EIGEN-SELECTION APPROACH FOR JOINT BEAMFORMING AND SPACE-FREQUENCY CODING IN MIMO-OFDM SYSTEMS WITH SPATIAL CORRELATION FEEDBACK

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ABSTRACT

This paper addresses the problem of joint optimization of transmit beamforming and space-frequency (SF) coding for MIMO-OFDM systems with spatial correlation feedback. This problem is challenging in the sense that the transmitter should be designed to beamform across multiple eigenspaces. The performance analysis for SF-coded MIMO-OFDM systems with beamforming is provided, and a general optimization problem for the beamforming design is formulated. Three suboptimal approaches to design the beamformer based on the derived design criteria are proposed. In terms of bit error rate, simulations show that improvement in the performance over SF coding without beamforming is about 3 dB for highly correlated channels.

1. INTRODUCTION

Recently there has been much interest in designing transmit-diversity schemes for multiple-input-multiple-output (MIMO) systems in the presence of partial channel state information (CSI) [1, 2]. However, these works have considered only the MIMO systems with flat fading channel model. For the case of frequency selective fading channels, which generally arises in broadband communication systems, few works have investigated the effect of the CSI feedback on MIMO-OFDM systems. In [4], the authors assumed perfect CSI at the transmitter, and they proposed a joint transmit-receive beamforming design for multicarrier frequency selective fading MIMO systems. A mean feedback model was adopted in [5], and an adaptive two-dimensional space-time coded beamformer was developed, based on a space-time (ST) coding strategy over each OFDM subcarrier. Note that utilizing ST coding on each subcarrier cannot exploit the frequency diversity available in the frequency selective fading environment [6].

In mobile scenarios, the channel may be quickly varying, and covariance feedback becomes an adequate approach to adopt as the channel statistics do not vary quickly. In this paper, we consider the problem of transmit beamforming design for MIMO-OFDM systems when the covariance matrix of the channel is available at the transmitter. We derive the average pairwise error probability of a MIMO OFDM system with arbitrary spatial correlation, and we formulate a general beamformer optimization problem in terms of minimizing the average pairwise error probability of MIMO-OFDM systems. For this joint optimization problem, we provide the criteria to design a SF-beamforming scheme. Then, we adopt a transmitting scheme in which we utilize a predesigned SF code and then design a beamformer that enhances the performance of the code under the knowledge of the channel covariance structure. The analysis reveals that in a MIMO multipath environment the eigenspace associated with the channel covariance matrices is larger than that of the beamformer, i.e., the beamformer has to match to multiple eigenspaces simultaneously, which makes it difficult to find a closed form solution for the optimal beamformer. This is different from the MIMO flat fading case, where the number of degrees of freedom available to design the beamformer is equal to the dimension of the channel covariance matrix that the beamformer should match to [2]. Based on the average pairwise error probability, we propose three suboptimal approaches to design the beamformer: i) Per-subcarrier scheme; ii) Eigenvalue selection scheme; and iii) Eigenspace selection scheme. Simulation results indicate that the Eigenvalue selection scheme provides the best performance among the proposed algorithms in terms of bit error rate (BER). This scheme locates the subspace associated with the largest eigenvalues in the eigenspaces of the channel covariance matrices. The corresponding eigenvectors are taken to be the beamformer's directions, and power is distributed along these directions proportional to the values of the corresponding channel eigenvalues. Simulation results show that this scheme has a 3 dB performance gain over SF coding without beamforming in highly correlated channels. For channels with lower correlations, it shows better performance in low to moderate SNR regions. This is due to the fact that at high enough SNR, diversity gain dominates.

The rest of the paper is organized as follows. In Section 2, the system model is described. In Section 3, we analyze the MIMO-OFDM system performance and formulate a general optimization problem for the beamformer design. In Section 4, we propose three approaches to design the beamformer. Simulation results are shown in Section 5, and finally, Section 6 concludes the paper. The following notations are used in the paper: the superscripts T, $\mathcal{H}$, and $*$ represent the transpose, conjugate transpose and element-wise conjugation respectively, and $\otimes$ represents the tensor product. Finally, vec$(\mathbf{C})$ transforms a matrix $\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_M]$ into a column vector vec$(\mathbf{C}) = \left[\mathbf{c}_1^T \dots \mathbf{c}_M^T\right]^T$.

