Diversity Analysis of Space-Time-Frequency Coded Broadband OFDM Systems

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***Abstract*: Multi-antenna communication systems with OFDM modulation have the potential to play an important role in the design of the next generation broadband wireless communication systems. In this paper, we propose a general framework for the performance analysis of space-time-frequency coded MIMO-OFDM systems. Our approach incorporates the space-time and space-frequency coding approaches as special cases. We also determine the maximum achievable diversity order of space-time-frequency codes, and a simple repetition coding approach is used to achieve full diversity. Moreover, we investigate the effect of temporal and frequency-domain correlation on the performance of MIMO-OFDM systems. The simulation results demonstrate that in some cases, considerable performance improvement can be achieved via space-time-frequency coding compared to the space-frequency coding approach.**

1. Introduction

Recent research results have shown that the adverse effects of the wireless propagation environment can be significantly reduced by employing multiple transmit and receive antennas, resulting in multiple-input-multiple-output (MIMO) communication systems. Combining MIMO systems with OFDM modulation, MIMO-OFDM systems have been proposed, and two coding approaches have been suggested for such systems: *space-frequency* (SF) coding, to exploit the spatial and frequency diversities, and *space-time-frequency* (STF) coding, to exploit the spatial, temporal, and frequency diversities available in frequency selective MIMO channels.

The first SF coding scheme was proposed in [2], in which previously existing *space-time* (ST) codes were used by replacing the time domain with frequency domain (OFDM tones). Later works [3]–[6] also described similar schemes, i.e. using ST codes directly as SF codes. The resulting SF codes achieved only the spatial diversity, and were not guaranteed to achieve the full spatial and frequency diversities. Later in [8] and [17],[18] systematic SF code design methods were proposed that can guarantee to achieve full diversity.

The STF coding strategy, by coding across multiple OFDM blocks, was first proposed in [12] for two transmit antennas and further developed in [10], [11], and [13] for multiple transmit antennas. Both [12] and [13] assumed that the MIMO channel stays constant over multiple OFDM blocks. In [11], an intuitive explanation on the equivalence between antennas and OFDM tones was presented from a capacity point of view. In [10], the performance criteria for STF codes were derived, and an upper bound on the maximum achievable diversity order was established. However, there was no discussion in [10] whether the upper bound can be achieved or not.

In this paper, we provide a general framework for the performance analysis of MIMO-OFDM systems, taking into account coding over the spatial, temporal and frequency domains. Our model incorporates the ST and SF coding approaches as special cases. We also derive an alternative form of the performance criteria for STF-coded MIMO-OFDM systems, based on the results of [14, 15]. We determine the maximum achievable diversity order for STF codes, and demonstrate that a simple repetition coding approach can be used to to achieve it. Finally, we investigate the effect of temporal and frequency-domain correlation on the performance of MIMO-OFDM systems and show that if the channel changes independently from OFDM block to OFDM block, significant performance improvement can be achieved by STF coding compared to the SF coding approach.

2. System Model

We consider a STF-coded MIMO-OFDM system with M_t transmit antennas, M_r receive antennas and N subcarriers. Suppose that the frequency selective fading channels between each pair of transmit and receive antennas have L independent delay paths and the same power delay profile. The MIMO channel is assumed to be constant over each OFDM block period, but it may vary from one OFDM block to another. At the k-th OFDM block, the channel impulse response from transmit antenna i to receive antenna j at time τ can be modeled as

$$h_{i,j}^k(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l)\delta(\tau - \tau_l), \tag{1}$$

where τ_l is the delay of the l-th path, and $\alpha_{i,j}^k(l)$ is the complex amplitude of the l-th path between transmit antenna i and receive antenna j at the k-th OFDM block. The $\alpha_{i,j}^k(l)$'s are modeled as zero-mean, complex Gaussian random variables with variances $E|\alpha_{i,j}^k(l)|^2 = \delta_l^2$, where E stands for the expectation. The powers of the L paths are normalized such that $\sum_{l=0}^{L-1} \delta_l^2 = 1$. From (1), the frequency response of the channel is given by

$$H_{i,j}^k(f) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l) e^{-\mathbf{j}2\pi f \tau_l}, \tag{2}$$

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where $\mathbf{j} = \sqrt{-1}$. We assume that the MIMO channel is spatially uncorrelated, i.e. the channel taps $\alpha_{i,j}^k(l)$ are independent for different indices (i, j).

