Towards Maximum Achievable Diversity in Space, Time, and Frequency: Performance Analysis and Code Design

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***Abstract*—Multiple input multiple output (MIMO) communication systems with orthogonal frequency division multiplexing (OFDM) modulation have a great potential to play an important role in the design of the next-generation broadband wireless communication systems. In this paper, we address the problem of performance analysis and code design for MIMO-OFDM systems when coding is applied over both spatial, temporal, and frequency domains. First, we provide an analytical framework for the performance analysis of MIMO-OFDM systems assuming arbitrary power delay profiles. Our general framework incorporates the space–time and space–frequency (SF) coding approaches as special cases. We also determine the maximum achievable diversity order, which is found to be the product of the number of transmit and receive antennas, the number of delay paths, and the rank of the temporal correlation matrix. Then, we propose two code design methods that are guaranteed to achieve the maximum diversity order. The first method is a repetition coding approach using full-diversity SF codes, and the second method is a block coding approach that can guarantee both full symbol rate and full diversity. Simulation results are also presented to support the theoretical analysis.**

***Index Terms*—Broadband wireless communications, maximum achievable diversity, MIMO-OFDM systems, multiple antennas, space–frequency coding, space–time–frequency coding.**

I. Introduction

Multiple input multiple output (MIMO) communication systems have a great potential to play an important role in the design of the next-generation wireless communication systems due to the advantages that such systems can offer. By employing multiple transmit and receive antennas, the adverse effects of the wireless propagation environment can be significantly reduced. In case of narrowband wireless communications, where the fading channel is frequency nonselective, many modulation and coding methods [1]–[9], termed as space–time (ST) codes, have been proposed to exploit the spatial and temporal diversities available in the multiantenna channel.

In case of broadband wireless communications, where the fading channel is frequency selective, orthogonal frequency division multiplexing (OFDM) modulation can be used to transform the frequency-selective channel into a set of parallel frequency flat channels, providing high spectral efficiency and eliminating the need for high-complexity equalization algorithms. To take advantage of both MIMO systems and OFDM modulation, MIMO-OFDM systems have been proposed, resulting in two major channel coding approaches for these systems. The first approach is space–frequency (SF) coding, where coding is applied within a single OFDM block to exploit the spatial and frequency diversities. The other approach is space–time–frequency (STF) coding, where the coding is applied across multiple OFDM blocks to exploit the spatial, temporal, and frequency diversities available in frequency-selective MIMO channels.

Early works on SF coding [10]–[15] used ST codes directly as SF codes, i.e., previously existing ST codes were used by replacing the time domain with the frequency domain (OFDM tones). The performance criteria for SF-coded MIMO-OFDM systems were derived in [15] and [16], and the maximum achievable diversity was found to be $LM_\mathrm{t}M_\mathrm{r}$, where M_t and M_r are the number of transmit and receive antennas, respectively, and L is the number of delay paths in the channel impulse response. Bölcskei and Paulraj [16] showed that, in general, systems using ST codes directly as SF codes can achieve only spatial diversity and are not guaranteed to achieve the full spatial and frequency diversity $LM_\mathrm{t}M_\mathrm{r}$. Later, in [17] and [18], systematic SF code design methods that could guarantee to achieve the maximum diversity were proposed.

To further improve the performance, one may consider STF coding across multiple OFDM blocks to exploit all of the available diversities in the spatial, temporal, and frequency domains. The STF coding strategy was first proposed in [19] for two transmit antennas and further developed in [20], [21], and [22] for multiple transmit antennas. Both [19] and [22] assumed that the MIMO channel stays constant over multiple OFDM blocks, and we will show later that in this case, STF coding cannot provide any additional diversity compared to the SF coding approach. In [21], an intuitive explanation on the equivalence between antennas and OFDM tones was presented from the viewpoint of channel capacity. In [20], the performance

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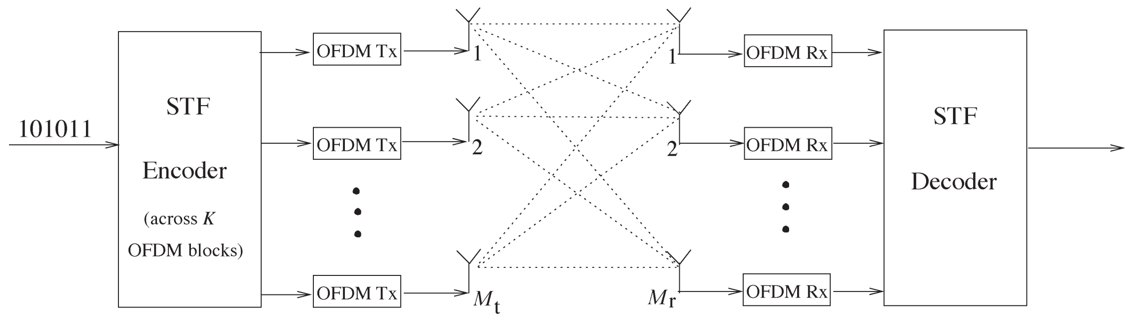

Fig. 1. STF-coded MIMO-OFDM system with M_{t} transmit and M_{r} receive antennas.

criteria for STF codes were derived, and an upper bound on the maximum achievable diversity order was established. However, there was no discussion in [20] whether the upper bound can be achieved or not, and the proposed STF codes were not guaranteed to achieve the full spatial, temporal, and frequency diversities.

In this paper, we consider the problem of performance analysis and full-diversity STF code design for MIMO-OFDM systems. We provide a general framework, taking into account coding over the spatial, temporal, and frequency domains. Our model incorporates the ST and SF coding approaches as special cases. First, we derive the performance criteria for STF-coded MIMO-OFDM systems, based on the results of [18], [23], and [24], and we show that the maximum achievable diversity order is $LM_{\mathrm{t}}M_{\mathrm{r}}T$, where T is the rank of the temporal correlation matrix of the channel. Then, we propose two STF code design methods that are guaranteed to achieve the maximum achievable diversity order. The first method is a repetition coding approach, which achieves full diversity at the price of symbol rate decrease. The advantage of this approach is that any full-diversity SF code (block or trellis) can be used to design full-diversity STF codes. The other STF code design method, a block coding approach, provides both data rate (full symbol rate) and performance (full diversity). In this case, the STF codes are constructed using existing results on signal constellation design for single-antenna fading channels.

The paper is organized as follows. In Section II, we describe the MIMO-OFDM system model with an arbitrary power delay profile. In Section III, we derive the STF code performance criteria and determine the maximum achievable diversity order. In Section IV, two STF code design methods are proposed. The simulation results are presented in Section V, and some conclusions are drawn in Section VI.

II. System Model

We consider an STF-coded MIMO-OFDM system with M_{t} transmit antennas, M_{r} receive antennas, and N subcarriers, as shown in Fig. 1. Suppose that the frequency-selective fading channels between each pair of transmit and receive antennas have L independent delay paths and the same power delay profile. The MIMO channel is assumed to be constant over each OFDM block period, but it may vary from one OFDM block to another. At the kth OFDM block, the channel impulse response from transmit antenna i to receive antenna j at time τ can be modeled as

$$h_{i,j}^{k}(\tau)=\sum_{l=0}^{L-1}\alpha_{i,j}^{k}(l)\delta(\tau-\tau_l) \tag{1}$$

where τ_l is the delay and $\alpha_{i,j}^{k}(l)$ is the complex amplitude of the lth path between transmit antenna i and receive antenna j. The $\alpha_{i,j}^{k}(l)$'s are modeled as zero-mean complex Gaussian random variables with variances $E|\alpha_{i,j}^{k}(l)|^2=\delta_l^2$, where E stands for the expectation. The powers of the L paths are normalized such that $\sum_{l=0}^{L-1}\delta_l^2=1$. We assume that the MIMO channel is spatially uncorrelated, so the channel coefficients $\alpha_{i,j}^{k}(l)$'s are independent for different indices (i,j). From (1), the frequency response of the channel is given by

$$H_{i,j}^{k}(f)=\sum_{l=0}^{L-1}\alpha_{i,j}^{k}(l)\mathrm{e}^{-\mathbf{j}2\pi f\tau_l} \tag{2}$$

where $\mathbf{j}=\sqrt{-1}$.

