0.27 - j0.05 \end{bmatrix}. \tag{36}$$

Fig. 2 depicts the performance of the SF code for these four cases. The figure shows that the performance curves for both the independent fading case and spatial correlation with full rank case have the same slope, thus they achieve equal diversity gain. Also, the slope of the performance curve decreases for the rank deficient cases.

Thus the SF code designed for the independent fading channel was able to achieve the maximum diversity offered in the spatially correlated channels irrespective of the channel spatial correlation matrix being full rank or rank deficient.

VI. Conclusion

In this paper, we analyzed the performance limits of the SF-coded MIMO-OFDM systems with arbitrary spatial correlation. We have determined the maximum achievable diversity order, and shown that any SF code designed to achieve full diversity in spatially independent MIMO-OFDM systems can be used to achieve the full diversity in spatially correlated scenario. Simulation results showed that the slopes of the performance curves (diversity order) depend on the rank of the spatial correlation matrix.

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