We consider STF coding across M_t transmit antennas, N OFDM sub-carriers and K OFDM blocks. Each STF codeword can be expressed as a $KN \times M_t$ matrix

$$C = [C_1^{\mathcal{T}} \ C_2^{\mathcal{T}} \ \cdots \ C_K^{\mathcal{T}}]^{\mathcal{T}}, \tag{3}$$

where

$$C_k = \begin{bmatrix} c_1^k(0) & c_2^k(0) & \cdots & c_{M_t}^k(0) \\ c_1^k(1) & c_2^k(1) & \cdots & c_{M_t}^k(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^k(N-1) & c_2^k(N-1) & \cdots & c_{M_t}^k(N-1) \end{bmatrix} \tag{4}$$

is the channel symbol matrix transmitted in the k-th OFDM block, and $c_i^k(n)$ is the channel symbol transmitted over the n-th sub-carrier by transmit antenna i at the k-th OFDM block. The STF code is assumed to satisfy the energy constraint $E||C||_F^2 = KNM_t$, where $||C||_F$ is the Frobenius norm of C. At the k-th OFDM block, the OFDM transmitter applies an N-point IFFT to each column of the matrix C_k. After appending a cyclic prefix, the OFDM symbol corresponding to the i-th $(i = 1, 2, \cdots, M_t)$ column of C_k is transmitted by transmit antenna i.

At the receiver, after matched filtering, removing the cyclic prefix, and applying FFT, the received signal at the n-th sub-carrier at receive antenna j in the k-th OFDM block is given by

$$y_j^k(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i^k(n) H_{i,j}^k(n) + z_j^k(n), \tag{5}$$

where

$$H_{i,j}^k(n) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l) e^{-\mathbf{j}2\pi n \Delta f \tau_l} \tag{6}$$

is the channel frequency response at the n-th sub-carrier between transmit antenna i and receive antenna j, $\Delta f = 1/T$ is the sub-carrier separation in the frequency domain, and T is the OFDM symbol period. We assume that the channel state information $H_{i,j}^k(n)$ is known at the receiver, but not at the transmitter. In (5), $z_j^k(n)$ denotes the additive complex Gaussian noise with zero mean and unit variance at the n-th sub-carrier at receive antenna j in the k-th OFDM block. The factor $\sqrt{\rho/M_t}$ in (5) ensures that ρ is the average signal to noise ratio (SNR) at each receive antenna, and it is independent of the number of transmit antennas.

3. Performance Criteria

In this section, we derive the performance criteria for STF-coded MIMO-OFDM systems, based on the results of [14, 15, 16]. This formulation will serve as the starting point for the results to be discussed later.

Using the notation $c_i((k-1)N + n) \triangleq c_i^k(n)$, $H_{i,j}((k-1)N + n) \triangleq H_{i,j}^k(n)$, $y_j((k-1)N + n) \triangleq y_j^k(n)$, and $z_j((k-1)N + n) \triangleq z_j^k(n)$ for $1 \le k \le K, 0 \le n \le N-1, 1 \le i \le M_t$ and $1 \le j \le M_r$, the received signal in (5) can be expressed as

$$y_j(m) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i(m) H_{i,j}(m) + z_j(m) \tag{7}$$

for $m = 0, 1, \cdots, KN-1$. We further rewrite the received signal in vector form as

$$\mathbf{Y} = \sqrt{\frac{\rho}{M_t}} \mathbf{DH} + \mathbf{Z}, \tag{8}$$

where $\mathbf{D}$ is a $KNM_r \times KNM_tM_r$ matrix constructed from the STF codeword C in (3) as follows:

$$\mathbf{D} = I_{M_r} \otimes [D_1 \ D_2 \ \cdots \ D_{M_t}], \tag{9}$$

where $\otimes$ denotes the tensor product, I_{M_r} is the identity matrix of size $M_r \times M_r$, and

$$D_i = diag\{c_i(0), \, c_i(1), \cdots, \, c_i(KN-1)\} \tag{10}$$

for any $i = 1, 2, \cdots, M_t$. Each D_i in (10) is related to the i-th column of the STF codeword C. The channel vector $\mathbf{H}$ of size $KNM_tM_r \times 1$ is formatted as

$$\mathbf{H} = [H_{1,1}^{\mathcal{T}} \ \cdots \ H_{M_t,1}^{\mathcal{T}} \ \ H_{1,2}^{\mathcal{T}} \ \cdots \ H_{M_t,2}^{\mathcal{T}} \ \cdots \\ \cdots H_{1,M_r}^{\mathcal{T}} \ \cdots \ H_{M_t,M_r}^{\mathcal{T}}]^{\mathcal{T}}, \tag{11}$$

where

$$H_{i,j} = [H_{i,j}(0) \ H_{i,j}(1) \cdots H_{i,j}(KN-1)]^{\mathcal{T}}. \tag{12}$$