We consider STF coding across M_{t} transmit antennas, N OFDM subcarriers, and K consecutive OFDM blocks. Each STF codeword can be expressed as a $KN\times M_{\mathrm{t}}$ matrix

$$C=[\,C_1^{\mathrm{T}}\quad C_2^{\mathrm{T}}\quad\cdots\quad C_K^{\mathrm{T}}\,]^{\mathrm{T}} \tag{3}$$

where the channel symbol matrix C_k is given by

$$C_k=\begin{bmatrix} c_1^k(0) & c_2^k(0) & \cdots & c_{M_{\mathrm{t}}}^k(0)\\ c_1^k(1) & c_2^k(1) & \cdots & c_{M_{\mathrm{t}}}^k(1)\\ \vdots & \vdots & \ddots & \vdots\\ c_1^k(N-1) & c_2^k(N-1) & \cdots & c_{M_{\mathrm{t}}}^k(N-1)\end{bmatrix} \tag{4}$$

and $c_i^k(n)$ is the channel symbol transmitted over the nth subcarrier by transmit antenna i in the kth OFDM block. The STF code is assumed to satisfy the energy constraint $E\|C\|_{\mathrm{F}}^2=KNM_{\mathrm{t}}$, where $\|C\|_{\mathrm{F}}$ is the Frobenius norm of C. During the kth OFDM block period, the transmitter applies an N-point IFFT to each column of the matrix C_k. After appending a cyclic prefix, the OFDM symbol corresponding to the ith $(i=1,2,\ldots,M_{\mathrm{t}})$ column of C_k is transmitted by transmit antenna i.

At the receiver, after matched filtering, removing the cyclic prefix, and applying FFT, the received signal at the nth subcarrier at receive antenna j in the kth OFDM block is given by

$$y_j^k(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i^k(n) H_{i,j}^k(n) + z_j^k(n) \qquad (5)$$

where

$$H_{i,j}^k(n) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l) \mathrm{e}^{-\mathbf{j}2\pi n \Delta f \tau_l} \qquad (6)$$

is the channel frequency response at the nth subcarrier between transmit antenna i and receive antenna j, $\Delta f = 1/T$ is the subcarrier separation in the frequency domain, and T is the OFDM symbol period. We assume that the channel state information $H_{i,j}^k(n)$ is known at the receiver but not at the transmitter. In (5), $z_j^k(n)$ denotes the additive white complex Gaussian noise with zero mean and unit variance at the nth subcarrier at receive antenna j in the kth OFDM block. The factor $\sqrt{\rho/M_t}$ in (5) ensures that ρ is the average signal-to-noise ratio (SNR) at each receive antenna.

III. Performance Criteria

In this section, we derive the performance criteria for STF-coded MIMO-OFDM systems, based on the results of [18], [23], and [24], and we also determine the maximum achievable diversity order for such systems.

A. Pairwise Error Probability

Using the notation $c_i[(k-1)N+n] \triangleq c_i^k(n)$, $H_{i,j}[(k-1)N+n] \triangleq H_{i,j}^k(n)$, $y_j[(k-1)N+n] \triangleq y_j^k(n)$, and $z_j[(k-1)N+n] \triangleq z_j^k(n)$ for $1 \le k \le K$, $0 \le n \le N-1$, $1 \le i \le M_t$, and $1 \le j \le M_r$, the received signal in (5) can be expressed as

$$y_j(m) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i(m) H_{i,j}(m) + z_j(m) \qquad (7)$$

for $m = 0, 1, \ldots, KN-1$. We further rewrite the received signal in vector form as

$$\mathbf{Y} = \sqrt{\frac{\rho}{M_t}} \mathbf{DH} + \mathbf{Z} \qquad (8)$$

where $\mathbf{D}$ is a $KNM_r \times KNM_tM_r$ matrix constructed from the STF codeword C in (3) as follows:

$$\mathbf{D} = I_{M_r} \otimes [\, D_1 \quad D_2 \quad \cdots \quad D_{M_t} \,] \qquad (9)$$

where $\otimes$ denotes the tensor product, I_{M_r} is the identity matrix of size $M_r \times M_r$, and

$$D_i = \mathrm{diag}\, \{c_i(0), c_i(1), \ldots, c_i(KN-1)\} \qquad (10)$$

for any $i = 1, 2, \ldots, M_t$. The channel vector $\mathbf{H}$ of size $KNM_tM_r \times 1$ is formatted as in (11) (at the bottom of the page) where

$$H_{i,j} = [\, H_{i,j}(0) \quad H_{i,j}(1) \quad \cdots \quad H_{i,j}(KN-1) \,]^{\mathrm{T}}. \qquad (12)$$

The received signal vector $\mathbf{Y}$ of size $KNM_r \times 1$ is given by (13) (at the bottom of the page) and the noise vector $\mathbf{Z}$, which has the same form as $\mathbf{Y}$, is given by (14) (at the bottom of the page).

Suppose that $\mathbf{D}$ and $\tilde{\mathbf{D}}$ are two matrices constructed from two different codewords C and $\tilde{C}$, respectively. Then, the pairwise error probability between $\mathbf{D}$ and $\tilde{\mathbf{D}}$ can be upper bounded as [23]

$$P(\mathbf{D} \to \tilde{\mathbf{D}}) \le \binom{2r-1}{r} \left(\prod_{i=1}^{r} \gamma_i \right)^{-1} \left(\frac{\rho}{M_t} \right)^{-r} \qquad (15)$$

where r is the rank of $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^{\mathrm{H}}$, $\gamma_1, \gamma_2, \ldots, \gamma_r$ are the nonzero eigenvalues of $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^{\mathrm{H}}$, and $\mathbf{R} = E\{\mathbf{HH}^{\mathrm{H}}\}$ is the correlation matrix of $\mathbf{H}$. The superscript H stands for the complex conjugate and transpose of a matrix. Based on the upper bound on the pairwise error probability in (15), two general STF code performance criteria can be proposed as follows.

1) Diversity (rank) criterion: The minimum rank of $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^{\mathrm{H}}$ over all pairs of different codewords C and $\tilde{C}$ should be as large as possible.

$$\mathbf{H} = [\, H_{1,1}^{\mathrm{T}} \quad \cdots \quad H_{M_t,1}^{\mathrm{T}} \quad H_{1,2}^{\mathrm{T}} \quad \cdots \quad H_{M_t,2}^{\mathrm{T}} \quad \cdots \quad H_{1,M_r}^{\mathrm{T}} \quad \cdots \quad H_{M_t,M_r}^{\mathrm{T}} \,]^{\mathrm{T}} \qquad (11)$$

$$\mathbf{Y} = [\, y_1(0) \quad \cdots \quad y_1(KN-1) \quad y_2(0) \quad \cdots \quad y_2(KN-1) \quad \cdots \quad y_{M_r}(0) \quad \cdots \quad y_{M_r}(KN-1) \,]^{\mathrm{T}} \qquad (13)$$

$$\mathbf{Z} = [\, z_1(0) \quad \cdots \quad z_1(KN-1) \quad z_2(0) \quad \cdots \quad z_2(KN-1) \quad \cdots \quad z_{M_r}(0) \quad \cdots \quad z_{M_r}(KN-1) \,]^{\mathrm{T}} \qquad (14)$$

2) Product criterion: The minimum value of the product $\prod_{i=1}^{r}\gamma_i$ over all pairs of different codewords C and $\tilde{C}$ should be maximized.

