SER Performance Analysis and Optimum Power Allocation for Decode-and-Forward Cooperation Protocol in Wireless Networks

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***Abstract*— In this paper, symbol-error-rate (SER) performance analysis is provided for a decode-and-forward cooperation protocol in wireless networks. We derive closed-form SER formulation for the cooperation system with PSK and QAM signals. Moreover, two SER upper bounds are established to show the asymptotic performance of the cooperation protocol, in which one of them is tight at high signal-to-noise ratio. Based on the SER performance analysis, we also determine the optimum power allocation for the cooperation systems. It turns out an equal power strategy is in general not optimum in the cooperation communications, and the optimum power allocation depends on the channel link quality. An interesting result is that in case that all channel links are available, the optimum power allocation does not depend on the direct link between source and destination, it depends only on the channel links related to the relay. Extensive simulations are performed to validate the theoretical results.**

I. Introduction

In the conventional point-to-point wireless communications, if the channel link is blocked or inactive, the receiver is not able to get the transmitted information, and therefore continuous communication is not guaranteed. Recently, the concept of cooperative communications was proposed for wireless networks, such as cellular networks and mobile ad hoc networks [1]–[7]. The basic idea of cooperative communications is that all mobile users or nodes in wireless networks help each other to send out information cooperatively. Each user's data information is sent out not only by the user, but also by other users. Thus, it is more reliable for destination to receive the transmitted information since from statistical point of view, the chance that all the channel links to the destination go down is low.

In [1], [2], various cooperation protocols were proposed for wireless networks. When a user helps other users to forward information, it serves as a relay, and it may decode the received information and then forward the decoded symbol or just simply amplify and forward it. Outage probability performance has been analyzed for such cooperation systems. The concept of user cooperation diversity was also proposed in [3], [4], in which a specific two-user cooperation scheme was investigated for CDMA systems. In [5]–[7], a coded two-user cooperation scheme was proposed by taking advantage of the existing channel codes, in which the coded information of each user is divided as two parts: one part is transmitted by the user itself and the other by its cooperator.

In this paper, we consider a decode-and-forward cooperation protocol in wireless networks as specified in [1], [2]. We derive closed-form symbol-error-rate (SER) for the decode-and-forward cooperation systems with PSK and QAM signals. Since the closed-form SER formulation is complicated, we establish two SER upper bounds to show the asymptotic performance of the cooperation system, in which one of them is tight at high signal-to-noise ratio (SNR). Based on the SER performance analysis, we are able to determine the optimum power allocation for the cooperation systems. It turns out the equal power strategy in [1], [2] is in general not optimum, and the optimum power allocation depends on the channel link quality. In case that all channel links are available, a surprising observation from the obtained result is that the optimum power allocation does not depend on the direct link between source and destination, it depends only on the channel links related to the relay. We also investigate SER performance and optimum power allocation for the cooperation protocol under some special channel conditions. Finally, simulation results validate the theoretical analysis.

II. System Model

We consider a cooperation strategy with two phases in a wireless network. In Phase 1, each mobile user (or node) in the wireless network sends information to its destination, and the information is also received by other users at the same time. In Phase 2, each user helps others by decoding the information that it received from other users in Phase 1 and sending out the decoded symbols. In both phases, all users transmit signals through orthogonal channels by using TDMA, FDMA or CDMA scheme [1]–[4]. For better understanding the cooperation concept, we will focus on a two-user cooperation scheme. Specifically, user 1 sends information to its destination in Phase 1, and user 2 also receives the information. User 2 decodes the information and helps user 1 to send out the information in Phase 2. Similarly, when user 2 sends its information to its destination in Phase 1, user 1 receives and decodes the information and will send it to user 2's destination in Phase 2. Due to the symmetry of the two users, we will analyze only user 1's performance. Without loss of generality,

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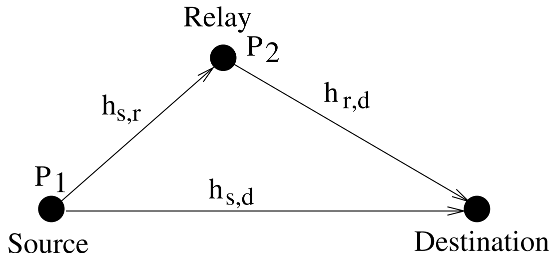

Fig. 1. A simplified cooperation model.

we will consider a concise model as shown in Fig. 1, in which source denotes user 1 and relay represents user 2.