The received signal vector $\mathbf{Y}$ of size $KNM_r \times 1$ is given by

$$\mathbf{Y} = [y_1(0) \cdots y_1(KN-1) \ \ y_2(0) \cdots \\ \cdots y_{M_r}(0) \cdots y_{M_r}(KN-1)]^{\mathcal{T}}, \tag{13}$$

and the noise vector $\mathbf{Z}$ has the same form as $\mathbf{Y}$, i.e.,

$$\mathbf{Z} = [z_1(0) \cdots z_1(KN-1) \ \ z_2(0) \cdots \\ \cdots z_{M_r}(0) \cdots z_{M_r}(KN-1)]^{\mathcal{T}}. \tag{14}$$

Suppose that $\mathbf{D}$ and $\tilde{\mathbf{D}}$ are two different matrices related to two different STF codewords C and $\tilde{C}$, respectively. Then, the pairwise error probability between $\mathbf{D}$ and $\tilde{\mathbf{D}}$ can be upper bounded as [14, 15]

$$P(\mathbf{D} \to \tilde{\mathbf{D}}) \le \binom{2r-1}{r} \left(\prod_{i=1}^{r} \gamma_i\right)^{-1} \left(\frac{\rho}{M_t}\right)^{-r}, \tag{15}$$

where r is the rank of $(\mathbf{D} - \tilde{\mathbf{D}})\mathbf{R}(\mathbf{D} - \tilde{\mathbf{D}})^{\mathcal{H}}$, $\gamma_1, \gamma_2, \cdots, \gamma_r$ are the non-zero eigenvalues of $(\mathbf{D} - \tilde{\mathbf{D}})\mathbf{R}(\mathbf{D} - \tilde{\mathbf{D}})^{\mathcal{H}}$, and $\mathbf{R} = E\{\mathbf{HH}^{\mathcal{H}}\}$ is the correlation matrix of $\mathbf{H}$. The superscript $\mathcal{H}$ stands for the complex conjugate and transpose of a matrix. Based on the upper bound on the pairwise error probability in (15), two general STF code performance criteria can be proposed as follows:

- *Diversity (rank) criterion:* The minimum rank of $(\mathbf{D} - \tilde{\mathbf{D}})\mathbf{R}(\mathbf{D} - \tilde{\mathbf{D}})^{\mathcal{H}}$ over all pairs of different codewords C and $\tilde{C}$ should be as large as possible.

- *Product criterion:* The minimum value of the product $\prod_{i=1}^{r} \gamma_i$ over all pairs of different codewords C and $\tilde{C}$ should be maximized.

4. Maximum Achievable Diversity

In case of spatially uncorrelated MIMO channels, i.e., the channel taps $\alpha_{i,j}^k(l)$ are independent for different transmit antenna i and receive antenna j, the correlation matrix $\mathbf{R}$ of size $KNM_tM_r \times KNM_tM_r$ becomes

$$\mathbf{R} = diag\Big(R_{1,1}, \cdots, R_{M_t,1},\ R_{1,2}, \cdots \cdots, R_{M_t,2}, \cdots, R_{1,M_r}, \cdots, R_{M_t,M_r}\Big), \quad (16)$$

where

$$R_{i,j} = E\left\{H_{i,j}H_{i,j}^{\mathcal{H}}\right\} \quad (17)$$

is the correlation matrix of the channel frequency response from transmit antenna i to receive antenna j. Using the notation $w = e^{-\mathrm{j}2\pi\Delta f}$, from (6) and (12), we have

$$H_{i,j} = (I_K \otimes W)A_{i,j}, \quad (18)$$

where

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix},$$

and

$$A_{i,j} = [\alpha_{i,j}^1(0)\ \alpha_{i,j}^1(1)\ \cdots\ \alpha_{i,j}^1(L-1)\ \cdots \cdots \alpha_{i,j}^K(0)\ \alpha_{i,j}^K(1)\ \cdots\ \alpha_{i,j}^K(L-1)]^{\mathcal{T}}.$$

