Differential Space-Frequency Modulation for MIMO-OFDM Systems via a "Smooth" Logical Channel

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***Abstract*— In this paper, a differential space-frequency modulation (DSFM) scheme is proposed for MIMO-OFDM systems with assumptions that the channel keeps constant only within each OFDM block, and it may change independently from one OFDM block to another. The transmitted/received signals are differentially en/decoded in the frequency dimension within each OFDM block. The performance of the proposed scheme depends on the channel power delay profile. Moreover, if the statistics of the power delay profile is known at the transmitter, we further propose to permute the channel frequency responses over subcarriers by using the Dijkstra's algorithm to increase the performance of the DSFM scheme. Simulation results show that the DSFM scheme with permutations performs very well for various channel profiles.**

I. Introduction

Multiple-input-multiple-output (MIMO) communication systems have shown great potential for the next generation wireless communications due to the capacity they provide. Among abundant space-time (ST) coding and modulation schemes, *differential space-time modulation* (DSTM) schemes [1]–[3] have been proposed for MIMO frequency-non-selective (flat) fading channels where both the transmitter and the receiver do not need to know the channel state information (CSI). With such schemes, there is only a 3 dB performance degradation compared with the case of coherent detection in which perfect CSI should be known at the receiver. The avoiding of channel estimations and the comparable performances make the DSTM schemes attractive. Many DSTM signal constellations have been designed since then, for example see [4]–[8].

In broadband wireless communications, the channel exhibits frequency selectivity due to the multiple delay paths that may introduce inter-symbol interference (ISI) at the receiver. In [9], two differential coding schemes were proposed for MIMO frequency-selective fading channels: one for single-carrier scenario with blind channel identification, and one for multi-carrier scenario with orthogonal frequency division multiplexing (OFDM). However, both schemes exploited only the spatial diversity, not the frequency diversity which is available in the frequency-selective fading channels. In [10], a differential encoding scheme was proposed, in which the frequency diversity was exploited, however, it is only for single-antenna OFDM systems. Later in [11], [12], [13], some differential schemes were proposed to achieve the maximum spatial and frequency diversities for MIMO-OFDM systems. Note that the differential schemes in [9], [13] followed the DSTM approach in [1] by taking advantage of the orthogonal ST block codes. They assumed that the fading channel stays constant over a transmission period of M_t, the number of transmit antennas considered in the systems, OFDM blocks. The differential schemes in [10], [11], [12], on the other hand, followed the DSTM schemes in [2], [3] by using the recursive differential en/decoding equations. The differential en/decoding was performed in the temporal dimension, i.e., the transmitted/received signals within a period of M_t OFDM blocks depend on those in the previous M_t OFDM blocks. These schemes also assumed that the fading channel keeps constant over M_t OFDM blocks, and furthermore assumed that the channel changes slowly from a period of M_t OFDM blocks to another. A differential scheme in the frequency dimension was also proposed in [11], in which the differential en/decoding process was performed over adjacent subcarriers. However, this scheme also depends on the assumption that the fading channel is constant over M_t OFDM blocks. In case of large number of transmit antennas, this assumption is not valid even for a typical slow fading scenario.

In this paper, we consider a problem of designing differential coding schemes for MIMO-OFDM systems with assumptions that the channel keeps constant only within each single OFDM block, and it may change independently from one OFDM block to another. First, we propose a *differential space-frequency modulation* (DSFM) scheme for MIMO-OFDM systems, in which the transmitted/received signals are differentially en/decoded in the frequency dimension within each single OFDM block. The performance of this scheme depends on the channel power delay profile. Second, if the statistics of the power delay profile is known at the transmitter, we are able to permute the channel frequency re-

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sponses over different subcarriers to create a "smooth" logical channel by using the the Dijkstra's algorithm. The proposed DSFM scheme with permutations performs very well even for a fading channel with severe variations in the spectrum of the channel impulse responses.

The rest of the paper is organized as follows. In Section II, we introduce the MIMO-OFDM system model and introduce the DSFM scheme. We use the Dijkstra's algorithm to create a "smooth" logical channel in Section III. The simulation results are presented in Section IV, and some conclusions are drawn in Section V.

II. Channel Model and DSFM Scheme

In this section, we specify the MIMO-OFDM channel model at first, and then propose the DSFM scheme within each single OFDM block.