In Phase 1, the source broadcasts its information to both the destination and the relay. The received signals $y_{s,d}$ and $y_{s,r}$ at the destination and the relay respectively can be written as

$$y_{s,d} = \sqrt{P_1}\, h_{s,d}\, x + \eta_{s,d}, \tag{1}$$

$$y_{s,r} = \sqrt{P_1}\, h_{s,r}\, x + \eta_{s,r}, \tag{2}$$

in which P_1 is the transmitted power at the source, x is the transmitted information symbol, and $\eta_{s,d}$ and $\eta_{s,r}$ are additive noise. In (1) and (2), $h_{s,d}$ and $h_{s,r}$ are the channel coefficients from the source to the destination and the relay respectively. If the relay is able to decode the transmitted symbol correctly, then in Phase 2, the relay forwards the decoded symbol with power P_2 to the destination, otherwise the relay does not send or idle. Thus, the received signal at the destination in Phase 2 can be modeled as

$$y_{r,d} = \sqrt{\tilde{P}_2}\, h_{r,d}\, x + \eta_{r,d}, \tag{3}$$

where $\tilde{P}_2 = P_2$ if the relay decodes the transmitted symbol correctly, otherwise $\tilde{P}_2 = 0$, and $h_{r,d}$ is the channel coefficient from the relay to the destination. The channel coefficients $h_{s,d}, h_{s,r}$ and $h_{r,d}$ are modeled as zero-mean, complex Gaussian random variables with variances $\delta^2_{s,d}, \delta^2_{s,r}$ and $\delta^2_{r,d}$ respectively. They are assumed to be known at the receiver, but not at the transmitter. The noise terms $\eta_{s,d}, \eta_{s,r}$ and $\eta_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$.

Jointly combining the received signal from the source directly in Phase 1 and that from the relay in Phase 2, the destination detects the transmitted symbols by use of the maximum-ratio combining (MRC) [8]. We fix the total transmitted power P such as

$$P_1 + P_2 = P. \tag{4}$$

Note that the power saving in case of $\tilde{P}_2 = 0$ is negligible, since at high SNR, the chance that the relay incorrectly decodes the symbol is rare as we will see later in the performance analysis. We assume that when the source sends out information, an ideal cyclic redundancy check (CRC) code [9] has been applied over the information symbols such that the relay is able to judge whether the transmitted symbol is correctly decoded or not.

III. SER Performance Analysis

In this section, we analyze the SER performance for the cooperative communication systems. We derive closed-form SER formulations for the systems with M-PSK and M-QAM[2] modulation, and also provide two SER upper bounds to reveal the asymptotic performance.

With knowledge of the channel coefficients $h_{s,d}$ (from the source to the destination) and $h_{r,d}$ (from the relay to the destination), the output of the MRC detector at the destination can be written as [8]

$$y = a_1 y_{s,d} + a_2 y_{r,d}, \tag{5}$$

where $a_1 = \sqrt{P_1} h^*_{s,d}/\mathcal{N}_0$ and $a_2 = \sqrt{\tilde{P}_2} h^*_{r,d}/\mathcal{N}_0$. Assume that the transmitted symbol x has average energy 1, then the SNR of the MRC output is [8]

$$\gamma = \frac{P_1|h_{s,d}|^2 + \tilde{P}_2|h_{r,d}|^2}{\mathcal{N}_0}. \tag{6}$$

If M-PSK modulation is used in the system, with the instantaneous SNR γ in (6), the conditional SER with the channel coefficients $h_{s,d}, h_{s,r}$ and $h_{r,d}$ can be written as [12]

$$P^{h_{s,d},h_{s,r},h_{r,d}}_{\mathrm{PSK}} = \Psi_{\mathrm{PSK}}(\gamma) \triangleq \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp(-\frac{b_{\mathrm{PSK}}\gamma}{\sin^2\theta})d\theta, \tag{7}$$

where $b_{\mathrm{PSK}} = \sin^2(\pi/M)$. If M-QAM ($M = 2^k$ with k even) signals are used in the system, the conditional SER can also be expressed as [12]

$$P^{h_{s,d},h_{s,r},h_{r,d}}_{\mathrm{QAM}} = \Psi_{\mathrm{QAM}}(\gamma), \tag{8}$$

where

$$\Psi_{\mathrm{QAM}}(\gamma) \triangleq 4KQ(\sqrt{b_{\mathrm{QAM}}\gamma}) - 4K^2Q^2(\sqrt{b_{\mathrm{QAM}}\gamma}), \tag{9}$$

in which $K = 1 - \frac{1}{\sqrt{M}}$, $b_{\mathrm{QAM}} = 3/(M-1)$, and $Q(u) = \frac{1}{\sqrt{2\pi}}\int_u^\infty \exp(-\frac{t^2}{2})dt$ is the Gaussian error function.