2. SYSTEM MODEL

We consider a MIMO frequency selective fading channel model with M_t transmit antennas and M_r receive antennas. OFDM is utilized as it provides an attractive means to lower the complexity of equalization and decoding in frequency selective environment, and it has N subcarriers. The multipath channel has L significant delay paths between each transmit-receive antenna pair. The path gains for different delays are assumed to be independent. The channel impulse response from transmit antenna i to receive an-

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tenna j can be modeled as $h_{ij}(\tau) = \sum_{l=0}^{L-1} \alpha_{ij}(l)\,\delta(\tau - \tau_l)$, where τ_l is the delay of the l-th path and $\alpha_{ij}(l)$ is the complex path gain between transmit antenna i and receive antenna j. The $\alpha_{ij}(l) \sim \mathcal{CN}(0, \beta_l^2)$ are modeled as zero mean, circularly symmetric complex Gaussian random variables with variance β_l^2. The channel gains are assumed jointly Gaussian. The power of the L paths are normalized such that $\sum_{l=0}^{L-1} \beta_l^2 = 1$. We consider MIMO-OFDM systems to have spatial correlation at the transmitter side, while different receive antennas are assumed independent and have the same fading statistics.

Each SF-beamformer symbol can be expressed as an $M_t \times N$ matrix $\mathbf{B} = \begin{bmatrix} \mathbf{b}(0) & \mathbf{b}(1) & \ldots & \mathbf{b}(N-1) \end{bmatrix}$, where $\mathbf{b}(n)$ is an $M_t \times 1$ column vector, and $\mathbf{B}$ is assumed to satisfy the energy constraint $E\left[\|\mathbf{B}\|_F^2\right] = NM_t$, where $E[\cdot]$ denotes expectation, and $\|\mathbf{B}\|_F$ is the Frobenius norm of $\mathbf{B}$. The OFDM transmitter applies IFFT to each row of the matrix $\mathbf{B}$. By appending a cyclic prefix, it transmits the i-th row of $\mathbf{B}$ at the i-th antenna. At the receiver, after matched filtering, removing the cyclic prefix, and applying FFT, the received signal at the n-th subcarrier at receive antenna j is given by

$$y_j(n) = \sqrt{\frac{\rho}{M_t}}\mathbf{h}_j^T(n)\,\mathbf{b}(n) + v_j(n), \qquad (1)$$

where $\mathbf{h}_j(n) = \begin{bmatrix} H_{1j}(n) & H_{2j}(n) & \ldots & H_{M_t j}(n) \end{bmatrix}^T$, in which $H_{ij}(n) = \sum_{l=0}^{L-1} \alpha_{ij}(l)\,e^{-j2\pi n\Delta f \tau_l}$, represents the channel frequency response at the n-th subcarrier between transmit antenna i and receive antenna j, where Δf is the subcarrier frequency separation. The term $v_j(n) \sim \mathcal{CN}(0,1)$ in (1) denotes the additive white circularly symmetric complex Gaussian noise, with zero mean and unit variance, at the n-th subcarrier at receive antenna j. ρ is the SNR per receive antenna.

3. PERFORMANCE ANALYSIS AND GENERAL OPTIMIZATION FORMULATION FOR BEAMFORMER DESIGN

First we derive an expression for the average pairwise error probability. We rewrite the received signal in (1) in matrix form as

$$\mathbf{y} = \sqrt{\frac{\rho}{M_t}}\mathbf{H}\mathrm{vec}(\mathbf{B}) + \mathbf{v}, \qquad (2)$$

where the $NM_r \times NM_t$ channel matrix $\mathbf{H}$ is formatted as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \ldots & \mathbf{H}_{M_r}^T \end{bmatrix}^T, \qquad (3)$$

in which $\mathbf{H}_j$ represents the channel frequency response to receive antenna j, and is formatted as an $N \times NM_t$ block diagonal matrix as follows $\mathbf{H}_j = \mathrm{diag}(\mathbf{h}_j^T(0), \mathbf{h}_j^T(1), \ldots, \mathbf{h}_j^T(N-1))$. We can show that the average pairwise error probability of a ML decoder is upper bounded by

$$Pr\left(\mathbf{B} \to \tilde{\mathbf{B}}\right) \le \left(\frac{\rho}{4M_t}\right)^{-r(\mathbf{R}_\Phi)}\left(\prod_{i=0}^{r(\mathbf{R}_\Phi)-1} \mu_i(\mathbf{R}_\Phi)\right)^{-1}, \qquad (4)$$