B. Performance Criteria and Maximum Achievable Diversity

In case of spatially uncorrelated MIMO channels, i.e., the channel taps $\alpha_{i,j}^k(l)$ are independent for different transmit antenna index i and receive antenna index j, the correlation matrix $\mathbf{R}$ of size $KNM_\mathrm{t}M_\mathrm{r} \times KNM_\mathrm{t}M_\mathrm{r}$ becomes

$$\mathbf{R} = \mathrm{diag}\,(R_{1,1},\ldots,R_{M_\mathrm{t},1},R_{1,2},\ldots,R_{M_\mathrm{t},2},\ldots,R_{1,M_\mathrm{r}},\ldots,R_{M_\mathrm{t},M_\mathrm{r}}) \quad (16)$$

where

$$R_{i,j} = E\left\{H_{i,j}H_{i,j}^\mathrm{H}\right\} \quad (17)$$

is the correlation matrix of the channel frequency response from transmit antenna i to receive antenna j. Using the notation $w = \exp(-\mathbf{j}2\pi\Delta f)$, from (6), we have

$$H_{i,j} = (I_K \otimes W)A_{i,j} \quad (18)$$

where

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix}$$

and $A_{i,j}$ is defined at the bottom of the page. Substituting (18) into (17), $R_{i,j}$ can be calculated as follows:

$$\begin{aligned} R_{i,j} &= E\{(I_K \otimes W)A_{i,j}A_{i,j}^\mathrm{H}(I_K \otimes W)^\mathrm{H}\} \\ &= (I_K \otimes W)E\{A_{i,j}A_{i,j}^\mathrm{H}\}\,(I_K \otimes W^\mathrm{H}). \end{aligned}$$

With the assumptions that the path gains $\alpha_{i,j}^k(l)$ are independent for different paths and different pairs of transmit and receive antennas, and that the second-order statistics of the time correlation is the same for all transmit and receive antenna pairs and all paths (i.e., the correlation values do not depend on i, j, and l), we can define the time correlation at lag m as $r_\mathrm{T}(m) = E\{\alpha_{i,j}^k(l)\alpha_{i,j}^{k+m^*}(l)\}$. Thus, the correlation matrix $E\{A_{i,j}A_{i,j}^\mathrm{H}\}$ can be expressed as

$$E\left\{A_{i,j}A_{i,j}^\mathrm{H}\right\} = R_\mathrm{T} \otimes \Lambda \quad (19)$$

where $\Lambda = \mathrm{diag}\,\{\delta_0^2,\delta_1^2,\ldots,\delta_{L-1}^2\}$, and R_T is the temporal correlation matrix of size $K \times K$, whose entry in the pth row and the qth column is given by $r_\mathrm{T}(q-p)$ for $1 \le p,\, q \le K$. We can also define the frequency correlation matrix, R_F, as $R_\mathrm{F} = E\{H_{i,j}^k H_{i,j}^{k^\mathrm{H}}\}$, where

$$H_{i,j}^k = \left[H_{i,j}^k(0),\ldots,H_{i,j}^k(N-1)\right]^\mathrm{T}.$$

Then, $R_\mathrm{F} = W\Lambda W^\mathrm{H}$. As a result, we arrive at

$$\begin{aligned} R_{i,j} &= (I_K \otimes W)(R_\mathrm{T} \otimes \Lambda)(I_K \otimes W^\mathrm{H}) \\ &= R_\mathrm{T} \otimes (W\Lambda W^\mathrm{H}) = R_\mathrm{T} \otimes R_\mathrm{F} \end{aligned} \quad (20)$$

yielding

$$\mathbf{R} = I_{M_\mathrm{t}M_\mathrm{r}} \otimes (R_\mathrm{T} \otimes R_\mathrm{F}). \quad (21)$$

Finally, combining (4), (9), (10), and (21), the expression for $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^\mathrm{H}$ in (15) can be rewritten as

$$\begin{aligned} &(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^\mathrm{H} \\ &= I_{M_\mathrm{r}} \otimes \left[\sum_{i=1}^{M_\mathrm{t}}(D_i-\tilde{D}_i)(R_\mathrm{T}\otimes R_\mathrm{F})(D_i-\tilde{D}_i)^\mathrm{H}\right] \\ &= I_{M_\mathrm{r}} \otimes \left\{\left[(C-\tilde{C})(C-\tilde{C})^\mathrm{H}\right] \circ (R_\mathrm{T}\otimes R_\mathrm{F})\right\} \end{aligned} \quad (22)$$

where $\circ$ denotes the Hadamard product.[1] Denote

$$\Delta \triangleq (C-\tilde{C})(C-\tilde{C})^\mathrm{H} \quad (23)$$

and $R \triangleq R_\mathrm{T} \otimes R_\mathrm{F}$. Then, substituting (22) into (15), the pairwise error probability between C and $\tilde{C}$ can be upper bounded as

$$P(C \to \tilde{C}) \le \binom{2\nu M_\mathrm{r}-1}{\nu M_\mathrm{r}}\left(\prod_{i=1}^{\nu}\lambda_i\right)^{-M_\mathrm{r}}\left(\frac{\rho}{M_\mathrm{t}}\right)^{-\nu M_\mathrm{r}} \quad (24)$$

where ν is the rank of $\Delta \circ R$, and $\lambda_1,\lambda_2,\ldots,\lambda_\nu$ are the nonzero eigenvalues of $\Delta \circ R$. The minimum value of the product $\prod_{i=1}^{\nu}\lambda_i$ over all pairs of distinct signals C and $\tilde{C}$ is termed as coding advantage, denoted by

$$\zeta_\mathrm{STF} = \min_{C \ne \tilde{C}}\prod_{i=1}^{\nu}\lambda_i. \quad (25)$$

As a consequence, we can formulate the performance criteria for STF codes as follows.

1) Diversity (rank) criterion: The minimum rank of $\Delta \circ R$ over all pairs of distinct codewords C and $\tilde{C}$ should be as large as possible.
2) Product criterion: The coding advantage or the minimum value of the product $\prod_{i=1}^{\nu}\lambda_i$ over all pairs of distinct signals C and $\tilde{C}$ should also be maximized.

[1] Suppose that $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$ are two matrices of size $m \times n$. The Hadamard product of A and B is defined as $A \circ B = \{a_{i,j}b_{i,j}\}_{1\le i\le m,\,1\le j\le n}$.

$$A_{i,j} = [\,\alpha_{i,j}^1(0) \quad \alpha_{i,j}^1(1) \quad \cdots \quad \alpha_{i,j}^1(L-1) \quad \cdots \quad \alpha_{i,j}^K(0) \quad \alpha_{i,j}^K(1) \quad \cdots \quad \alpha_{i,j}^K(L-1)\,]^\mathrm{T}$$

If the minimum rank of $\Delta \circ R$ is ν for any pair of distinct STF codewords C and $\tilde{C}$, we say that the STF code achieves a diversity order of $\nu M_{\rm r}$. For a fixed number of OFDM blocks K, transmit antennas $M_{\rm t}$, and correlation matrices $R_{\rm T}$ and $R_{\rm F}$, the maximum achievable diversity or full diversity is defined as the maximum diversity order that can be achieved by STF codes of size $KN \times M_{\rm t}$.

According to the rank inequalities on Hadamard products and tensor products [38], we have

$$\mathrm{rank}(\Delta \circ R) \leq \mathrm{rank}(\Delta)\mathrm{rank}(R_{\rm T})\mathrm{rank}(R_{\rm F}).$$

Since the rank of Δ is at most $M_{\rm t}$ and the rank of $R_{\rm F}$ is at most L, we obtain

$$\mathrm{rank}(\Delta \circ R) \leq \min\left\{LM_{\rm t}\mathrm{rank}(R_{\rm T}), KN\right\}. \qquad (26)$$

Thus, the maximum achievable diversity is at most $\min\left\{LM_{\rm t}M_{\rm r}\mathrm{rank}(R_{\rm T}), KNM_{\rm r}\right\}$, in agreement with the results of [20]. However, there is no discussion in [20] whether this upper bound can be achieved or not. In the following sections, we show that this upper bound can indeed be achieved. We can also observe that if the channel stays constant over multiple OFDM blocks $(\mathrm{rank}(R_{\rm T}) = 1)$, the maximum achievable diversity is only $\min\{LM_{\rm t}M_{\rm r}, KNM_{\rm r}\}$. In this case, STF coding cannot provide additional diversity advantage compared to the SF coding approach.

Note that the proposed analytical framework includes ST and SF codes as special cases. If we consider only one subcarrier $(N = 1)$ and one delay path $(L = 1)$, the channel becomes a single-carrier time-correlated flat fading MIMO channel. The correlation matrix R simplifies to $R = R_{\rm T}$, and the code design problem reduces to that of ST code design, as described in [24]. In the case of coding over a single OFDM block $(K = 1)$, the correlation matrix R becomes $R = R_{\rm F}$, and the code design problem simplifies to that of SF codes, as discussed in [18].

IV. FULL-DIVERSITY STF CODE DESIGN METHODS

We propose two STF code design methods to achieve the maximum achievable diversity order $\min\left\{LM_{\rm t}M_{\rm r}\mathrm{rank}(R_{\rm T}), KNM_{\rm r}\right\}$ in this section. Without loss of generality, we assume that the number of subcarriers N is not less than $LM_{\rm t}$, so the maximum achievable diversity order is $LM_{\rm t}M_{\rm r}\mathrm{rank}(R_{\rm T})$.