If the relay decodes the transmitted symbol correctly, then the relay forwards the decoded symbol with power P_2 to the destination, i.e., $\tilde{P}_2 = P_2$; otherwise the relay does not send, i.e., $\tilde{P}_2 = 0$. If an M-PSK symbol is sent from the source, then at the relay, the chance of incorrect decoding is $\Psi_{\mathrm{PSK}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$, and the chance of correct decoding is $1 - \Psi_{\mathrm{PSK}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$. Similarly, if an M-QAM symbol is sent out at the source, then the chance of incorrect decoding at the relay is $\Psi_{\mathrm{QAM}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$, and the chance of correct decoding is $1 - \Psi_{\mathrm{QAM}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$.

Let us focus on the SER performance analysis in case of M-PSK modulation at first. Taking into account the two scenarios of $\tilde{P}_2 = P_2$ and $\tilde{P}_2 = 0$, we further calculate the conditional SER in (7) as follows:

$$P^{h_{s,d},h_{s,r},h_{r,d}}_{\mathrm{PSK}} = \Psi_{\mathrm{PSK}}(\gamma)|_{\tilde{P}_2=0}\Psi_{\mathrm{PSK}}(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}) + \Psi_{\mathrm{PSK}}(\gamma)|_{\tilde{P}_2=P_2}\left[1 - \Psi_{\mathrm{PSK}}(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0})\right].$$

[2]Throughout the paper, QAM stands for a square QAM constellation whose size is given by $M = 2^k$ with k even.

Averaging over the Rayleigh fading channels $h_{s,d}, h_{s,r}$ and $h_{r,d}$, the SER of the cooperation system with M-PSK modulation can be given by

$$\begin{aligned} P_{\mathrm{PSK}} &= F_1\left(1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)F_1\left(1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right) \\ &+ F_1\left((1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta})(1+\frac{b_{\mathrm{PSK}}P_2\delta_{r,d}^2}{\mathcal{N}_0\sin^2\theta})\right) \\ &\times \left[1-F_1\left(1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right)\right], \end{aligned} \tag{10}$$

where

$$F_1(x(\theta)) = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\frac{1}{x(\theta)}d\theta. \tag{11}$$

Similarly, with M-QAM modulation, the conditional SER in (8) with the channel coefficients $h_{s,d}, h_{s,r}$ and $h_{r,d}$ can be determined as

$$\begin{aligned} P_{\mathrm{QAM}}^{h_{s,d},h_{s,r},h_{r,d}} &= \Psi_{\mathrm{QAM}}(\gamma)|_{\tilde{P}_2=0}\Psi_{\mathrm{QAM}}(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}) \\ &+ \Psi_{\mathrm{QAM}}(\gamma)|_{\tilde{P}_2=P_2}\left[1-\Psi_{\mathrm{QAM}}(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0})\right]. \end{aligned}$$

By substituting (9) into the above formulation and averaging over the fading channels $h_{s,d}, h_{s,r}$ and $h_{r,d}$, the SER of the system with M-QAM modulation can be given by

$$\begin{aligned} P_{\mathrm{QAM}} &= F_2\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)F_2\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,r}^2}{2\mathcal{N}_0\sin^2\theta}\right) \\ &+ F_2\left((1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta})(1+\frac{b_{\mathrm{QAM}}P_2\delta_{r,d}^2}{2\mathcal{N}_0\sin^2\theta})\right) \\ &\times \left[1-F_2\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,r}^2}{2\mathcal{N}_0\sin^2\theta}\right)\right], \end{aligned} \tag{12}$$

where

$$F_2(x(\theta)) = \frac{4K}{\pi}\int_0^{\pi/2}\frac{1}{x(\theta)}d\theta - \frac{4K^2}{\pi}\int_0^{\pi/4}\frac{1}{x(\theta)}d\theta. \tag{13}$$

In order to get the SER formulation in (12), we use two special properties of the Gaussian Q-function as $Q(u) = \frac{1}{\pi}\int_0^{\pi/2}\exp(-\frac{u^2}{2\sin^2\theta})d\theta$ and $Q^2(u) = \frac{1}{\pi}\int_0^{\pi/4}\exp(-\frac{u^2}{2\sin^2\theta})d\theta$ for $u \geq 0$ [11], [12].