where $\Phi = \mathbf{H}\left(\mathrm{vec}(\mathbf{B}) - \mathrm{vec}\left(\tilde{\mathbf{B}}\right)\right)$ is an $NM_r \times 1$ vector, and

$$\mathbf{R}_\Phi = E\left[\Phi\Phi^{\mathcal{H}}\right]. \qquad (5)$$

In (4), $r(\mathbf{R}_\Phi)$ and $\mu_i(\mathbf{R}_\Phi)$ are the rank and the i-th eigenvalue of the covariance matrix $\mathbf{R}_\Phi$, respectively. We can show that the covariance matrix $\mathbf{R}_\Phi$ in (5) can be decomposed as follows

$$\mathbf{R}_\Phi = \mathbf{I}_{M_r} \otimes \mathbf{F}\mathrm{diag}\left[\mathbf{R}_{\alpha,0}, \mathbf{R}_{\alpha,1}, \cdots, \mathbf{R}_{\alpha,L-1}\right]\mathbf{F}^{\mathcal{H}}, \qquad (6)$$

where $\mathbf{R}_{\alpha,l} = E[\alpha_j(l)\alpha_j^{\mathcal{H}}(l)]$, $\alpha_i(l) = [\alpha_{1i}(l), \cdots, \alpha_{M_t i}(l)]^T$, and $\mathbf{F} = [\mathbf{D}^{\tau_0}\left(\mathbf{B} - \tilde{\mathbf{B}}\right)^T, \cdots, \mathbf{D}^{\tau_{L-1}}\left(\mathbf{B} - \tilde{\mathbf{B}}\right)^T]$, in which $\mathbf{D} = \mathrm{diag}\left(1, e^{-j2\pi\Delta f}, \ldots, e^{-j2\pi(N-1)\Delta f}\right)$. The proof is omitted for space limitations.

We state the optimization problem as follows. We try to jointly design a general SF-beamformer matrix $\mathbf{B}_o$ that minimizes the system pairwise error probability (4), i.e.,

$$\mathbf{B}_o = \arg\min_{\mathbf{B}}\left\{\left(\frac{\rho}{4M_t}\right)^{-r(\mathbf{R}_\Phi)}\left(\prod_{i=0}^{r(\mathbf{R}_\Phi)-1} \mu_i(\mathbf{R}_\Phi)\right)^{-1}\right\}, \qquad (7)$$

with the energy constraint $E\left[\|\mathbf{B}\|_F^2\right] = NM_t$, and where the matrix $\mathbf{R}_\Phi$ is specified in (6). The objective function (7) suggests two criteria to design a general SF-beamformer symbol $\mathbf{B}$: (i) Maximize the rank of the matrix $\mathbf{R}_\Phi$, which corresponds to maximizing the diversity gain of the system. (ii) Maximize the product of the nonzero eigenvalues of $\mathbf{R}_\Phi$, which corresponds to maximizing the coding gain of the system. We emphasize at this point that the above design criteria for SF-beamformer are general in the sense that we do not impose any structure on the SF-beamformer $\mathbf{B}$. If there is no spatial correlation at the transmitter side, i.e., the spatial correlation matrices $\mathbf{R}_{\alpha,l}$ are identity, the above conditions reduces to the design criteria of SF codes [7]. The general optimization problem is difficult to tackle analytically. To overcome this problem, we will adopt another transmitting scheme in which a SF code is already designed to achieve full diversity for a spatial correlation-free channel, and then we try to design a beamformer $\mathbf{W}$ to match to the channel correlation matrix.

In the sequel, we denote the SF code by a $M_t \times N$ matrix $\mathbf{C}$. The linear transformation, or beamformer $\mathbf{W}$, can take various forms, for example: i) $\mathrm{vec}(\mathbf{B}) = \mathbf{W}\mathrm{vec}(\mathbf{C})$; ii)$\mathbf{B} = \mathbf{WC}$. In general, we represent the relation between the SF-beamformer symbol $\mathbf{B}$ and the SF-code $\mathbf{C}$ as follows $\mathbf{f}(\mathbf{B}) = \mathbf{W}\mathbf{f}(\mathbf{C})$. For space limitations, we only consider the conventional definition of the beamformer $\mathbf{B} = \mathbf{WC}$, however, the analysis can be extended for the general case. Substituting this definition into (6), we get

$$\mathbf{R}_\Phi = \mathbf{I}_{M_r} \otimes \hat{\mathbf{F}}\mathrm{diag}\left(\mathbf{W}^T\mathbf{R}_{\alpha,0}\mathbf{W}^*, \mathbf{W}^T\mathbf{R}_{\alpha,1}\mathbf{W}^*, \cdots, \mathbf{W}^T\mathbf{R}_{\alpha,L-1}\mathbf{W}^*\right)\hat{\mathbf{F}}^{\mathcal{H}}, \qquad (8)$$

where $\hat{\mathbf{F}} = \left[\mathbf{D}^{\tau_0}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)^T, \;\cdots,\; \mathbf{D}^{\tau_{L-1}}\left(\mathbf{C} - \tilde{\mathbf{C}}\right)^T\right]$.