A. Repetition-Coded STF Code Design

In [18], we proposed a systematic approach to design full-diversity SF codes. Suppose that $C_{\rm SF}$ is a full-diversity SF code of size $N \times M_{\rm t}$. We now construct a full-diversity STF code $C_{\rm STF}$ by repeating $C_{\rm SF} K$ times (over K OFDM blocks) as follows:

$$C_{\rm STF} = \mathbf{1}_{k\times 1} \otimes C_{\rm SF} \qquad (27)$$

where $\mathbf{1}_{k\times 1}$ is an all one matrix of size $k \times 1$. Let

$$\Delta_{\rm STF} = (C_{\rm STF} - \tilde{C}_{\rm STF})(C_{\rm STF} - \tilde{C}_{\rm STF})^{\rm H}$$

and

$$\Delta_{\rm SF} = (C_{\rm SF} - \tilde{C}_{\rm SF})(C_{\rm SF} - \tilde{C}_{\rm SF})^{\rm H}.$$

Then, we have

$$\begin{aligned}\Delta_{\rm STF} &= \left[\mathbf{1}_{k\times 1} \otimes (C_{\rm SF} - \tilde{C}_{\rm SF})\right]\left[\mathbf{1}_{1\times k} \otimes (C_{\rm SF} - \tilde{C}_{\rm SF})^{\rm H}\right] \\ &= \mathbf{1}_{k\times k} \otimes \Delta_{\rm SF}.\end{aligned}$$

Thus,

$$\begin{aligned}\Delta_{\rm STF} \circ R &= (\mathbf{1}_{k\times k} \otimes \Delta_{\rm SF}) \circ (R_{\rm T} \otimes R_{\rm F}) \\ &= R_{\rm T} \otimes (\Delta_{\rm SF} \circ R_{\rm F}).\end{aligned}$$

Since the SF code $C_{\rm SF}$ achieves full diversity in each OFDM block, the rank of $\Delta_{\rm SF} \circ R_{\rm F}$ is $LM_{\rm t}$. Therefore, the rank of $\Delta_{\rm STF} \circ R$ is $LM_{\rm t}\mathrm{rank}(R_{\rm T})$, so $C_{\rm STF}$ in (27) is guaranteed to achieve a diversity order of $LM_{\rm t}M_{\rm r}\mathrm{rank}(R_{\rm T})$.

We observe that the maximum achievable diversity depends on the rank of the temporal correlation matrix $R_{\rm T}$. If the fading channels are constant during K OFDM blocks, i.e., $\mathrm{rank}(R_{\rm T}) = 1$, the maximum achievable diversity order for STF codes (coding across several OFDM blocks) is the same as that for SF codes (coding within one OFDM block). Moreover, if the channel changes independently in time, i.e., $R_{\rm T} = I_K$, the repetition structure of STF code $C_{\rm STF}$ in (27) is sufficient, but not necessary to achieve the full diversity. In this case

$$\Delta \circ R = \mathrm{diag}\left(\Delta_1 \circ R_{\rm F}, \Delta_2 \circ R_{\rm F}, \ldots, \Delta_K \circ R_{\rm F}\right)$$

where $\Delta_k = (C_k - \tilde{C}_k)(C_k - \tilde{C}_k)^{\rm H}$ for $1 \leq k \leq K$. Thus, in this case, the necessary and sufficient condition to achieve full-diversity $KLM_{\rm t}M_{\rm r}$ is that each matrix $\Delta_k \circ R_{\rm F}$ be of rank $LM_{\rm t}$ over all pairs of distinct codewords simultaneously for all $1 \leq k \leq K$.

The proposed repetition-coded STF code design ensures full diversity at the price of symbol rate decrease by a factor of $1/K$ (over K OFDM blocks) compared to the symbol rate of the underlying SF code. The advantage of this approach is that any full-diversity SF code (block or trellis) can be used to design full-diversity STF codes.

B. Full-Rate STF Code Design

We can also design a class of STF codes that can achieve a diversity order of $\Gamma M_{\rm t}M_{\rm r}\mathrm{rank}(R_{\rm T})$ for any fixed integer Γ $(1 \leq \Gamma \leq L)$ by extending the full-rate full-diversity SF code construction method (coding over one OFDM block, i.e., the $K = 1$ case) proposed in [25].

We consider an STF code structure consisting of STF codewords C of size KN by $M_{\rm t}$

$$C = \left[\, C_1^{\rm T} \quad C_2^{\rm T} \quad \cdots \quad C_K^{\rm T} \,\right]^{\rm T} \qquad (28)$$

where

$$C_k = \left[\, G_{k,1}^{\rm T} \quad G_{k,2}^{\rm T} \quad \cdots \quad G_{k,P}^{\rm T} \quad \mathbf{0}_{N-P\Gamma M_{\rm t}}^{\rm T} \,\right]^{\rm T} \qquad (29)$$

for $k = 1, 2, \ldots, K$. In (29), $P = \lfloor N/(\Gamma M_\mathrm{t}) \rfloor$, and each matrix $G_{k,p}$ $(1 \le k \le K,\ 1 \le p \le P)$ is of size $\Gamma M_\mathrm{t} \times M_\mathrm{t}$. The zero padding in (29) is used if the number of subcarriers N is not an integer multiple of ΓM_t. For each p $(1 \le p \le P)$, we design the code matrices $G_{1,p}, G_{2,p}, \ldots, G_{K,p}$ jointly, but the design of G_{k_1,p_1} and G_{k_2,p_2}, $p_1 \neq p_2$, is independent of each other. For a fixed p $(1 \le p \le P)$, let

$$G_{k,p} = \sqrt{M_\mathrm{t}}\, \mathrm{diag}\left(X_{k,1}, X_{k,2}, \ldots, X_{k,M_\mathrm{t}}\right), \quad k = 1, 2, \ldots, K \tag{30}$$

where $\mathrm{diag}\left(X_{k,1}, X_{k,2}, \ldots, X_{k,M_\mathrm{t}}\right)$ is a block diagonal matrix, $X_{k,i} = [x_{k,(i-1)\Gamma+1} \quad x_{k,(i-1)\Gamma+2} \quad \cdots \quad x_{k,i\Gamma}]^\mathrm{T}$, $i = 1, 2, \cdots, M_\mathrm{t}$, and all $x_{k,j}$, $j = 1, 2, \cdots, \Gamma M_\mathrm{t}$, are complex symbols and will be specified later. The energy normalization condition is

$$E\left(\sum_{k=1}^{K} \sum_{j=1}^{\Gamma M_\mathrm{t}} |x_{k,j}|^2\right) = K\Gamma M_\mathrm{t}.$$

The symbol rate of the proposed scheme is $P\Gamma M_\mathrm{t}/N$, ignoring the cyclic prefix. If N is a multiple of ΓM_t, the symbol rate is 1. If not, the rate is less than 1, but since usually N is much greater than ΓM_t, the symbol rate is very close to 1. We define full rate as one channel symbol per subcarrier per OFDM block period, so the proposed method can either achieve full symbol rate, or it can perform very close to it. Note that this scheme includes the code design method proposed in [25] as a special case when $K = 1$.

The following theorem provides a sufficient condition for the STF codes described above to achieve a diversity order of $\Gamma M_\mathrm{t} M_\mathrm{r} \mathrm{rank}(R_\mathrm{T})$. For simplicity, we use the notation $\mathbf{X} = [x_{1,} \quad \cdots \quad x_{1,\Gamma M_\mathrm{t}} \quad \cdots \quad x_{K,1} \quad \cdots \quad x_{K,\Gamma M_\mathrm{t}}]$ and $\tilde{\mathbf{X}} = [\tilde{x}_{1,1} \quad \cdots \quad \tilde{x}_{1,\Gamma M_\mathrm{t}} \quad \cdots \quad \tilde{x}_{K,1} \quad \cdots \quad \tilde{x}_{K,\Gamma M_\mathrm{t}}]$. Moreover, for any $n \times n$ nonnegative definite matrix A, we denote its eigenvalues in a nonincreasing order as: $\mathrm{eig}_1(A) \ge \mathrm{eig}_2(A) \ge \cdots \ge \mathrm{eig}_n(A)$.