Since the closed-form SER formulations in (10) and (12) involve integrations even though they can be calculated by computer efficiently, we provide some SER upper bounds in the following theorem to get some insight understanding.

Theorem *1:* The SER of the cooperation systems with M-PSK or M-QAM modulation can be upper-bounded as

$$\begin{aligned} P_s &\leq \frac{(M-1)\mathcal{N}_0^2}{M^2} \\ &\times \frac{MbP_1\delta_{s,r}^2+(M-1)bP_2\delta_{r,d}^2+M\mathcal{N}_0}{(\mathcal{N}_0+bP_1\delta_{s,d}^2)(\mathcal{N}_0+bP_1\delta_{s,r}^2)(\mathcal{N}_0+bP_2\delta_{r,d}^2)}, \end{aligned} \tag{14}$$

where $b = b_{\mathrm{PSK}}$ for M-PSK signals and $b = b_{\mathrm{QAM}}/2$ for M-QAM signals. Furthermore, if all of the channel links $h_{s,d}, h_{s,r}$ and $h_{r,d}$ are available, i.e., $\delta_{s,d}^2 \neq 0, \delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$, then the SER of the systems with M-PSK or M-QAM modulation can be upper-bounded as

$$P_s \leq \frac{\mathcal{N}_0^2}{b^2}\cdot\frac{1}{P_1\delta_{s,d}^2}\left(\frac{A^2}{P_1\delta_{s,r}^2}+\frac{B}{P_2\delta_{r,d}^2}\right), \tag{15}$$

where in case of M-PSK signals, $b = b_{\mathrm{PSK}}$ and

$$A = \frac{M-1}{2M}+\frac{\sin\frac{2\pi}{M}}{4\pi}, \tag{16}$$

$$B = \frac{3(M-1)}{8M}+\frac{\sin\frac{2\pi}{M}}{4\pi}-\frac{\sin\frac{4\pi}{M}}{32\pi}; \tag{17}$$

while in case of M-QAM signals, $b = b_{\mathrm{QAM}}/2$ and

$$A = \frac{M-1}{2M}+\frac{K^2}{\pi}, \tag{18}$$

$$B = \frac{3(M-1)}{8M}+\frac{K^2}{\pi}. \tag{19}$$

Proof: Since $0 \leq \sin^2\theta \leq 1$, we obtain the upper bound in (14) by substituting $\sin^2\theta = 1$ into the SER formulations in (10) and (12), respectively.

In the following we show the upper bound in (15). First, let us consider the M-PSK modulation. In the SER expression (10), by removing the negative term and ignoring all 1's in denominator, we have

$$P_{\mathrm{PSK}} \leq \frac{A^2\mathcal{N}_0^2}{b_{\mathrm{PSK}}^2P_1^2\delta_{s,d}^2\delta_{s,r}^2}+\frac{B\mathcal{N}_0^2}{b_{\mathrm{PSK}}^2P_1P_2\delta_{s,d}^2\delta_{r,d}^2},$$

where

$$A = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\sin^2\theta d\theta = \frac{M-1}{2M}+\frac{\sin\frac{2\pi}{M}}{4\pi},$$

$$B = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\sin^4\theta d\theta = \frac{3(M-1)}{8M}+\frac{\sin\frac{2\pi}{M}}{4\pi}-\frac{\sin\frac{4\pi}{M}}{32\pi}.$$

Thus, the upper bound in (15) holds for the M-PSK modulation. In case of M-QAM signals, we observe that the function $F_2(x(\theta))$ in (13) can be rewritten as

$$F_2(x(\theta)) = \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2}\frac{1}{x(\theta)}d\theta+\frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2}\frac{1}{x(\theta)}d\theta, \tag{20}$$

which does not contain negative term. Therefore, by substituting (20) into the SER formulation (12), removing the negative term and ignoring all 1's in denominator, we have

$$P_{\mathrm{QAM}} \leq \frac{4A^2\mathcal{N}_0^2}{b_{\mathrm{QAM}}^2P_1^2\delta_{s,d}^2\delta_{s,r}^2}+\frac{4B\mathcal{N}_0^2}{b_{\mathrm{QAM}}^2P_1P_2\delta_{s,d}^2\delta_{r,d}^2},$$

where

$$\begin{aligned} A &= \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2}\sin^2\theta d\theta+\frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2}\sin^2\theta d\theta \\ &= \frac{M-1}{2M}+\frac{K^2}{\pi}, \\ B &= \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2}\sin^4\theta d\theta+\frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2}\sin^4\theta d\theta \\ &= \frac{3(M-1)}{8M}+\frac{K^2}{\pi}. \end{aligned}$$

Thus, the upper bound (15) also holds for the M-QAM signals.