In order to simplify the notations, let the $LM_t \times LM_t$ matrix $\tilde{\mathbf{R}}$ denote the block diagonal matrix in (8) as

$$\tilde{\mathbf{R}} = \mathrm{diag}\left(\mathbf{W}^T\mathbf{R}_{\alpha,0}\mathbf{W}^*, \mathbf{W}^T\mathbf{R}_{\alpha,1}\mathbf{W}^*, \cdots, \mathbf{W}^T\mathbf{R}_{\alpha,L-1}\mathbf{W}^*\right). \qquad (9)$$

Assuming that the SF code is designed to achieve full diversity in the case of no spatial correlation, we rewrite $\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}$ after row and column reordering in the form

$$\mathbf{J} = \begin{bmatrix} \hat{\mathbf{F}}_1 \\ \hat{\mathbf{F}}_2 \end{bmatrix}\tilde{\mathbf{R}}\begin{bmatrix} \hat{\mathbf{F}}_1^{\mathcal{H}} & \hat{\mathbf{F}}_2^{\mathcal{H}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{F}}_1\tilde{\mathbf{R}}\hat{\mathbf{F}}_1^{\mathcal{H}} & \hat{\mathbf{F}}_1\tilde{\mathbf{R}}\hat{\mathbf{F}}_2^{\mathcal{H}} \\ \hat{\mathbf{F}}_2\tilde{\mathbf{R}}\hat{\mathbf{F}}_1^{\mathcal{H}} & \hat{\mathbf{F}}_2\tilde{\mathbf{R}}\hat{\mathbf{F}}_2^{\mathcal{H}} \end{bmatrix}, \qquad (10)$$

where $\mathbf{J}$ is the reordered matrix, $\hat{\mathbf{F}}_1$ is of size $LM_t \times LM_t$ and is full rank, and the matrix $\hat{\mathbf{F}}_2$ takes the rest of the matrix. Since the ordered singular values of a matrix are not smaller than the corresponding singular values of any square submatrix obtained by deleting equal number of rows and columns of the original matrix [8], we get

$$\mu_i(\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}) \geq \mu_i(\hat{\mathbf{F}}_1\tilde{\mathbf{R}}\hat{\mathbf{F}}_1^{\mathcal{H}}), \tag{11}$$

where $\mu_i(\cdot)$ denotes the i-th eigenvalue of a matrix and are ordered in non-increasing order.

According to Ostrowski [9], the eigenvalues of $\hat{\mathbf{F}}_1\tilde{\mathbf{R}}\hat{\mathbf{F}}_1^{\mathcal{H}}$ are given by $\mu_i(\hat{\mathbf{F}}_1\tilde{\mathbf{R}}\hat{\mathbf{F}}_1^{\mathcal{H}}) = \theta_i\mu_i(\tilde{\mathbf{R}})$, where θ_i is a nonnegative real number such that $\mu_{min}(\hat{\mathbf{F}}_1\hat{\mathbf{F}}_1^{\mathcal{H}}) \leq \theta_i \leq \mu_{max}(\hat{\mathbf{F}}_1\hat{\mathbf{F}}_1^{\mathcal{H}})$, and μ_{min} and μ_{max} denote the smallest and largest eigenvalues, respectively. Applying Ostrowski's theorem along with (11), the eigenvalues of the matrix $\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}$ can be lower bounded as follows

$$\mu_i(\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}) \geq \mu_{min}(\hat{\mathbf{F}}_1\hat{\mathbf{F}}_1^{\mathcal{H}})\mu_i(\tilde{\mathbf{R}}). \tag{12}$$