Theorem 1: For any STF code constructed by (28)–(30), if $\prod_{k=1}^{K} \prod_{j=1}^{\Gamma M_\mathrm{t}} |x_{k,j} - \tilde{x}_{k,j}| \neq 0$ for any pair of distinct symbols $\mathbf{X}$ and $\tilde{\mathbf{X}}$, the STF code achieves a diversity order of $\Gamma M_\mathrm{t} M_\mathrm{r} \mathrm{rank}(R_\mathrm{T})$, and the coding advantage is bounded by

$$(M_\mathrm{t}\delta_{\min})^{\Gamma M_\mathrm{t} \mathrm{rank}(R_\mathrm{T})} \Phi \le \zeta_\mathrm{STF} \le (M_\mathrm{t}\delta_{\max})^{\Gamma M_\mathrm{t} \mathrm{rank}(R_\mathrm{T})} \Phi \tag{31}$$

where

$$\delta_{\min} = \min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \min_{1 \le k \le K,\ 1 \le j \le \Gamma M_\mathrm{t}} |x_{k,j} - \tilde{x}_{k,j}|^2 \tag{32}$$

$$\delta_{\max} = \max_{\mathbf{X} \neq \tilde{\mathbf{X}}} \max_{1 \le k \le K,\ 1 \le j \le \Gamma M_\mathrm{t}} |x_{k,j} - \tilde{x}_{k,j}|^2 \tag{33}$$

$$\Phi = |\det(Q_0)|^{M_\mathrm{t} \mathrm{rank}(R_\mathrm{T})} \prod_{i=1}^{\mathrm{rank}(R_\mathrm{T})} [\mathrm{eig}_i(R_\mathrm{T})]^{\Gamma M_\mathrm{t}} \tag{34}$$

and

$$Q_0 = W_0\, \mathrm{diag}\left(\delta_0^2, \delta_1^2, \cdots, \delta_{L-1}^2\right) W_0^\mathrm{H} \tag{35}$$

$$W_0 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(\Gamma-1)\tau_0} & w^{(\Gamma-1)\tau_1} & \cdots & w^{(\Gamma-1)\tau_{L-1}} \end{bmatrix}_{\Gamma \times L}. \tag{36}$$

Furthermore, if the temporal correlation matrix R_T is of full rank, i.e., $\mathrm{rank}(R_\mathrm{T}) = K$, the coding advantage is

$$\zeta_\mathrm{STF} = \delta\, M_\mathrm{t}^{K\Gamma M_\mathrm{t}} |\det(R_\mathrm{T})|^{\Gamma M_\mathrm{t}} |\det(Q_0)|^{KM_\mathrm{t}} \tag{37}$$

where

$$\delta = \min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \prod_{k=1}^{K} \prod_{j=1}^{\Gamma M_\mathrm{t}} |x_{k,j} - \tilde{x}_{k,j}|^2. \tag{38}$$

Proof: Suppose that C and $\tilde{C}$ are two distinct STF codewords that are constructed from $G_{k,p}$ and $\tilde{G}_{k,p}$ $(1 \le k \le K,\ 1 \le p \le P)$, respectively. We would like to determine the rank of $\Delta \circ R$, where $\Delta = (C - \tilde{C})(C - \tilde{C})^\mathrm{H}$ and $R = R_\mathrm{T} \otimes R_\mathrm{F}$. For convenience, let

$$\mathbf{G}_p = [\, G_{1,p}^\mathrm{T} \quad G_{2,p}^\mathrm{T} \quad \cdots \quad G_{K,p}^\mathrm{T} \,]^\mathrm{T}$$

for each $p = 1, 2, \cdots, P$. For two distinct codewords C and $\tilde{C}$, there exists at least one index p_0 $(1 \le p_0 \le P)$ such that $\mathbf{G}_{p_0} \neq \tilde{\mathbf{G}}_{p_0}$. We may further assume that $\mathbf{G}_p = \tilde{\mathbf{G}}_p$ for any $p \neq p_0$ since the rank of $\Delta \circ R$ does not decrease if $\mathbf{G}_p \neq \tilde{\mathbf{G}}_p$ for some $p \neq p_0$ [38, Corollary 3.1.3, p. 149].

Note that the frequency correlation matrix R_F is a Toeplitz matrix. With the assumption that $\mathbf{G}_p = \tilde{\mathbf{G}}_p$ for any $p \neq p_0$, we observe that the nonzero eigenvalues of $\Delta \circ R$ are the same as those of $[(\mathbf{G}_{p_0} - \tilde{\mathbf{G}}_{p_0})(\mathbf{G}_{p_0} - \tilde{\mathbf{G}}_{p_0})^\mathrm{H}] \circ (R_\mathrm{T} \otimes Q)$, where $Q = \{q_{i,j}\}_{1 \le i,\ j \le \Gamma M_\mathrm{t}}$ is also a Toeplitz matrix whose entries are

$$q_{i,j} = \sum_{l=0}^{L-1} \delta_l^2 w^{(i-j)\tau_l}, \qquad 1 \le i, j \le \Gamma M_\mathrm{t}. \tag{39}$$

Note that Q is independent of the index p_0, i.e., it is independent of the position of $\mathbf{G}_{p_0} - \tilde{\mathbf{G}}_{p_0}$ in $C - \tilde{C}$. For any $1 \le k \le K$, we have

$$\begin{aligned} G_{k,p_0} - \tilde{G}_{k,p_0} &= \sqrt{M_\mathrm{t}}\, \mathrm{diag}\Big(X_{k,1} - \tilde{X}_{k,1}, X_{k,2} - \tilde{X}_{k,2}, \\ &\qquad \ldots, X_{k,M_\mathrm{t}} - \tilde{X}_{k,M_\mathrm{t}}\Big) \\ &= \sqrt{M_\mathrm{t}}\, \mathrm{diag}(x_{k,1} - \tilde{x}_{k,1}, \ldots, x_{k,\Gamma M_\mathrm{t}} - \tilde{x}_{k,\Gamma M_\mathrm{t}}) \\ &\quad \times (I_{M_\mathrm{t}} \otimes \mathbf{1}_{\Gamma \times 1}) \end{aligned}$$

so the difference matrix between $\mathbf{G}_{p_0}$ and $\tilde{\mathbf{G}}_{p_0}$ is defined at the bottom of the page, where

$$\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}}) \triangleq \operatorname{diag}(x_{1,1}-\tilde{x}_{1,1},\ldots,x_{1,\Gamma M_t}-\tilde{x}_{1,\Gamma M_t}, \ldots, x_{K,1}-\tilde{x}_{K,1},\ldots,x_{K,\Gamma M_t}-\tilde{x}_{K,\Gamma M_t}).$$

Thus, we have

$$\begin{aligned}
&\left[\left(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0}\right)\left(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0}\right)^{\mathrm{H}}\right]\circ(R_{\mathrm{T}}\otimes Q)\\
&= M_{\mathrm{t}}\Big\{\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left[\mathbf{1}_{K\times1}\otimes(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1})\right]\\
&\qquad\times\left[\mathbf{1}_{K\times1}\otimes(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1})\right]^{\mathrm{H}}\\
&\qquad\times\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\Big\}\circ(R_{\mathrm{T}}\otimes Q)\\
&= M_{\mathrm{t}}\Big[\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left(\mathbf{1}_{K\times K}\otimes I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times\Gamma}\right)\\
&\qquad\times\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\Big]\circ(R_{\mathrm{T}}\otimes Q)\\
&= M_{\mathrm{t}}\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left\{R_{\mathrm{T}}\otimes\left[(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times\Gamma})\circ Q\right]\right\}\\
&\quad\times\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\\
&= M_{\mathrm{t}}\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0\right)\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}
\end{aligned}\tag{40}$$

where $Q_0=\{q_{i,j}\}_{1\le i,j\le\Gamma}$ and $q_{i,j}$ is given by (39). In the above derivation, the second equality follows from the identities $[\mathbf{1}_{K\times1}\otimes(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1})]^{\mathrm{H}}=\mathbf{1}_{1\times K}\otimes I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{1\times\Gamma}$ and $(A_1\otimes B_1)(A_2\otimes B_2)(A_3\otimes B_3)=(A_1A_2A_3)\otimes(B_1B_2B_3)$ [38, p. 251], and the third equality follows from a property of the Hadamard product [38, p. 304].