Substituting (18) into (17), $R_{i,j}$ can be calculated as follows:

$$\begin{aligned} R_{i,j} &= E\left\{(I_K \otimes W)A_{i,j}A_{i,j}^{\mathcal{H}}(I_K \otimes W)^{\mathcal{H}}\right\} \\ &= (I_K \otimes W)E\left\{A_{i,j}A_{i,j}^{\mathcal{H}}\right\}(I_K \otimes W^{\mathcal{H}}). \end{aligned}$$

With the assumptions that the path gains $\alpha_{i,j}^k(l)$ are independent for different paths and different pairs of transmit and receive antennas, and that the second order statistics of the time correlation is the same for all transmit and receive antenna pairs and all paths (i.e. the correlation values do not depend on i, j and l), we can define the time correlation at lag m as

$$r_T(m) = E\{\alpha_{i,j}^k(l)\alpha_{i,j}^{k+m^*}(l)\}. \quad (19)$$

Thus, the correlation matrix $E\left\{A_{i,j}A_{i,j}^{\mathcal{H}}\right\}$ can be expressed as

$$E\left\{A_{i,j}A_{i,j}^{\mathcal{H}}\right\} = R_T \otimes \Lambda, \quad (20)$$

where $\Lambda = diag\{\delta_0^2, \delta_1^2, \cdots, \delta_{L-1}^2\}$, and R_T is the temporal correlation matrix of size $K \times K$, whose entry in the p-th row and the q-th column is given by $r_T(q-p)$ for $1 \leq p, q \leq K$. We can also define the frequency correlation matrix, R_F, as

$$R_F = E\{H_{i,j}^k H_{i,j}^{k\ \mathcal{H}}\}, \quad (21)$$

where $H_{i,j}^k = [H_{i,j}^k(0), ..., H_{i,j}^k(N-1)]^{\mathcal{T}}$. Then, $R_F = W\Lambda W^{\mathcal{H}}$. As a result, we arrive at

$$\begin{aligned} R_{i,j} &= (I_K \otimes W)(R_T \otimes \Lambda)(I_K \otimes W^{\mathcal{H}}) \\ &= R_T \otimes (W\Lambda W^{\mathcal{H}}) = R_T \otimes R_F, \end{aligned} \quad (22)$$

yielding

$$\mathbf{R} = I_{M_tM_r} \otimes (R_T \otimes R_F). \quad (23)$$

Finally, combining (4), (9), (10) and (23), the expression for $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^{\mathcal{H}}$ in (15) can be rewritten as

$$\begin{aligned} &(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^{\mathcal{H}} \\ &= I_{M_r} \otimes \left[\sum_{i=1}^{M_t}(D_i - \tilde{D}_i)(R_T \otimes R_F)(D_i - \tilde{D}_i)^{\mathcal{H}}\right] \\ &= I_{M_r} \otimes \left\{\left[(C-\tilde{C})(C-\tilde{C})^{\mathcal{H}}\right] \circ (R_T \otimes R_F)\right\}, \end{aligned} \quad (24)$$

where $\circ$ denotes the Hadamard product[1]. Denote $\Delta \triangleq (C-\tilde{C})(C-\tilde{C})^{\mathcal{H}}$, and $R \triangleq R_T \otimes R_F$. Then, substituting (24) into (15), the pairwise error probability between C and $\tilde{C}$ can be upper bounded as

$$P(C \to \tilde{C}) \leq \binom{2\nu M_r - 1}{\nu M_r}\left(\prod_{i=1}^{\nu}\lambda_i\right)^{-M_r}\left(\frac{\rho}{M_t}\right)^{-\nu M_r}, \quad (25)$$

where ν is the rank of $\Delta \circ R$, and $\lambda_1, \lambda_2, \cdots, \lambda_\nu$ are the non-zero eigenvalues of $\Delta \circ R$. As a consequence, we can formulate the performance criteria for STF codes as follows:

- *Diversity (rank) criterion:* The minimum rank of $\Delta \circ R$ over all pairs of distinct codewords C and $\tilde{C}$ should be as large as possible.
- *Product criterion:* The minimum value of the product $\prod_{i=1}^{\nu}\lambda_i$ over all pairs of distinct signals C and $\tilde{C}$ should also be maximized.

If the minimum rank of $\Delta \circ R$ is ν for any pair of distinct STF codewords C and $\tilde{C}$, we say that the STF code achieves a *diversity order* of νM_r. For a fixed the number of OFDM blocks K, the number of transmit antennas M_t, and the correlation matrices R_T and R_F, the *maximum achievable diversity* or *full diversity* is defined as the maximum diversity order that can be achieved by STF codes of size $KN \times M_t$.