Note that, maximizing the coding gain of the system corresponds to maximizing the product of the nonzero eigenvalues of the matrix $\mathbf{R}_{\Phi}$ which is equivalent to maximizing the product of the nonzero eigenvalues of $\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}$. From (12), this product can be lower bounded as follows

$$\prod_{i=0}^{r(\tilde{\mathbf{R}})-1} \mu_i(\hat{\mathbf{F}}\tilde{\mathbf{R}}\hat{\mathbf{F}}^{\mathcal{H}}) \geq \gamma \prod_{i=0}^{r(\tilde{\mathbf{R}})-1} \mu_i(\tilde{\mathbf{R}}). \tag{13}$$

where γ is a constant that depends on $\mu_{min}(\hat{\mathbf{F}}_1\hat{\mathbf{F}}_1^{\mathcal{H}})$. If the matrix $\tilde{\mathbf{R}}$ is full rank, the product of its eigenvalues corresponds to its determinant. According to Hadamard's inequality [9], the determinant of the matrix $\tilde{\mathbf{R}}$ is upper bounded by the product of its diagonal elements, i.e., $\det(\tilde{\mathbf{R}}) \leq \prod_{i=1}^{LM_t} \tilde{\mathbf{R}}_{ii}$, where $\tilde{\mathbf{R}}_{ii}$ is the i-th diagonal element of the matrix $\tilde{\mathbf{R}}$. The equality holds when the matrix $\tilde{\mathbf{R}}$ is diagonalized and this corresponds to choosing $\mathbf{W}$ to daigonalize $\mathbf{W}^T\mathbf{R}_{\alpha,l}\mathbf{W}^*$, for all $0 \leq l \leq (L-1)$.

According to (8) and the above discussion, the beamformer $\mathbf{W}$ should match the multiple eigenspaces of the spatial correlation matrices $\mathbf{R}_{\alpha,l}$, $0 \leq l \leq L-1$ simultaneously. This can not be achieved, in general, except for the special cases when all of the L delay paths have the same spatial correlation matrix, or when $L = 1$ which corresponds to the flat fading case. As a result, it is very difficult, if not impossible, to find a closed form solution for the optimal beamformer. The above optimization problem is challenging, as it is different from the problem of beamforming in a MIMO flat fading channel in the sense that the transmitter should beamform across multiple eigenspaces simultaneously. In order to provide some insights, we will render to suboptimal solutions for the problem.

4. SUBOPTIMAL DESIGNS OF BEAMFORMERS

In this section, three different approaches for designing the beamformer are proposed. The proposed approaches, although suboptimal, are well motivated by the derived performance criteria and the understanding of the underlying physics of the problem.

4.1. Per-subcarrier Solution

First, we consider designing a beamformer independently for each subcarrier. Analyzing the system performance, we can show that the corresponding matrix that determines the performance in this case is given by

$$\mathbf{R}_{\Phi}(n) = \mathbf{I}_{M_r} \otimes \Big[(\mathbf{c}(n) - \tilde{\mathbf{c}}(n))^T \mathbf{W}^T(n) \sum_{l=0}^{L-1} \mathbf{R}_{\alpha,l} \cdot \mathbf{W}^*(n)(\mathbf{c}(n) - \tilde{\mathbf{c}}(n))^*\Big]. \tag{14}$$

$\mathbf{W}(n)$ denotes the beamformer at the n-th subcarrier. Equation (14) asserts that independent of the subcarrier, the beamformer should match to the same matrix which is given by $\sum_{l=0}^{L-1} \mathbf{R}_{\alpha,l}$. The interpretation for this result is that the channel transfer function at any subcarrier is given by the FFT of the channel gains at all delays, and thus they all have the same spatial information about the channel.

To get an expression for the beamformer, let the eigendecomposition of the matrix $\sum_{l=0}^{L-1} \mathbf{R}_{\alpha,l}$ be given by $\sum_{l=0}^{L-1} \mathbf{R}_{\alpha,l} = \mathbf{V}\Lambda\mathbf{V}^{\mathcal{H}}$. Two main components constitutes any beamformer: the directions along which the information are being sent, and the power loading along each of these directions. We represent the beamformer $\mathbf{W}$ in the following way

$$\mathbf{W} = \mathbf{U}\Gamma, \tag{15}$$

where the i-th column in $\mathbf{U}$ corresponds to the i-th direction, and Γ is a diagonal matrix with the i-th diagonal element representing the power loading along this direction. We can show that in order to whiten the effect of the spatial correlation, the eigenbeams in $\mathbf{U}$ should be chosen such that

$$\mathbf{U} = \mathbf{V}^*. \tag{16}$$

Substituting (16) and (15) into (14) the matrix $\mathbf{R}_{\Phi}(n)$ can be written as