A. Channel Model

We consider a MIMO-OFDM system with M_t transmit antennas, M_r receive antennas and N subcarriers. Both the transmitter and the receiver do not know the channel state information. The MIMO channel is assumed to be constant within each OFDM block period, and it may change independently from one OFDM block to another. Suppose that the frequency selective fading channels between different transmit and receive antenna pairs have L paths and the same power delay profile.

In the frequency domain, the received signal at the n-th $(0 \le n \le N-1)$ subcarrier at receive antenna j can be written as

$$y_j(n) = \sqrt{\rho} \sum_{i=1}^{M_t} x_i(n) H_{i,j}(n) + z_j(n), \qquad (1)$$

where $x_i(n)$ is the channel symbol transmitted over the n-th subcarrier by transmit antenna i, $H_{i,j}(n)$ is the channel frequency response at the n-th subcarrier between transmit antenna i and receive antenna j, and $z_j(n)$ denotes the additive complex Gaussian noise with zero mean and unit variance at the n-th subcarrier at receive antenna j. The channel frequency response $H_{i,j}(n)$ can be further specified as

$$H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-\mathbf{j}2\pi n \Delta f \tau_l}, \qquad (2)$$

where $\alpha_{i,j}(l)$ is the complex amplitude of the l-th path, τ_l is the delay of the l-th path, $\Delta f = 1/T$ is the subcarrier separation in the frequency domain in which T is the OFDM symbol period, and $\mathbf{j} = \sqrt{-1}$ is the imaginary unit. The $\alpha_{i,j}(l)$'s are modeled as zero-mean, complex Gaussian random variables with variances $E|\alpha_{i,j}(l)|^2 = \delta_l^2$, where E stands for the expectation. The powers of the L paths are normalized such that $\sum_{l=0}^{L-1} \delta_l^2 = 1$. The transmitted signal $x_i(n)$ is assumed to satisfy the energy constraint $E \sum_{n=0}^{N-1} \sum_{i=1}^{M_t} |x_i(n)|^2 = N$. Thus, ρ in (1) is the average transmitted power at the transmitter.

B. DSFM Scheme

We follow a full-diversity signal transmission method proposed in [14], [15], in which a class of space-frequency signals were designed to achieve a diversity order of $\Gamma M_t M_r$ for any fixed integer $\Gamma \, (1 \le \Gamma \le L)$ when a coherent detection is applied at the receiver. Specifically, denote $P = \lfloor N/(\Gamma M_t) \rfloor$, which is the largest integer not greater than $N/(\Gamma M_t)$, and denote $B_p = (p-1)\Gamma M_t$ for any $p = 1, 2, \cdots, P$. We specify the transmitted signal $x_i(n)$ in (1) as follows: i) $x_i(n) \neq 0$ for any

$$n = B_p + (i-1)\Gamma + \gamma - 1, \qquad (3)$$

in which $1 \le p \le P, 1 \le i \le M_t$ and $1 \le \gamma \le \Gamma$; and ii) $x_i(n) = 0$ for other n. It means that at each subcarrier, there is only one transmit antenna that sends non-zero symbol. Therefore, the received signal in (1) can be written as

$$y_j(n) = \sqrt{\rho}\, x_i(n) H_{i,j}(n) + z_j(n), \qquad (4)$$

for any $n = B_p + (i-1)\Gamma + \gamma - 1$ with $1 \le p \le P, 1 \le i \le M_t$ and $1 \le \gamma \le \Gamma$.

We format the received signals in (4) in a compact matrix form as follows:

$$\mathbf{Y}_p = \sqrt{\rho}\, \mathbf{X}_p \mathbf{H}_p + \mathbf{Z}_p, \quad p = 1, 2, \cdots, P, \qquad (5)$$

where $\mathbf{X}_p$ is is a $\Gamma M_t \times \Gamma M_t$ diagonal matrix as

$$\mathbf{X}_p = diag\Big(x_i(B_p + (i-1)\Gamma + \gamma - 1) : 1 \le i \le M_t, 1 \le \gamma \le \Gamma \Big). \qquad (6)$$

In (5), the received signal matrix $\mathbf{Y}_p$ is of size ΓM_t by M_r whose (k, j)-th $(1 \le k \le \Gamma M_t, 1 \le j \le M_r)$ entry is $y_j(B_p + k - 1)$, the channel matrix $\mathbf{H}_p$ also has size ΓM_t by M_r whose $((i-1)\Gamma + \gamma, j)$-th $(1 \le i \le M_t, 1 \le \gamma \le \Gamma, 1 \le j \le M_t)$ entry is $H_{i,j}(B_p + (i-1)\Gamma + \gamma - 1)$, and the noise matrix $\mathbf{Z}_p$ has the same format as $\mathbf{Y}_p$.