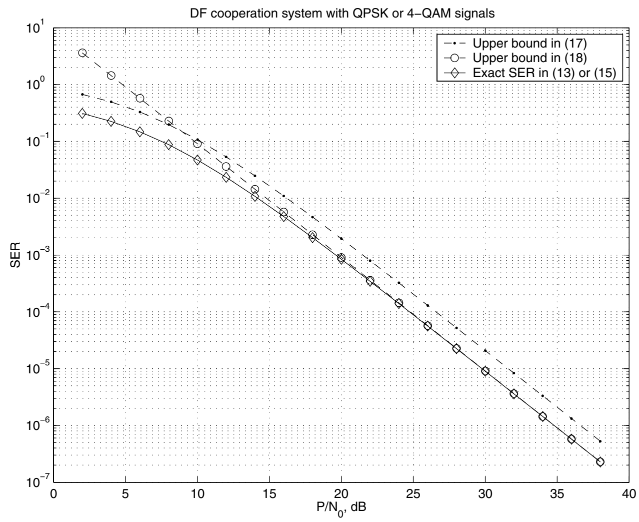

Fig. 2. Comparison of the exact SER formulation and the upper bounds for the cooperation system with QPSK or 4-QAM signals. We assumed that $\delta_{s,d}^2 = \delta_{s,r}^2 = \delta_{r,d}^2 = 1$, $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P/2$.

It is not difficult to check that in case of QPSK modulation, the SER formulation in (10) is consistent with that in (12) for 4-QAM modulation. For both QPSK and 4-QAM, the parameters b, A and B in the upper bounds (14) and (15) are the same respectively. In such a case, those parameters are $b = 1, A = \frac{3}{8} + \frac{1}{4\pi}$ and $B = \frac{9}{32} + \frac{1}{4\pi}$. In Fig. 2, we compare the two upper bounds (14) and (15) with the exact SER in (10) or (12), we can see that the upper bound (14) is asymptotically parallel with the exact SER curve at high SNR, and the upper bound (15) is loose at low SNR, but it is tight at high SNR.

IV. Optimum Power Allocation

In this section, we determine an asymptotic optimum power allocation for the cooperation protocol based on the tight SER upper bound we obtained in the previous section. Specifically, we will determine the optimum transmitted power P_1 at the source and P_2 at the relay for a fixed total transmission power $P_1 + P_2 = P$.

Theorem *2:* In the cooperation systems, if all of the channel links $h_{s,d}, h_{s,r}$ and $h_{r,d}$ are available, i.e., $\delta_{s,d}^2 \neq 0, \delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$, then for enough high SNR, the optimum power allocation is

$$P_1 = \frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}} P, \tag{21}$$

$$P_2 = \frac{2\delta_{s,r}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + (8A^2/B)\delta_{r,d}^2}} P, \tag{22}$$

where A and B are specified in (16–19) for M-PSK and M-QAM signals respectively.

Proof: Since the BER upper bound in (15) is tight for high SNR, we determine an asymptotic optimum power allocation based on this upper bound. Sufficiently, we just need to minimize the term $(A^2 P_2 \delta_{r,d}^2 + BP_1\delta_{s,r}^2)/(P_1^2 P_2)$ in (15) under the power constraint $P_1 + P_2 = P$. By taking derivative over P_1 and setting the resulting derivation as 0, we have

$$B\delta_{s,r}^2(P_1^2 - P_1P_2) - 2A^2\delta_{r,d}^2 P_2^2 = 0.$$

By solving the above equation with the power constraint $P_1 + P_2 = P$, we can obtain the optimum power allocation in (21) and (22). $\square$

The result in Theorem 2 is somewhat surprising since the asymptotic optimum power allocation does not depend on the channel link between source and destination, it depends only on the channel link between source and relay and the channel link between relay and destination. Specifically, we can see that the optimum ratio of the transmitted power P_1 at the source over the total power P is less than 1 and larger than $1/2$, while the optimum ratio of the power P_2 used at the relay over the total power P is larger than 0 and less than $1/2$. Furthermore, if the link quality between source and relay is much less than that between relay and destination, i.e., $\delta_{s,r}^2 << \delta_{r,d}^2$, then P_1 goes to P and P_2 goes to 0. It means we just need to put the total power P at the source and do not use the relay. On the other hand, if the link quality between source and relay is much larger than that between relay and destination, i.e., $\delta_{s,r}^2 >> \delta_{r,d}^2$, then P_1 goes to $P/2$ and P_2 also goes to $P/2$. It means we need to put equal power at the source and the relay in this case.