$$\mathbf{R}_{\Phi}(n) = \mathbf{I}_{M_r} \otimes \sum_{i=1}^{M_t} |\, c_i(n) - \tilde{c}_i(n)\,|^2\, \sigma_i^2\lambda_i, \tag{17}$$

where σ_i and λ_i are the i-th diagonal entries in the matrices Γ and Λ respectively. To maximize the diagonal entries in (17) irrespective of the SF code design, which corresponds to maximizing the eigenvalues of $\mathbf{R}_{\Phi}(n)$, we can upper and lower bound it as follows

$$C_{min} \sum_{i=1}^{M_t} \sigma_i^2\lambda_i \leq \sum_{i=1}^{M_t} |\, c_i(n) - \tilde{c}_i(n)\,|^2\, \sigma_i^2\lambda_i \leq C_{max} \sum_{i=1}^{M_t} \sigma_i^2\lambda_i, \tag{18}$$

where C_{min} and C_{max} are the minimum and maximum of the quantity $|\, c_i(n) - \tilde{c}_i(n)\,|^2$, respectively. We try to maximize $\sum_{i=1}^{M_t} \sigma_i^2\lambda_i$, which appears in both the lower and upper bounds shown in (18). However, there is an energy constraint given by

$$\mathbf{B} = \frac{\mathbf{W}\mathbf{C}}{\|\mathbf{W}\mathbf{C}\|_F}\sqrt{NM_t}, \tag{19}$$

which guarantees that the energy of the SF-beamformer symbol $\mathbf{B}$ does not exceed NM_t. This optimization can in general be done numerically, however we found through simulations an efficient way to do power loading. More specifically, applying Schwartz inequality to the quantity $\sum_{i=1}^{M_t} \sigma_i^2\lambda_i$ we get

$$\sigma_i^2 = \frac{\lambda_i}{\sum_{l=0}^{L-1} \lambda_i} P, \tag{20}$$

where P is a constant and depends on the energy constraint (19). The rationale behind doing the power loading in this way is that in general the available power should be distributed according to the channel conditions, i.e., more power should be allocated to channels with better quality.

4.2. Eigenvalue Selection Scheme

In this subsection, we design the beamformer jointly for all subcarriers, and propose the Eigenvalue selection scheme. As stated before, it is very difficult, if not impossible, to design the optimal $\mathbf{W}$ to match to all of the L channel covariance matrices simultaneously. One intuitive way to overcome this problem is to design the beamformer in order to span the best M_t directions from the L eigenspaces. In the following, we propose an Eigenvalue selection approach, in which we choose the largest M_t eigenvalues λ_i from the LM_t eigenvalues available from the eigendecomposition of the L covariance matrices, and the corresponding M_t eigenvectors $\mathbf{v}_i$. The algorithm can be summarized in the following steps:

1) Let the eigendecomposition of the spatial correlation matrix at the l-th path be given by $\mathbf{R}_{\alpha,l} = \mathbf{V}_l\Lambda_l\mathbf{V}_l^{\mathcal{H}}$, where $0 \leq l \leq L-1$.
2) Choose the largest M_t eigenvalues and the corresponding eigenvectors from the LM_t available eigenvalues and eigenvectors in Λ_l and $\mathbf{V}_l$, $l = 0, 1, \cdots, L-1$.
3) Arrange the M_t selected pairs in matrix format as follows
$$\Lambda = diag(\lambda_1, \cdots, \lambda_{M_t}), \qquad \mathbf{V} = [\mathbf{v}_1, \cdots, \mathbf{v}_{M_t}]. \tag{21}$$
4) The beamformer $\mathbf{W}$ is determined as $\mathbf{W} = \mathbf{U}\Gamma$, in which
$$\mathbf{U} = \mathbf{V}^*, \qquad \Gamma = diag(\sigma_1, \cdots, \sigma_{M_t}), \tag{22}$$
and $\sigma_i^2 = \frac{\lambda_i}{\sum_{l=0}^{L-1}\lambda_i}P$.

It can be expected that choosing the directions with the largest eigenvalues, i.e., with the most reliable channel conditions, enhance the coding gain. However, since the directions associated with the beamformer belong to different eigenspaces, they are no more orthogonal. Hence, full diversity is not guaranteed in this Eigenvalue selection approach. We will explore this more in another approach described next.