With the transceiver model in (5), we encode the transmitted signals differentially in the frequency dimension within each OFDM block as follows:

$$\mathbf{X}_p = V_{l_p} \mathbf{X}_{p-1}, \quad p = 2, 3, \cdots, P, \qquad (7)$$

where $\mathbf{X}_1 = I_{\Gamma M_t \times \Gamma M_t}$, and V_{l_p} carries the transmitted information and is a unitary diagonal matrix chosen from a cyclic signal constellation designed in [3]. Specifically,

$$V_{l_p} \in \left\{ V_l = diag(e^{\mathbf{j}u_1\theta_l}, e^{\mathbf{j}u_2\theta_l}, \cdots, e^{\mathbf{j}u_{\Gamma M_t}\theta_l}) : l = 0, 1, \cdots, L_0 - 1 \right\}, \quad (8)$$

where $\theta_l = 2\pi l / L_0, 0 \le l \le L_0 - 1$, and $u_1, u_2, \cdots, u_{\Gamma M_t} \in \{0, 1, \cdots, L_0 - 1\}$. The spectral efficiency of this differential scheme is $log(L_0)(P-1)/N$ bits/s/Hz, ignoring the cyclic prefix. We consider differential decoding over two received matrices $\mathbf{Y}_p$ and $\mathbf{Y}_{p-1}$ for any $p = 2, 3, \cdots, P$ as follows. Since

$$\mathbf{Y}_p = \sqrt{\rho}\,\mathbf{X}_p\mathbf{H}_p + \mathbf{Z}_p \quad (9)$$

$$= \sqrt{\rho}\,V_{l_p}\mathbf{X}_{p-1}\mathbf{H}_p + \mathbf{Z}_p, \quad (10)$$

$$\mathbf{Y}_{p-1} = \sqrt{\rho}\,\mathbf{X}_{p-1}\mathbf{H}_{p-1} + \mathbf{Z}_{p-1}, \quad (11)$$

so, we have

$$\mathbf{Y}_p = V_{l_p}\mathbf{Y}_{p-1} + \sqrt{\rho}\mathbf{X}_p\mathbf{\Delta}_p + \mathbf{Z}'_p, \quad (12)$$

where $\mathbf{\Delta}_p = \mathbf{H}_p - \mathbf{H}_{p-1}$ is the channel difference matrix between $\mathbf{H}_p$ and $\mathbf{H}_{p-1}$, and $\mathbf{Z}'_p = \mathbf{Z}_p - V_{l_p}\mathbf{Z}_{p-1}$ is a noise matrix whose each entry is an independent complex Gaussian random variable with mean zero and variance $\sqrt{2}$ since V_{l_p} is unitary. If $\mathbf{H}_p \approx \mathbf{H}_{p-1}$, or the Frobenius norm of the channel difference $||\mathbf{\Delta}_p||_F$ is small enough such that $\sqrt{\rho}||\mathbf{\Delta}_p||_F$ is much less than $||\mathbf{Z}'_p||_F$, then the maximum-likelihood (ML) decoding can be performed as

$$\hat{l_p} = \arg \min_{0 \le l_p \le L_0 - 1} ||\mathbf{Y}_p - V_{l_p}\mathbf{Y}_{p-1}||_F. \quad (13)$$

If the delay spread of the multiple paths is small with respect to the OFDM block period, the assumption of $\mathbf{H}_p \approx \mathbf{H}_{p-1}$ is valid and the differential decoding in (13) would be successful. However, if the delay spread of the multiple paths is not small, the performance of the differential decoding will degrade and it depends on the channel mismatch $\mathbf{\Delta}_p$.

In case that the delay spread of the multiple paths is large with respect to the OFDM block period, there will be severe variations in the spectrum of the channel impulse responses. It is hard to assume that the changing of the channel frequency responses is slow over two adjacent subcarriers. However, if the statistics of the power delay profile is known at the transmitter, we are able to permute the transmitted signals over different subcarriers to create a "smooth" logical channel. Denote the permuted channel as $\tilde{\mathbf{H}}_p$, and accordingly denote the received signal matrix and the noise matrix as $\tilde{\mathbf{Y}}_p$ and $\tilde{\mathbf{Z}}'_p$ respectively. Similar to (12), we have