We interpret the result in Theorem 2 as follows. Since we assume that all of the channel links $h_{s,d}, h_{s,r}$ and $h_{r,d}$ are available in the system, the goal of the cooperation strategy is to achieve a performance diversity of order two. The performance of the system is guaranteed to have the first order diversity due to the channel link between source and destination. However, in order to achieve the second order diversity, the channel link between source and relay and the channel link between relay and destination are more important. If the link quality between source and relay is bad, then it is difficult for the relay to correctly decode the transmitted symbol. Thus, the forwarding role of the relay is not important and it makes sense to put all of the power at the source. On the other hand, if the link quality between source and relay is very good, the relay can always decode the transmitted symbol correctly, so the decoded symbol at the relay is almost the same as that at the source. We may consider the relay as a copy of the source, so we may put power on them almost equally. Finally, we want to emphasize that it is an extreme case to put power equally on the source and the relay. In general, this is not the case, even for the special scenario that the link quality between source and relay is the same as that between relay and destination. From (21) and (22), we can see that if $\delta_{s,r}^2 = \delta_{r,d}^2$, then the optimum power allocation is

$$P_1 = \frac{1 + \sqrt{1 + 8A^2/B}}{3 + \sqrt{1 + 8A^2/B}} P, \tag{23}$$

$$P_2 = \frac{2}{3 + \sqrt{1 + 8A^2/B}} P, \tag{24}$$

where A and B depend on specific modulation signals.

We determine the optimum power allocation in Theorem 2 for the case that all of the channel links $h_{s,d}, h_{s,r}$ and $h_{r,d}$ are available. In the rest of this section, we consider some special cases that some of the channel links are not available.

i When the channel link between relay and destination is not available, i.e., $\delta_{r,d}^2 = 0$, according to (10), the SER of the system with M-PSK modulation can be given by

$$P_{\text{PSK}} = F_1\left(1 + \frac{b_{\text{PSK}} P_1 \delta_{s,d}^2}{\mathcal{N}_0 \sin^2\theta}\right) \le \frac{A\mathcal{N}_0}{b_{\text{PSK}} P_1 \delta_{s,d}^2}, \quad (25)$$

where $F_1(\cdot)$ is defined in (11), and A is specified in (16). Similarly, by (12), the SER of the system with M-QAM modulation is

$$P_{\text{QAM}} = F_2\left(1 + \frac{b_{\text{QAM}} P_1 \delta_{s,d}^2}{2\mathcal{N}_0 \sin^2\theta}\right) \le \frac{2A\mathcal{N}_0}{b_{\text{QAM}} P_1 \delta_{s,d}^2}, \quad (26)$$

where $F_2(\cdot)$ is defined in (13), and A is specified in (18). From (25) and (26), we can see that for both M-PSK and M-QAM signals, the optimum power allocation is $P_1 = P$ and $P_2 = 0$. It means that we should use the direct transmission from source to destination in this case.

ii When the channel link between source and relay is not available, i.e., $\delta_{s,r}^2 = 0$, according to (10) and (12), the SER of the system with M-PSK or M-QAM signals can be upper bounded as

$$P_s \le \frac{2A\mathcal{N}_0}{bP_1\delta_{s,d}^2},$$

where in case of M-PSK modulation, $b = b_{\text{PSK}}$ and A is specified in (16), while in case of M-QAM modulation, $b = b_{\text{QAM}}/2$ and A is specified in (18). Therefore, the optimum power allocation in this case is $P_1 = P$ and $P_2 = 0$.