From (40), we observe that if $\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})$ is of full rank, i.e., $x_{k,j}-\tilde{x}_{k,j}\neq0$ for any $1\le k\le K$ and $1\le j\le\Gamma M_{\mathrm{t}}$, then the rank of $[(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0})(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0})^{\mathrm{H}}]\circ(R_{\mathrm{T}}\otimes Q)$ can be determined as $\operatorname{rank}(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0)$, which is equal to $M_{\mathrm{t}}\operatorname{rank}(R_{\mathrm{T}})\operatorname{rank}(Q_0)$. Similar to the correlation matrix R_{F} in (20), Q_0 can be expressed as

$$Q_0=W_0\operatorname{diag}\left(\delta_0^2,\delta_1^2,\ldots,\delta_{L-1}^2\right)W_0^{\mathrm{H}}$$

where W_0 is defined in (36). Note that W_0 is a $\Gamma\times L$ matrix consisting of Γ rows of a Vandermonde matrix [38], so with $\tau_0<\tau_1<\cdots<\tau_{L-1}$, W_0 is nonsingular. Thus, Q_0 is of full rank (rank Γ). Therefore, if $\prod_{k=1}^{K}\prod_{j=1}^{\Gamma M_{\mathrm{t}}}|x_{k,j}-\tilde{x}_{k,j}|\neq0$, the rank of $\Delta\circ R$ is $\Gamma M_{\mathrm{t}}\operatorname{rank}(R_{\mathrm{T}})$.

The assumption that $\mathbf{G}_p=\tilde{\mathbf{G}}_p$ for any $p\neq p_0$ is also sufficient to calculate the coding advantage since the nonzero eigenvalues of $\Delta\circ R$ do not decrease if $\mathbf{G}_p\neq\tilde{\mathbf{G}}_p$ for some $p\neq p_0$ [38, Corollary 3.1.3, p. 149]. Using the notation $\nu_0=\Gamma M_{\mathrm{t}}\operatorname{rank}(R_{\mathrm{T}})$, the coding advantage can be calculated as

$$\begin{aligned}
\zeta_{\mathrm{STF}} &= \min_{\mathbf{X}\neq\tilde{\mathbf{X}}}\prod_{i=1}^{\nu_0}\operatorname{eig}_i\Big\{\left[\left(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0}\right)\left(\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0}\right)^{\mathrm{H}}\right]\\
&\qquad\circ(R_{\mathrm{T}}\otimes Q)\Big\}\\
&= \min_{\mathbf{X}\neq\tilde{\mathbf{X}}}\prod_{i=1}^{\nu_0}\operatorname{eig}_i\Big[M_{\mathrm{t}}\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0\right)\\
&\qquad\times\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\Big] \qquad (41)\\
&= \min_{\mathbf{X}\neq\tilde{\mathbf{X}}}\prod_{i=1}^{\nu_0}\theta_iM_{\mathrm{t}}\operatorname{eig}_i\left(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0\right) \qquad (42)
\end{aligned}$$

where $\operatorname{eig}_{K\Gamma M_{\mathrm{t}}}[\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}]\le\theta_i\le\operatorname{eig}_1[\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}]$ for $i=1,2,\ldots,\nu_0$. In the above derivation, the second equality follows by (40), and the last equality follows by Ostrowski's theorem [37, p. 224]. Since

$$\begin{aligned}
&\prod_{i=1}^{\nu_0}\operatorname{eig}_i\left(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0\right)\\
&= \prod_{i=1}^{\Gamma\operatorname{rank}(R_{\mathrm{T}})}\left[\operatorname{eig}_i\left(R_{\mathrm{T}}\otimes Q_0\right)\right]^{M_{\mathrm{t}}}\\
&= \left|\det(Q_0)\right|^{M_{\mathrm{t}}\operatorname{rank}(R_{\mathrm{T}})}\prod_{i=1}^{\operatorname{rank}(R_{\mathrm{T}})}\left[\operatorname{eig}_i(R_{\mathrm{T}})\right]^{\Gamma M_{\mathrm{t}}}
\end{aligned}$$

and

$$\begin{aligned}
&\operatorname{eig}_1\left(\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\right)\\
&\quad=\max_{\mathbf{X}\neq\tilde{\mathbf{X}}}\ \max_{1\le k\le K,\ 1\le j\le\Gamma M_{\mathrm{t}}}|x_{k,j}-\tilde{x}_{k,j}|^2\\
&\operatorname{eig}_{K\Gamma M_{\mathrm{t}}}\left(\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\right)\\
&\quad=\min_{\mathbf{X}\neq\tilde{\mathbf{X}}}\ \min_{1\le k\le K,\ 1\le j\le\Gamma M_{\mathrm{t}}}|x_{k,j}-\tilde{x}_{k,j}|^2
\end{aligned}$$

we have the lower and upper bounds in (31).

$$\begin{aligned}
\mathbf{G}_{p_0}-\tilde{\mathbf{G}}_{p_0} &= \sqrt{M_{\mathrm{t}}}\begin{bmatrix}\operatorname{diag}(x_{1,1}-\tilde{x}_{1,1},\ldots,x_{1,\Gamma M_{\mathrm{t}}}-\tilde{x}_{1,\Gamma M_{\mathrm{t}}})\left(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1}\right)\\ \operatorname{diag}(x_{2,1}-\tilde{x}_{2,1},\ldots,x_{2,\Gamma M_{\mathrm{t}}}-\tilde{x}_{2,\Gamma M_{\mathrm{t}}})\left(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1}\right)\\ \vdots\\ \operatorname{diag}(x_{K,1}-\tilde{x}_{K,1},\ldots,x_{K,\Gamma M_{\mathrm{t}}}-\tilde{x}_{K,\Gamma M_{\mathrm{t}}})\left(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1}\right)\end{bmatrix}\\
&= \sqrt{M_{\mathrm{t}}}\operatorname{diag}(\mathbf{X}-\tilde{\mathbf{X}})\left[\mathbf{1}_{K\times1}\otimes\left(I_{M_{\mathrm{t}}}\otimes\mathbf{1}_{\Gamma\times1}\right)\right]
\end{aligned}$$

Finally, if R_{T} is of full rank, $\nu_0 = K\Gamma M_{\mathrm{t}}$. From (41), the coding advantage is

$$\begin{aligned}\zeta_{\mathrm{STF}} &= \min_{\mathbf{X}\neq\tilde{\mathbf{X}}} \det\Big[M_{\mathrm{t}}\,\mathrm{diag}\,(\mathbf{X}-\tilde{\mathbf{X}})\,(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0)\\ &\qquad\qquad \times \mathrm{diag}\,(\mathbf{X}-\tilde{\mathbf{X}})^{\mathrm{H}}\Big]\\ &= M_{\mathrm{t}}^{K\Gamma M_{\mathrm{t}}} \det\,(R_{\mathrm{T}}\otimes I_{M_{\mathrm{t}}}\otimes Q_0)\\ &\qquad \times \min_{\mathbf{X}\neq\tilde{\mathbf{X}}} \prod_{k=1}^{K}\prod_{j=1}^{\Gamma M_{\mathrm{t}}} |x_{k,j}-\tilde{x}_{k,j}|^2\\ &= \delta\, M_{\mathrm{t}}^{K\Gamma M_{\mathrm{t}}} \left|\det\,(R_{\mathrm{T}})\right|^{\Gamma M_{\mathrm{t}}} \left|\det\,(Q_0)\right|^{K M_{\mathrm{t}}}\end{aligned}$$

where δ is given by (38). Thus, we have proven Theorem 1 completely. ■

From Theorem 1, we observe that with the code structure specified in (28)–(30), it is not difficult to achieve the diversity order of $\Gamma M_{\mathrm{t}} M_{\mathrm{r}}\,\mathrm{rank}(R_{\mathrm{T}})$. The remaining problem is to design a set of complex symbol vectors, $\mathbf{X} = [\,x_{1,1} \;\; \cdots \;\; x_{1,\Gamma M_{\mathrm{t}}} \;\; \cdots \;\; x_{K,1} \;\; \cdots \;\; x_{K,\Gamma M_{\mathrm{t}}}\,]$, such that the coding advantage ζ_{STF} is as large as possible. One approach is to maximize $\delta_{\min}$ and $\delta_{\max}$ in (31) according to the lower and upper bounds of the coding advantage. Another approach is to maximize δ in (38). We follow the latter for two reasons. First, the coding advantage ζ_{STF} in (37) is determined by δ in closed form although this closed form only holds with the assumption that the temporal correlation matrix R_{T} is of full rank. Second, the problem of designing $\mathbf{X}$ to maximize δ is related to the problem of constructing signal constellations for Rayleigh fading channels, which has been well solved [9]–[26]. In the literature, δ is called the minimum product distance of the set of symbols $\mathbf{X}$ [26], [27].