According to the rank inequalities on Hadamard products and tensor products [20], we have

$$rank(\Delta \circ R) \leq rank(\Delta)rank(R_T)rank(R_F).$$

Since the rank of Δ is at most M_t, the rank of R_F is at most L, and the rank of $\Delta \circ R$ is at most KN, we obtain

$$rank(\Delta \circ R) \leq \min\{LM_t rank(R_T), KN\}. \quad (26)$$

[1]Suppose that $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$ are two matrices of size $m \times n$. The *Hadamard product* of A and B is defined as

$$A \circ B = \begin{bmatrix} a_{1,1}b_{1,1} & \cdots & a_{1,n}b_{1,n} \\ \cdots & \cdots & \cdots \\ a_{m,1}b_{m,1} & \cdots & a_{m,n}b_{m,n} \end{bmatrix}.$$

Thus, the maximum achievable diversity is at most $\min\{LM_tM_r rank(R_T), KNM_r\}$, in agreement with the results of [10]. However, there is no discussion in [10] whether this upper bound can be achieved or not. In the rest of the section, we show that this upper bound can indeed be achieved. Without loss of generality, we assume that the number of sub-carriers, N, is not less than LM_t, so our objective is to show that the maximum achievable diversity order is $LM_tM_r rank(R_T)$.

In [17], [18] we have proposed a systematic approach to design full-diversity SF codes. Suppose that C_{SF} is a full diversity SF code of size $N \times M_t$. We now construct a STF code by repeating C_{SF} K times (over K OFDM blocks) as follows:

$$C_{STF} = \mathbf{1}_{k\times 1} \otimes C_{SF}, \qquad (27)$$

where $\mathbf{1}_{k\times 1}$ is an all one matrix of size $k \times 1$. Let $\Delta_{STF} = (C_{STF} - \tilde{C}_{STF})(C_{STF} - \tilde{C}_{STF})^{\mathcal{H}}$ and $\Delta_{SF} = (C_{SF} - \tilde{C}_{SF})(C_{SF} - \tilde{C}_{SF})^{\mathcal{H}}$. Then we have

$$\Delta_{STF} = \Big[\mathbf{1}_{k\times 1} \otimes (C_{SF} - \tilde{C}_{SF})\Big] \times \Big[\mathbf{1}_{1\times k} \otimes (C_{SF} - \tilde{C}_{SF})^{\mathcal{H}}\Big] = \mathbf{1}_{k\times k} \otimes \Delta_{SF}.$$

Thus,

$$\begin{aligned} \Delta_{STF} \circ R &= (\mathbf{1}_{k\times k} \otimes \Delta_{SF}) \circ (R_T \otimes R_F) \\ &= R_T \otimes (\Delta_{SF} \circ R_F). \end{aligned}$$

Since the SF code C_{SF} achieves full diversity in each OFDM block, the rank of $\Delta_{SF} \circ R_F$ is LM_t. Therefore, the rank of $\Delta_{STF} \circ R$ is $LM_t rank(R_T)$, so C_{STF} in (27) is guaranteed to achieve a diversity order of $LM_tM_r rank(R_T)$.

We observe that the maximum achievable diversity depends on the rank of the temporal correlation matrix R_T. If the fading channels are constant during K OFDM blocks, i.e. $rank(R_T) = 1$, the maximum achievable diversity order for STF codes (coding across several OFDM blocks) is the same as that for SF codes (coding within one OFDM block). Moreover, if the channel changes independently in time, i.e. $R_T = I_K$, the repetition structure of STF code C_{STF} in (27) is sufficient, but not necessary to achieve the full diversity. In this case,

$$\Delta \circ R = diag(\Delta_1 \circ R_F,\ \Delta_2 \circ R_F,\ \cdots,\ \Delta_K \circ R_F),$$

where $\Delta_k = (C_k - \tilde{C}_k)(C_k - \tilde{C}_k)^{\mathcal{H}}$ for $1 \le k \le K$. Thus, the necessary and sufficient condition to achieve full diversity KLM_tM_r is to make $\Delta_k \circ R_F$ of rank LM_t over all pairs of distinct codewords simultaneously for all $1 \le k \le K$.

Note that the proposed analytical framework includes ST and SF codes as special cases. If we consider only one sub-carrier ($N = 1$), and one delay path ($L = 1$), the channel becomes a single-carrier, time-correlated, flat fading MIMO channel. The correlation matrix R simplifies to $R = R_T$, and the code design problem reduces to that of ST code design, as described in [16]. In the case of coding over a single OFDM block ($K = 1$), the correlation matrix R becomes $R = R_F$, and the code design problem simplifies to that of SF codes, as discussed in [17],[18] .