$$\tilde{\mathbf{Y}}_p = V_{l_p}\tilde{\mathbf{Y}}_{p-1} + \sqrt{\rho}\mathbf{X}_p\tilde{\mathbf{\Delta}}_p + \tilde{\mathbf{Z}}'_p, \quad (14)$$

where $\tilde{\mathbf{\Delta}}_p = \tilde{\mathbf{H}}_p - \tilde{\mathbf{H}}_{p-1}$ and $\tilde{\mathbf{Z}}'_p = \tilde{\mathbf{Z}}_p - V_{l_p}\tilde{\mathbf{Z}}_{p-1}$. The differential ML decoding is

$$\hat{l_p} = \arg \min_{0 \le l_p \le L_0 - 1} ||\tilde{\mathbf{Y}}_p - V_{l_p}\tilde{\mathbf{Y}}_{p-1}||_F. \quad (15)$$

The permutation will be designed in the next section such that the norm of the difference between two logic adjacent subcarriers $||\tilde{\mathbf{\Delta}}_p||_F$ is as small as possible to ensure the successful differential decoding.

III. OBTAINING A "SMOOTH" LOGICAL CHANNEL BY PERMUTATIONS

Assume in this section that the statistics of the channel power delay profile is known at the transmitter. We want to obtain a "smooth" logical channel by permutations such that the differential decoding in the DSFM scheme can perform well even if the channel frequency responses may vary severely over two adjacent subcarriers. Intuitively, we try to sort the channel in such a way that any two adjacent sorted subchannels are approximately the same.

A. Average Signal to Noise Ratio

From the system performance point of view, one should optimize the permutations by maximizing the bit-error-rate (BER) performance of the ML decoding in (15). However, this approach is intractable, if not impossible. We consider an approach of maximizing the average signal-to-noise ratio (SNR) which is defined as

$$SNR_{average} = \frac{\sum_{p=2}^{P}\left(E||V_{l_p}\tilde{\mathbf{Y}}_{p-1}||_F^2\right)}{\sum_{p=2}^{P}\left(E||\sqrt{\rho}\mathbf{X}_p\tilde{\mathbf{\Delta}}_p + \tilde{\mathbf{Z}}'_p||_F^2\right)}. \quad (16)$$

Note that both V_{l_p} and $\mathbf{X}_p$ are unitary matrices. Assume that the transmitted signals, the channel realizations and the noise are independent to each other, then we have

$$E||V_{l_p}\tilde{\mathbf{Y}}_{p-1}||_F^2 = (\rho + 1)\Gamma M_t M_r,$$

and

$$E||\sqrt{\rho}\mathbf{X}_p\tilde{\mathbf{\Delta}}_p + \tilde{\mathbf{Z}}'_p||_F^2 = \rho E||\tilde{\mathbf{\Delta}}_p||_F^2 + 2\,\Gamma M_t M_r.$$

Therefore, the average SNR in (16) is

$$SNR_{average} = \frac{\rho + 1}{\rho\Phi + 2}, \quad (17)$$

in which Φ is specified as

$$\Phi = \frac{1}{(P-1)\Gamma M_t M_r}\sum_{p=2}^{P} E||\tilde{\mathbf{\Delta}}_p||_F^2. \quad (18)$$

We can see that Φ is an average channel mismatch. It depends on the power delay profile and the permutation. If adjacent two channels are approximately the same, i.e., $\mathbf{H}_p \approx \mathbf{H}_{p-1}$ for any $p = 2, 3, \cdots, P$, then the average channel mismatch Φ is zero, which means that the average SNR is $(\rho + 1)/2$. It goes to $\rho/2$ for large transmitted power ρ. This implies that there is a 3 dB performance loss of the DSFM scheme compared with the coherent detection.

In order to maximize the average SNR, we try to minimize the average channel mismatch Φ in (18). For simplicity, denote $n_{p,k} = B_p + k$ for any $2 \le p \le P$ and $0 \le k \le \Gamma M_t - 1$. We assume that with permutation, the $n_{p,k}$-th subcarrier of the original channel is permuted to the $\sigma(n_{p,k})$-th subcarrier in the logic channel. Since the frequency selective fading channels between different transmit and receive antenna pairs are assumed to have the same power delay profile, we can determine $E||\tilde{\mathbf{\Delta}}_p||_F^2$ for any $2 \le p \le P$ as follows:

$$\begin{aligned} & E||\tilde{\mathbf{\Delta}}_p||_F^2 \\ = \; & M_r \sum_{k=0}^{\Gamma M_t - 1} E\left|H_{1,1}(\sigma(n_{p,k}) - H_{1,1}(\sigma(n_{p-1,k})\right|^2 \\ = \; & M_r \sum_{k=0}^{\Gamma M_t - 1} \sum_{l=0}^{L-1} E|\alpha_{1,1}(l)|^2 \left| e^{-\mathbf{j}2\pi\sigma(n_{p,k})\tau_l/T} \right. \\ & \qquad \left. - e^{-\mathbf{j}2\pi\sigma(n_{p-1,k})\tau_l/T} \right|^2 \\ = \; & M_r \sum_{k=0}^{\Gamma M_t - 1} \sum_{l=0}^{L-1} 4\sin^2[\pi(\sigma(n_{p,k}) - \sigma(n_{p-1,k}))\tau_l/T]\delta_l^2. \end{aligned}$$

Denote

$$d(i,j) = \sum_{l=0}^{L-1} 4\sin^2[\pi(i-j)\tau_l/T]\delta_l^2, \tag{19}$$

for any pair of i and j $(1 \le i,j \le P\Gamma M_t)$. Then, the average channel mismatch Φ in (18) is

$$\Phi = \frac{1}{(P-1)\Gamma M_t} \sum_{p=2}^{P} \sum_{k=0}^{\Gamma M_t - 1} d(\sigma(n_{p,k}), \sigma(n_{p-1,k})). \tag{20}$$

B. The Dijkstra's Algorithm

We minimize the average channel mismatch Φ in (20) by using the Dijkstra's algorithm [16], [17]. Assume that we have a graph with nodes $\{1, 2, \cdots, P\Gamma M_t\}$, in which the distance between node i and node j $(1 \le i,j \le P\Gamma M_t)$ is $d(i,j)$ From (20), we can see that for any permutation, Φ is the length of a path that goes over all nodes in the graph. Thus, the problem of minimizing Φ is equivalent to the *shortest path problem* [17], i.e., to find a shortest path that goes over all nodes in the graph.

The shortest path problem is a classical network optimization problem, which has been intensively studied since 1950's (see [16], [17], and the references therein). There is the so-called *1-to-all* shortest path problem of finding the shortest path from one specific node to all other nodes in the network. A related problem is the *all-to-all* problem of finding the shortest path that goes over all nodes in the network. A well-known solution to the 1-to-all problem is the Dijkstra's algorithm [16], [17]. For the all-to-all problem, one can simply apply the Dijkstra's algorithm repeatedly by choosing each node in the network as a starting node, and then choose the minimum path according to the results from several running of the Dijkstra's algorithm. For a network with $\mathcal{N}$ nodes, the complexity of the Dijkstra's algorithm is $O(\mathcal{N}^2)$, thus the searching complexity for an all-to-all problem is $O(\mathcal{N}^3)$.

The problem of minimizing the average channel mismatch Φ in (20) is related to the all-to-all shortest path problem. First, we apply the Dijkstra's algorithm by choosing node 1 as the starting node as follows:

- *Let* $V_0 = \{1, 2, \cdots, P\Gamma M_t\}$. *Starts from node* 1 *by setting* $e_1 = 1$ *and* $V_1 = V_0 - \{1\}$.
- *For* $i = 2 \,:\, P\Gamma M_t$
  - i) *Find a node* j *in* V_{i-1} *which is nearest to node* $i-1$, *i.e., to minimize* $d(i-1, j)$*;*
  - ii) *Denote* $e_i = j$ *and* $V_i = V_{i-1} - \{j\}$*;*

  end;

With the above algorithm, we obtain a shortest path that starts at node 1 and goes over all other nodes in the graph. Then, we repeat the Dijkstra's algorithm by choosing another node as a starting node. Finally with comparison, we are able to get a shortest path, denoted as $e_1 \to e_2 \to \cdots \to e_{P\Gamma M_t}$. Therefore, the permutation $\sigma(i) = e_i$, $i = 1, 2, \cdots, P\Gamma M_t$ is the desired permutation that minimizes Φ in (20).

IV. SIMULATION RESULTS

We simulated the proposed DSFM scheme for a system with $M_t = 2$ transmit and $M_r = 1$ receive antennas. The OFDM modulation had $N = 128$ subcarriers, and the total bandwidth was 1 MHz. Thus, the OFDM block duration was $T = 128\mu s$ without the cyclic prefix. We considered a two-ray, power delay profile ($L = 2$), with a delay of τ μs between the two rays. Each ray was modeled as a zero mean complex Gaussian random variable with variance 0.5. We simulated two cases: i) $\tau = 5\mu s$ and ii) $\tau = 20\mu s$, and set the length of the cyclic prefix to $20\mu s$ for both cases. We present average bit error rate (BER) curves as functions of the average energy per bit E_b/N_0.