We summarize some existing results on designing $\mathbf{X}$ in order to maximize the minimum product distance δ as follows. For simplicity, denote $\mathcal{L} = K\Gamma M_{\mathrm{t}}$ and assume that Ω is a constellation such as quadratic-amplitude modulation (QAM), pulse-amplitude modulation, and so on. The set of complex symbol vectors is obtained by applying a transform over a $\mathcal{L}$-dimensional signal set $\Omega^{\mathcal{L}}$ [9], [28], [29]. Specifically

$$\mathbf{X} = S \cdot \frac{1}{\sqrt{\mathcal{L}}} V(\theta_1, \theta_2, \cdots, \theta_{\mathcal{L}}) \tag{43}$$

where $S = [\,s_1 \;\; s_2 \;\; \cdots \;\; s_{\mathcal{L}}\,] \in \Omega^K$ is a vector of arbitrary channel symbols to be transmitted, and $V(\theta_1, \theta_2, \ldots, \theta_{\mathcal{L}})$ is a Vandermonde matrix with variables $\theta_1, \theta_2, \ldots, \theta_{\mathcal{L}}$ [37]

$$V(\theta_1, \theta_2, \ldots, \theta_{\mathcal{L}}) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \theta_1 & \theta_2 & \cdots & \theta_{\mathcal{L}} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^{\mathcal{L}-1} & \theta_2^{\mathcal{L}-1} & \cdots & \theta_{\mathcal{L}}^{\mathcal{L}-1} \end{bmatrix}. \tag{44}$$

The optimum θ_l's, $1 \le l \le \mathcal{L}$, have been specified for different $\mathcal{L}$ and Ω. For example, if Ω is a QAM constellation and $\mathcal{L} = 2^s$ $(s \ge 1)$, the optimum θ_l's were given by [28], [29]

$$\theta_l = \mathrm{e}^{\mathbf{j}\frac{4l-3}{2\mathcal{L}}\pi}, \quad l = 1, 2, \ldots, \mathcal{L}. \tag{45}$$

In case of $\mathcal{L} = 2^s \cdot 3^t$ $(s \ge 1, t \ge 1)$, a class of θ_l's were given in [29] as

$$\theta_l = \mathrm{e}^{\mathbf{j}\frac{6l-5}{3\mathcal{L}}\pi}, \quad l = 1, 2, \ldots, \mathcal{L}. \tag{46}$$

For more details and other cases of Ω and $\mathcal{L}$, we refer the reader to [9], [28], and [29].

The STF code design discussed in this subsection achieves full symbol rate, which is much larger than that of the repetition coding approach. However, the maximum-likelihood decoding complexity of this approach is high. Its complexity increases exponentially with the number of OFDM blocks K while the decoding complexity of the repetition-coded STF codes increases only linearly with K. Fortunately, sphere decoding methods [30]–[32] can be used to reduce the complexity.

V. Simulation Results

We simulated the proposed two STF code design methods for different fading channel models and compared their performance. The OFDM modulation had $N = 128$ subcarriers, and the total bandwidth was 1 MHz. Thus, the OFDM block duration was 128 μs. We set the length of the cyclic prefix to 20 μs for all cases. We present average bit error rate (BER) curves as functions of the average SNR.

A. Performance of the Repetition-Coded STF Codes

We simulated a block code and a trellis code example. The simulated communication system had $M_{\mathrm{r}} = 1$ receive antenna. We considered a two-ray equal power delay profile ($L = 2$), with a delay of 20 μs between the two rays. Each ray was modeled as a zero-mean complex Gaussian random variable with variance 0.5.

The full-diversity STF block codes were obtained by repeating a full-diversity SF block code via (27) across $K = 1, 2, 3, 4$ OFDM blocks. The used full-diversity SF block code for $M_{\mathrm{t}} = 2$ transmit antennas was constructed from the Alamouti scheme [4] with quaternary phase-shift keying (QPSK) modulation via mapping described in [18]. The spectral efficiency of the resulting STF codes were 1, 0.5, 0.33, and 0.25 bit/s/Hz (omitting the cyclic prefix) for $K = 1, 2, 3, 4$, respectively. We simulated the full-diversity STF block code without temporal correlation (R_{T} was an identity matrix). In Fig. 2, we can see that by repeating the SF code over multiple OFDM blocks, the achieved diversity order can be increased.

The simulated full-diversity STF trellis code was obtained from a full-diversity SF trellis code via (27) with $K = 1, 2, 3, 4$, respectively. The used full-diversity SF trellis code for $M_{\mathrm{t}} = 3$ transmit antennas was constructed by applying the repetition mapping [18] to the 16-state QPSK ST trellis code proposed in [33]. Since the modulation was the same in all four cases, the spectral efficiency of the resulting STF codes were 1, 0.5, 0.33, and 0.25 bit/s/Hz (omitting the cyclic prefix) for $K = 1, 2, 3, 4$, respectively. Similar to the previous case, we assumed that the channel changes independently from OFDM block to OFDM block. The obtained BER curves can be observed in Fig. 3. As

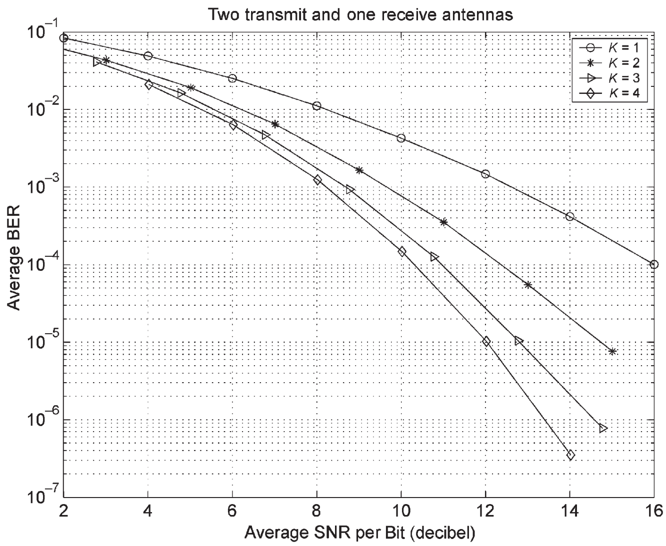

Fig. 2. Performance of the repetition block codes.

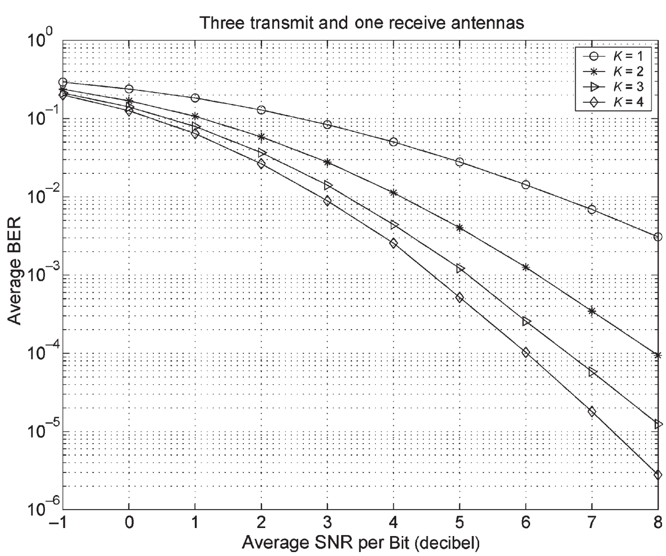

Fig. 3. Performance of the repetition trellis codes.

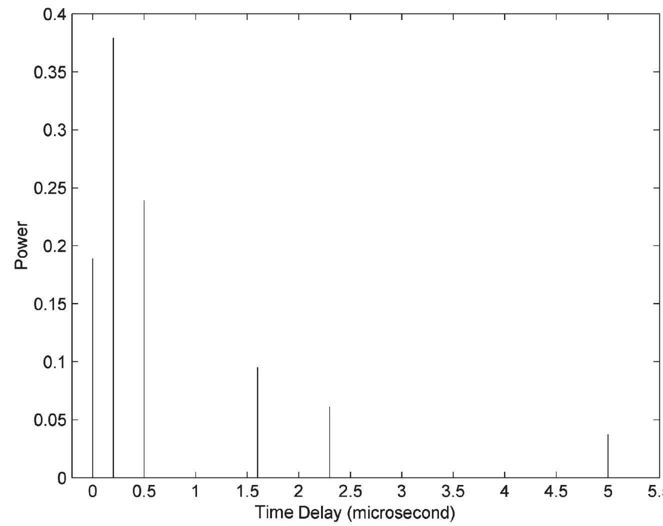

Fig. 4. Six-ray power delay profile of the TU channel model.