5. Simulation Results

To illustrate the above theoretical results, we have done some computer simulations. The simulated communication system had $M_r = 1$ receive antenna. The OFDM modulation had $N = 128$ sub-carriers, and the total bandwidth was 1 MHz. Thus, the OFDM block duration was $128\mu s$ without the cyclic prefix. We considered a two-ray, equal power delay profile ($L = 2$), with a delay of $20\mu s$ between the two rays. Each ray was modeled as a zero mean complex Gaussian random variable with variance 0.5. We have simulated a block code and a trellis code example.

The full-diversity SF block code for $M_t = 2$ transmit antennas was constructed from the Alamouti scheme [1] with BPSK modulation via the repetition mapping described in [17],[18]. The full-diversity STF block code was obtained from a full-diversity SF code via (27) across two OFDM blocks ($K = 2$) with QPSK modulation. Thus, both schemes had the same spectral efficiency of 0.5 bit/s/Hz (omitting the cyclic prefix for simplicity). We simulated the full-diversity STF block code for the independently fading channel model ($R_T = I_2$) and compared it with the SF block code. Figure 1 depicts the obtained average bit error rate (BER) curves as a function of the average signal to noise ratio. From the figure, we can see that in case of independent fading, the STF code achieves higher diversity order than the SF code.

In case of the full-diversity SF trellis code, we applied the repetition mapping to the 3-antenna, 16-state, QPSK ST trellis code proposed in [19]. The full-diversity STF trellis code was obtained from this SF code via (27) with $K = 2$. Since the modulation was the same in both cases, the spectral efficiencies of the SF code and the STF code were 1 bit/s/Hz and 0.5 bit/s/Hz, respectively, omitting the cyclic prefix. Similarly to the previous case, we assumed that the channel changes independently from OFDM block to OFDM block. The BER curves of the two schemes can be observed in Figure 2. As apparent from the figure, the STF code achieved higher diversity.

6. Conclusion

We proposed a general framework for the performance analysis of STF-coded MIMO-OFDM systems. We determined the maximum achievable diversity order of the STF codes and a simple repetition coding approach was used to achieve it. Moreover, we analyzed the effect of temporal and frequency-domain correlation on the performance of MIMO-OFDM systems. Assuming independent fading between OFDM blocks, the simulation results showed that by coding across multiple OFDM blocks, the achieved diversity order can be significantly increased.

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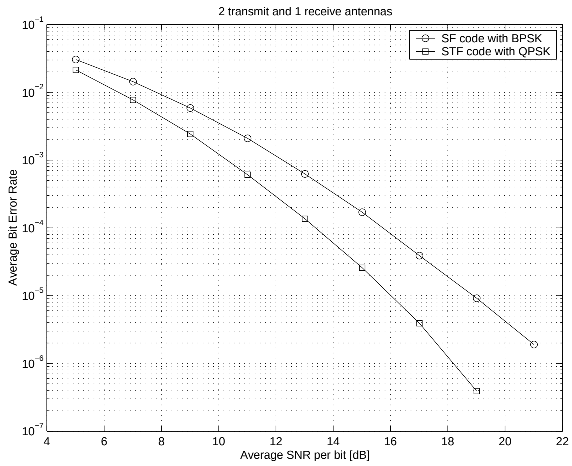

Figure 1: The performance of the block codes

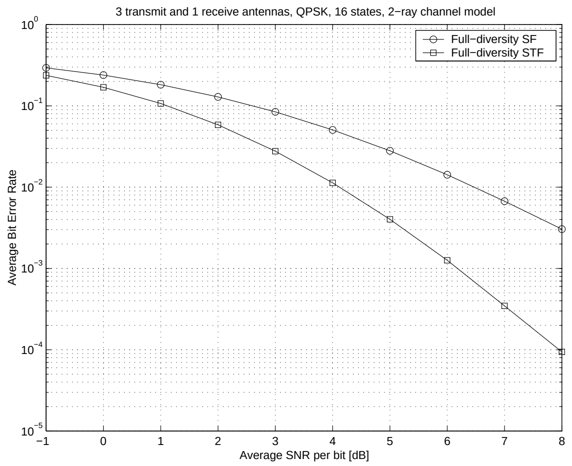

Figure 2: The performance of the trellis codes

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