The performance of the proposed DSFM scheme is shown in Figure 1 for the case of $\tau = 5\mu s$. Since the $5\mu s$ separation of the two paths is small compared with the $128\mu s$ duration of the OFDM block, the channel frequency responses change smooth over different subcarriers. We can see that the DSFM scheme performs successfully in this case, and the performance of the scheme with permutation is a little better than that of the scheme without permutation. With permutation, the performance of the DSFM scheme is about 3 dB away

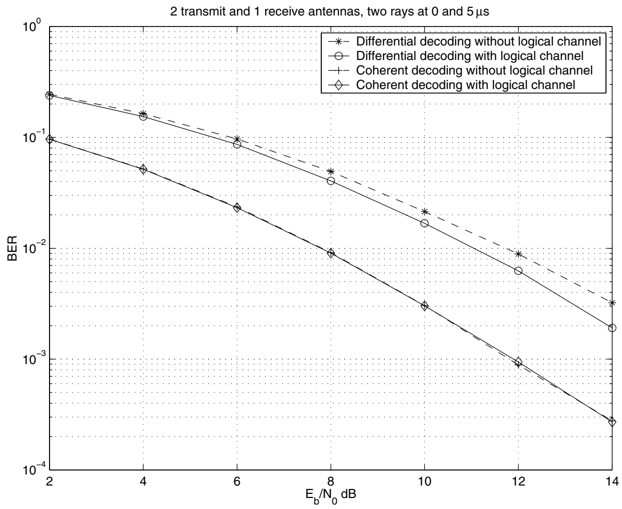

Fig. 1. Performances of the DSFM scheme in case of $\tau = 5\mu s$

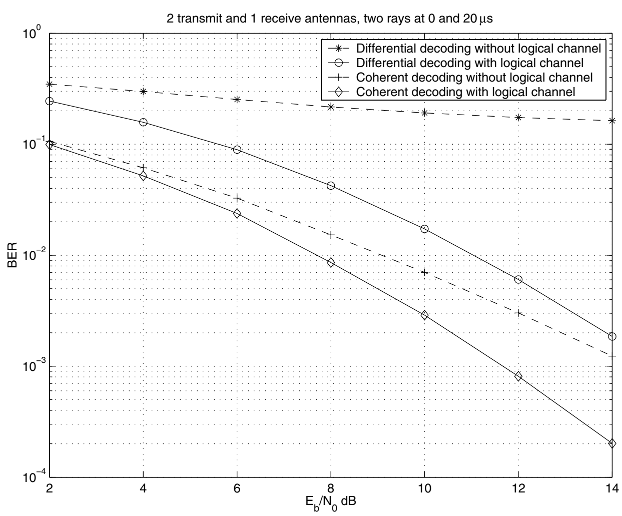

Fig. 2. Performances of the DSFM scheme in case of $\tau = 20\mu s$

from that of the coherent detection. Note that with or without permutation, the performance of the coherent detection is almost the same.

Figure 2 depicts the performance for the case of $\tau = 20\mu s$. In this case, there are severe variations in the spectrum of the channel impulse responses, resulting the failure of the DSFM scheme without channel permutation. However, if we permute the channel over different subcarriers properly, we are able to obtain a "smooth" logical channel that can guarantee the successful differential decoding as shown in the solid curve with circles. With permutation, the performance of the DSFM scheme is about 3 dB away from that of the coherent detection, which is not observed in case without channel permutation.

V. Conclusion

In this paper, a differential modulation scheme was proposed for MIMO-OFDM systems. The transmitted/received signals were differentially en/decoded in the frequency dimension within each OFDM block. Thus, the proposed DSFM scheme can be applied to the scenarios that the fading channel may changes from one OFDM block to another independently. The performance of the proposed scheme depends on the power delay profile. Moreover, if the statistics of the channel power delay profile is known at the transmitter, we further proposed to permute the channel frequency responses over subcarriers to create a "smooth" logical channel by using the the Dijkstra's algorithm. Simulation results showed that the performance of the DSFM scheme with permutations is almost 3 dB away from that of the coherent detection, which validates the theoretical analysis.

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