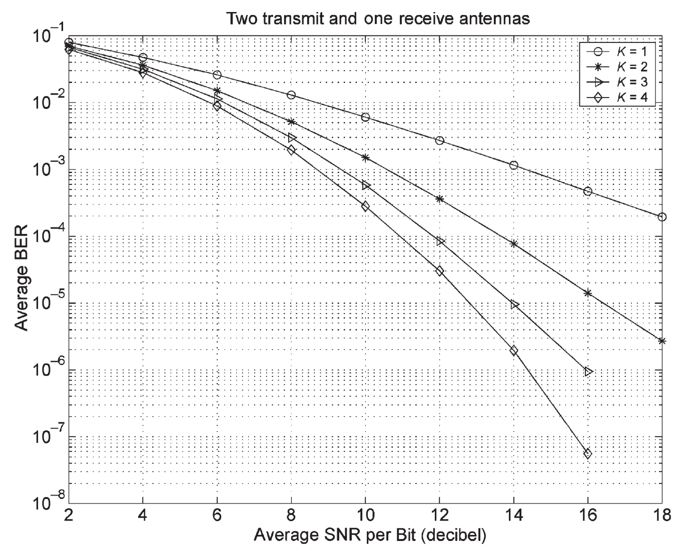

Fig. 5. Performance of the full-rate STF codes, $\varepsilon = 0$.

apparent in the figure, the STF codes $(K > 1)$ achieved higher diversity order than the SF code $(K = 1)$.

B. Performance of the Full-Rate STF Codes

In this part, we used a more realistic six-ray typical urban (TU) power delay profile [36], which is shown in Fig. 4, and simulated the fading channel with different temporal correlations. We assumed that the fading channel is constant within each OFDM block period but varies from one OFDM block period to another according to a first-order Markovian model [34], [35]

$$\alpha_{i,j}^{k}(l) = \varepsilon\, \alpha_{i,j}^{k-1}(l) + \eta_{i,j}^{k}(l), \qquad 0 \le l \le L-1 \tag{47}$$

where the constant ε $(0 \le \varepsilon \le 1)$ determines the amount of the temporal correlation and $\eta_{i,j}^{k}(l)$ is a zero-mean complex Gaussian random variable with variance $\delta_l(1-\varepsilon^2)^{1/2}$. If $\varepsilon = 0$, there is no temporal correlation (independent fading), while if $\varepsilon = 1$, the channel stays constant over multiple OFDM blocks. We considered three temporal correlation scenarios: $\varepsilon = 0$; $\varepsilon = 0.8$; and $\varepsilon = 0.95$.

The simulated full-rate STF codes were constructed by (28)–(30) for $M_{\rm t} = 2$ transmit antennas with $\Gamma = 2$. The set of complex symbol vectors $\mathbf{X}$ was obtained via (43) by applying Vandermonde transforms over a signal set Ω^{4K} for $K = 1, 2, 3, 4$. The Vandermonde transforms were determined for different K values according to (45) and (46). The constellation Ω was chosen to be binary phase-shift keying. Thus, the spectral efficiency of the resulting STF codes was 1 bit/s/Hz (omitting the cyclic prefix), which is independent of the number of jointly encoded OFDM blocks K.

The performance of the full-rate STF codes are depicted in Figs. 5–7 for the three different temporal correlation scenarios. From the figures, we observe that the diversity order of the STF

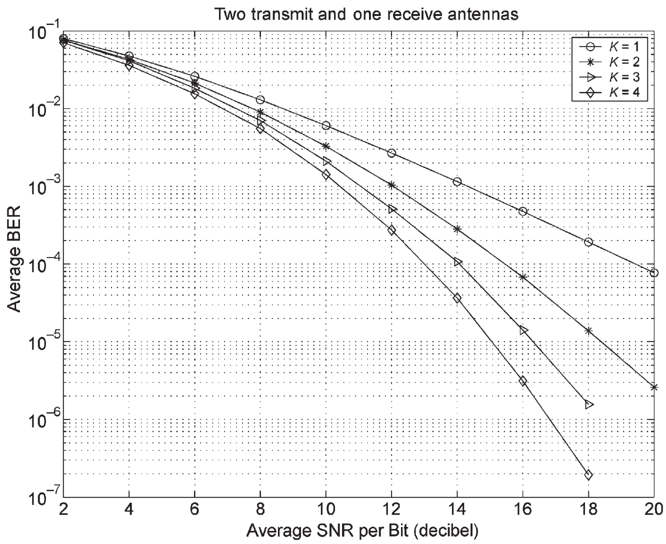

Fig. 6. Performance of the full-rate STF codes, $\varepsilon = 0.8$.

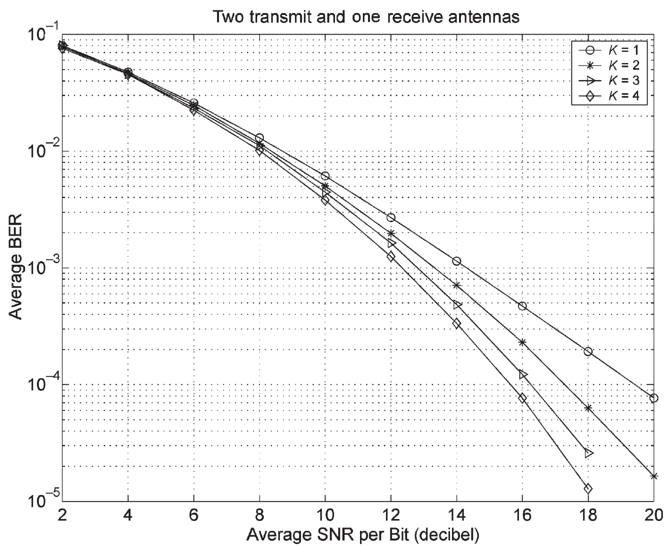

Fig. 7. Performance of the full-rate STF codes, $\varepsilon = 0.95$.

codes increases with the number of jointly encoded OFDM blocks K. However, the improvement of the diversity order depends on the temporal correlation. The performance gain obtained by coding across multiple OFDM blocks decreases as the correlation factor ε increases. For example, without temporal correlation ($\varepsilon = 0$), the STF code with $K = 4$ achieves an average BER of about 6.0×10^{-8} at an SNR of 16 dB. In case of the correlated channel model and $\varepsilon = 0.8$, the STF code with $K = 4$ has an average BER of only 3.0×10^{-6} at an SNR of 16 dB. Finally, in case of the correlated channel model and $\varepsilon = 0.95$, the STF code with $K = 4$ has an average BER of around 10^{-4} at an SNR of 16 dB.

VI. Conclusion

In narrowband MIMO wireless communications, the maximum achievable diversity order is $M_{\mathrm{t}}M_{\mathrm{r}}$ for quasi-static fading channels, while in the SF-coded broadband MIMO-OFDM systems, the maximum achievable diversity order is $LM_{\mathrm{t}}M_{\mathrm{r}}$. The factor L comes from the additional frequency diversity due to the spread fading. In this paper, we explored different channel coding approaches for MIMO-OFDM systems by taking into account all opportunities for performance improvement in both the spatial, the temporal, and the frequency domains in terms of the achievable diversity order. First, we developed a general framework for the performance analysis of STF-coded MIMO-OFDM systems, incorporating the ST and SF coding approaches as special cases. Then, we derived the code performance criteria and showed that the maximum achievable diversity order is $LM_{\mathrm{t}}M_{\mathrm{r}}T$, where T is the rank of the temporal correlation matrix of the channel. Moreover, we proposed two STF code design methods that are guaranteed to achieve the maximum achievable diversity order. The simulation results showed that by coding across multiple OFDM blocks, the diversity order of the code can be increased significantly, and the achieved improvement depends on the temporal correlation.

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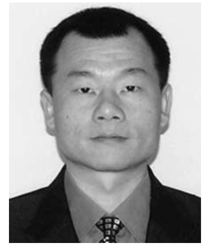

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