4.3. Eigenspace Selection Scheme

In this scheme, we try to jointly select the eigenvalues and eigenvectors, not only based on coding gain, but also based on the diversity order of the system. The criteria that we suggest is maximizing the volume occupied by the beamformer matrix, which is given by the absolute value of the determinant of the beamformer matrix
$$\mathbf{W} = \underset{\substack{\lambda_i, \mathbf{v}_i \\ i=1,\cdots,LM_t}}{\operatorname{argmax}} \ |\det(\mathbf{W})|. \tag{23}$$

To understand the intuition behind using this cost function, let us investigate the $M_t = 2$ case, in which the beamformer can be written as follows
$$\mathbf{W} = [\mathbf{u}_1 \mathbf{u}_2] \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}. \tag{24}$$

In this case, the criteria is proportional to the area spanned by the matrix $\mathbf{W}$. This area is given by $\sigma_1\sigma_2 \sin(<\mathbf{u}_1, \mathbf{u}_2>)$, where $\mathbf{u}_i$ and σ_i are the i-th eigenbeam and associated allocated power respectively, and $< \cdot, \cdot >$ denotes the angle between the two vectors. Clearly, the coding gain is controlled by the part $\sigma_1\sigma_2$, which corresponds to the power loading and the magnitude of the channel eigenvalues. The diversity gain is controlled by $\sin(<\mathbf{u}_1, \mathbf{u}_2>)$. Note that full diversity corresponds to the case when the two eigenvectors $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal, while diversity order one results when these two vectors are parallel.

In a higher dimensional space, the volume occupied by the beamformer, given by $\det(\mathbf{W})$, is merely the volume spanned by a parallelepiped in an M_t-dimensional space. Maximizing $\det(\mathbf{W})$ provides a tradeoff between the coding gain and the diversity order achieved by the system. We summarize the algorithm for the Eigenspace selection scheme in the following steps:

1) Let the eigendecomposition of the spatial correlation matrix at the l-th path be given by $\mathbf{R}_{\alpha,l} = \mathbf{V}_l\Lambda_l\mathbf{V}_l^{\mathcal{H}}$, where $0 \leq l \leq L-1$.
2) Choose every possible combination of M_t eigenvalue and eigenvector pairs from the LM_t pairs available from the eigendecomposition in the previous step.
3) Arrange the M_t selected pairs in matrix format as in (21).
4) The beamformer $\mathbf{W}$ is determined as in (22).
5) Calculate $|\det(\mathbf{W})|$.
6) From among all possible combinations, choose $\mathbf{W}$ with the largest determinant.

Similar to the Eigenvalue selection scheme, the columns of the matrix $\mathbf{U}$ in the Eigenspace selection algorithm are not orthogonal. Hence, we need to normalize the energy of the new SF-beamformer symbol as $\mathbf{B} = \frac{\mathbf{WC}}{\|\mathbf{WC}\|}\sqrt{NM_t}$.

5. SIMULATION RESULTS

To demonstrate the performance improvement due to applying the proposed algorithms compared to that of SF coding without beamforming, we performed some computer simulations. The channel model used is a two-ray, equal-power delay profile, with a delay of $20\mu s$ between the two rays. The MIMO-OFDM system has $N = 128$ subcarriers, and QPSK modulation is used. The total bandwidth of the system is $1MHz$. $M_t = 2$ and $M_r = 2$ antennas are used throughout the simulations. We choose the full-diversity SF code via mapping from [7] to conduct the simulations. In our simulations we use the 2×2 Alamouti's code [3] with repetition two times. To generate the spatial correlation channel coefficients, we use the following model $\alpha_l = \mathbf{A}_l\tilde{\alpha}_l$, where $l \in [0, \cdots, L-1]$, the vector α_l is defined as $\alpha_l = [\alpha_1^T(l), \cdots, \alpha_{M_r}^T(l)]^T$, $\tilde{\alpha}_l$ is an $M_rM_t \times 1$ vector with i.i.d entries chosen from a complex Gaussian distribution with zero mean and variance β_l^2, and the matrix $\mathbf{A}_l$ contains the correlation coefficients. For space limitations, two channel scenarios are considered in the simulation experiments: (i) Channel 1: The eigenvalues of the 2×2 matrix $\mathbf{R}_{\alpha,l}$, $l \in \{0, 1\}$, has one non-zero eigenvalue. This can be considered as a highly correlated scenario. (ii) Channel 2: The eigenvalues for $\mathbf{R}_{\alpha,0}$ are 0.13 and 0.8, and for $\mathbf{R}_{\alpha,1}$ are 0.7 and 0.2. This corresponds to a channel with low spatial correlation.