iii When the channel link between source and destination is not available, i.e., $\delta_{s,d}^2 = 0$, from (10) and (12), the SER of the system with M-PSK or M-QAM signals can be given by

$$\begin{aligned} P_s &= F_i\left(1 + \frac{bP_1\delta_{s,r}^2}{\mathcal{N}_0 \sin^2\theta}\right) + F_i\left(1 + \frac{bP_2\delta_{r,d}^2}{\mathcal{N}_0 \sin^2\theta}\right) \\ &\quad \times \left[1 - F_i\left(1 + \frac{bP_1\delta_{s,r}^2}{\mathcal{N}_0 \sin^2\theta}\right)\right], \quad (27) \end{aligned}$$

where $i = 1$ and $b = b_{\text{PSK}}$ for M-PSK modulation, and $i = 2$ and $b = b_{\text{QAM}}/2$ for M-QAM modulation. If $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$, i.e., the cases in i and ii do not happen, then by the same procedure as we obtained the upper bound in (15), the SER in (27) can be upper bounded as

$$P_s \le \frac{A\mathcal{N}_0^2}{b^2} \cdot \frac{P_2\delta_{r,d}^2 + P_1\delta_{s,r}^2}{P_1P_2\delta_{s,r}^2\delta_{r,d}^2}, \quad (28)$$

where in case of M-PSK modulation, $b = b_{\text{PSK}}$ and A is specified in (16), while in case of M-PSK modulation, $b = b_{\text{QAM}}/2$ and A is specified in (18). From (28), we can see that with the total power $P_1 + P_2 = P$, the optimum power allocation in this case is

$$P_1 = \frac{\delta_{r,d}}{\delta_{s,r} + \delta_{r,d}} P \quad (29)$$

$$P_2 = \frac{\delta_{s,r}}{\delta_{s,r} + \delta_{r,d}} P \quad (30)$$

for both M-PSK and M-QAM signals.

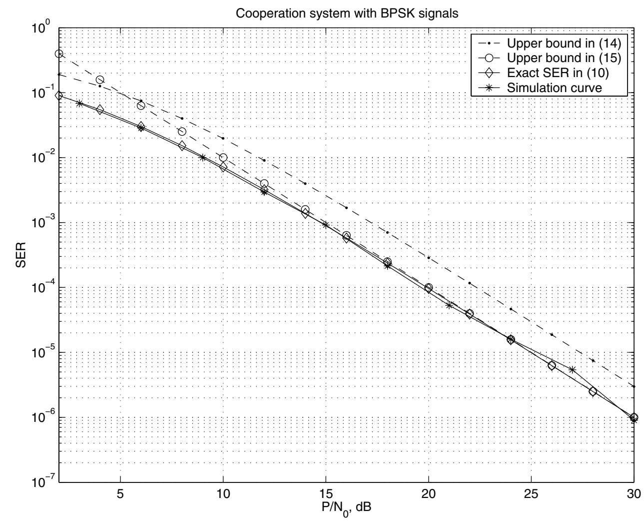

Fig. 3. Simulation of a cooperation system with BPSK signals, $\delta_{s,r}^2 = \delta_{r,d}^2 = 1$, $P_1/P = 0.5931$ and $P_2/P = 0.4069$.

Note that when the channel link between source and destination is not available ($\delta_{s,d}^2 = 0$), the system reduces to a two-hop scenario [10]. It is worth noting that the optimum power allocation in (29) and (30) that we obtained from an approach of minimizing the SER bound (28) is consistent with the result in [10], in which the optimum power allocation was determined for multi-hop systems from a minimizing outage probability point of view.

V. Simulation Results

To illustrate the above theoretical analysis, we performed some computer simulations. In all simulations, we assumed that the variance of the noise is 1 (i.e., $\mathcal{N}_0 = 1$), and the variance of the channel link between source and destination is 1 (i.e., $\delta_{s,d}^2 = 1$). For fair comparison, we present average SER curves as functions of $P/\mathcal{N}_0$.

We simulated at first a cooperation system with BPSK signals. We assumed that the variances of the channel link between source and relay and that between relay and destination are 1, i.e., $\delta_{s,r}^2 = \delta_{r,d}^2 = 1$. In such a case, by Theorem 2, the optimum power ratios are $P_1/P = 0.5931$ and $P_2/P = 0.4069$. We also plotted the exact SER calculation in (10) and the two upper bounds in (14) and (15). From Fig. 3, we can see that the exact SER calculation (solid line with "◇") fits to the simulation curve (solid line with "$*$"). The upper bound in (14) (dashed line with "·") goes parallel along the exact SER curve with a 2 dB gap. The upper bound in (15) (dashed line with "○") is tight at high SNR, and it merges with the exact SER curve at a SER of 10^{-3}.