Fig. 1 depicts the results for channel 1, in which both the Eigenvalue and Eigenspace selection schemes choose the same eigenvector pair. As shown in the results, the proposed algorithms have better performance compared to that of SF coding without beamforming. Also, it can be seen that the performance curves are approximately parallel, as the beamformer achieves coding-gain, and does not incur any diversity loss in this case, due to the fact that Channel 1 is highly correlated. The performance of the Eigenvalue selection scheme is better than that of the per-subcarrier algorithm, and it has approximately 3 dB gain over SF coding without beamforming. This is due to the fact that the Eigenvalue selection algorithm beamforms in the two non-zero directions, while SF coding distributes its power equally among all directions, hence losing half its power along the two non-reliable directions with zero eigenvalues.

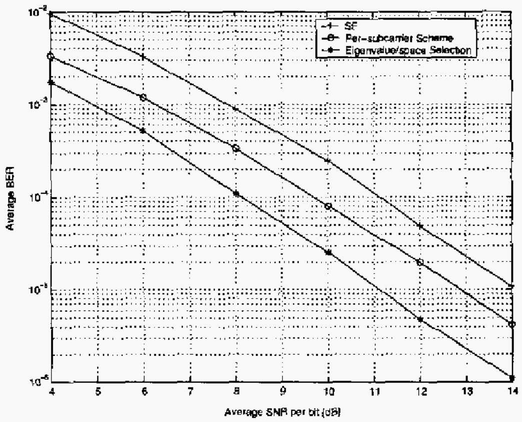

Fig. 1. Bit error rate performance comparison: Channel 1 with $M_t = 2$ transmit and $M_r = 2$ receive antennas.

Fig. 2 depicts the results for channel 2, which is a low correlated channel. In this scenario, Eigenvalue and Eigenspace beamforming choose different eigenvector pairs. It can be seen that at low to medium SNR, Eigenvalue selection still gives the best performance, while the performance becomes in favor of SF coding at high SNR regions. This can be interpreted as follows: Since Channel 2 is less correlated than Channel 1, the eigenvalues are more spread. In the region of low and medium SNR, sending information on the most reliable channels gives the best performance, this is achieved by the Eigenvalue selection scheme as it chooses the directions with largest eigenvalues. While in high SNR region, diversity gain dominates the performance, which corresponds to equal power loading along all directions, and it is achieved by SF coding. As expected, Eigenspace selection beamforming provides the tradeoff between the two extreme cases: Eigenvalue selection scheme, which corresponds to optimizing the coding gain, and SF coding without beamforming which achieves full diversity gain.

6. CONCLUSION

In this paper, we derived the performance analysis for a MIMO-OFDM system with arbitrary spatial correlation, and we provided the criteria to jointly design an optimum SF-beamformer at the transmitter. The analysis revealed that finding a closed form for the optimal beamformer design is not tractable due to the fact that the beamformer need to match to multiple eigenspaces simultaneously. Based on our analytical results, we proposed three suboptimal transmitting schemes. For any SF code designed for a spatially uncorrelated channel, we designed beamformers that match to the spatial covariance structure of the channel. Simulation results indicated that the Eigenvalue selection scheme provides the best performance among the proposed schemes in terms of the system BER. The Eigenvalue selection scheme can achieve a 3 dB gain over SF coding without beamforming in highly correlated channels. For channels with lower correlations, it results in a better performance in low to moderate SNR regions.

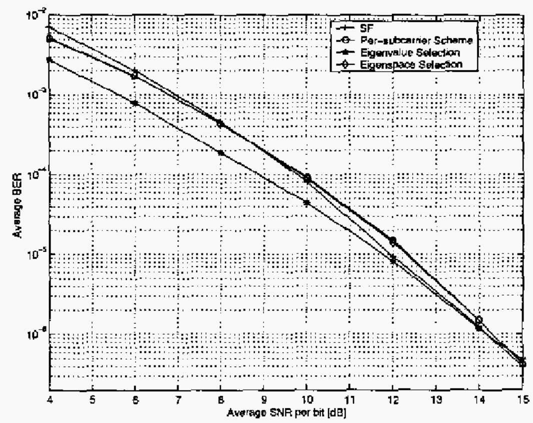

Fig. 2. Bit error rate performance comparison: Channel 2 with $M_t = 2$ transmit and $M_r = 2$ receive antennas.

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