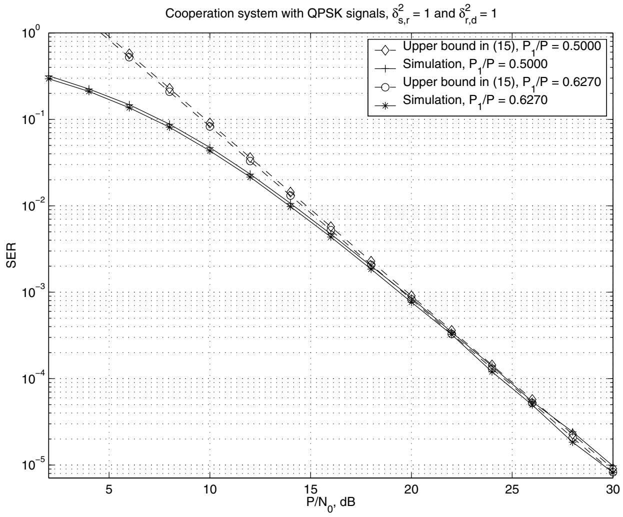

Fig. 4. Simulation of a cooperation system with QPSK signals for two power allocation schemes, assuming $\delta^2_{s,r} = \delta^2_{r,d} = 1$.

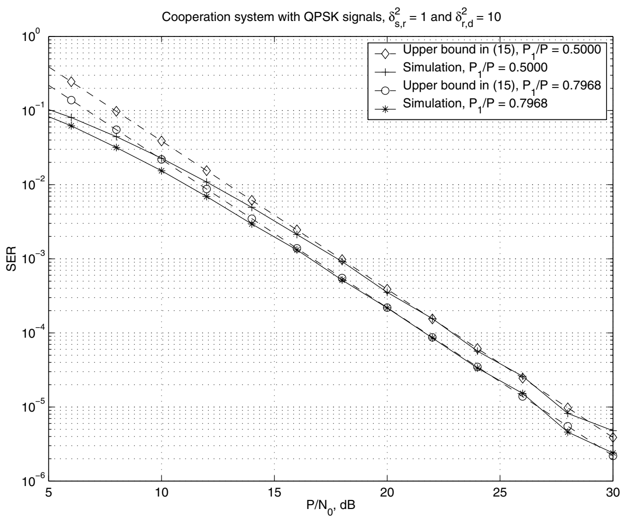

Fig. 5. Simulation of a cooperation system with QPSK signals for two power allocation schemes, assuming $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$.

We also simulated a cooperation system with QPSK signals, as shown in Figs. 4 and 5. We compared the performance of the optimum power allocation with that of the equal power case. In Fig. 4, we assume $\delta^2_{s,r} = \delta^2_{r,d} = 1$, and the optimum power ratios in this case are $P_1/P = 0.6270$ and $P_2/P = 0.3730$ by Theorem 2. From the figure, we observe that the performance of the optimum power allocation in this case is almost the same as that of the equal power case ($P_1/P = P_2/P = 1/2$), and the two bounds are consistent with the simulation curves at high SNR respectively. In the simulations in Fig. 5, we assume that $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$. In this case, by Theorem 2, the optimum power ratios are $P_1/P = 0.7968$ and $P_2/P = 0.2032$. From Fig. 5, we can see that the optimum power allocation (solid line with "$*$") outperforms the equal power case (solid line with "$+$") with a performance improvement of more than 1 dB. Also, the two bounds merge with the simulation curves at high SNR respectively. Note that if the ratio of the link quality $\delta^2_{r,d}/\delta^2_{s,r}$ becomes larger, we will observe more performance improvement of the optimum power allocation over the equal power case. In all of the above simulations, we can see that the SER bound in (15) is tight enough at high SNR.

VI. Conclusion

In this paper, we analyzed the SER performance for the decode-and-forward cooperation systems. Closed-form SER formulation was given explicitly for cooperation systems with PSK and QAM signals. For better understanding the asymptotic performance of the systems, two SER upper bounds were established, in which one of them is tight at high SNR. Furthermore, based on the SER performance analysis, we determined the optimum power allocation for the cooperation systems. From the theoretical and simulation results, we can draw the following conclusions. First, the equal power strategy is in general not optimum in the cooperative communications, and the optimum power allocation depends on the channel link quality. Second, in case that all channel links are available, the optimum power allocation does not depend on the direct link between source and destination, it depends only on the channel link between source and relay and that between relay and destination. Finally, if the link quality between source and relay is much less than that between relay and destination, i.e., $\delta^2_{s,r} << \delta^2_{r,d}$, then we should put the total power at the source and do not use the relay. On the other hand, if the link quality between source and relay is much larger than that between relay and destination, i.e., $\delta^2_{s,r} >> \delta^2_{r,d}$, then the equal power strategy at the source and the relay tends to be optimum.

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