Systematic Design of Space-Frequency Codes with Full Rate and Full Diversity

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***Abstract*— In this paper, a general space-frequency block code structure is proposed that can guarantee full-rate and full-diversity transmission in MIMO-OFDM systems. The proposed method can be used to construct space-frequency codes for an arbitrary number of transmit antennas, any memoryless modulation and arbitrary power delay profiles. Moreover, assuming that the power delay profile is known at the transmitter, we devise an interleaving method to maximize the overall performance of the code. The simulation results show that under the simulated environment, the proposed SF codes outperform the existing SF codes from orthogonal designs by about 3-5 dB, and that the proposed interleaving method results in about 1-3 dB performance improvement compared to random interleaving.**

I. Introduction

Multiple-input-multiple-output (MIMO) wireless communication systems have attracted considerable attention recently, as they can exploit the spatial diversity present in MIMO channels. In case of frequency selective channels, there is an additional source of diversity, frequency diversity, due to the existence of multiple propagation paths between each transmit and receive antenna pair. By combining the orthogonal frequency division multiplexing (OFDM) modulation with MIMO systems, *space-frequency* (SF) codes have been proposed to exploit both spatial and frequency diversity [1]–[9]. The strategy of SF coding is to distribute the channel symbols over different transmit antennas and OFDM tones within one OFDM block.

The first SF coding scheme was proposed in [1], in which previously existing ST codes were used by replacing the time domain with the frequency domain. Similar schemes were described later in [2]–[5]. The performance criteria for SF coded MIMO-OFDM systems were derived in [6], [7]. The authors in [7] showed that, in general, existing ST codes cannot exploit the frequency diversity available in frequency selective MIMO channels. Later, in [8], they constructed a class of full-diversity SF codes relying on the assumption that all of the path delays are located exactly at the sampling instances of the receiver. Recently in [9], a systematic design method to obtain full-diversity SF codes was proposed for arbitrary power delay profiles. The resulting SF codes provide higher data rates than the approach described in [8], but they still cannot achieve full rate (one channel symbol per subcarrier) transmission. Therefore, it is of interest to devise new SF code design methods that can guarantee both performance (full diversity) and high data rate (full symbol rate).

The detrimental effects of the correlation between adjacent subcarriers can be reduced by interleaving, or *permuting*, the subcarriers. Assuming that the delay paths are equally spaced and fall onto the sampling instances of the receiver, an optimum subcarrier grouping method was proposed in [11]. However, the proposed grouping method was not guaranteed to be optimum for arbitrary power delay profiles.

In this paper, we consider the problem of SF block code design for MIMO-OFDM systems. We propose a SF code design approach that offers full symbol rate and guarantees full diversity for an arbitrary number of transmit antennas, any memoryless modulation method and arbitrary power delay profiles. First, we describe a general SF code structure and show that the combination of this code structure and the algebraically rotated signal constellations [14]–[18] or the diagonal cyclic space-time constellations [12] can guarantee full-rate full-diversity transmission. Second, assuming that the statistics of the channel (the power delay profile) is known at the transmitter, we devise a subcarrier permutation (or interleaving) method to maximize the overall performance of the code. We decompose the diversity product as the product of the "intrinsic" and the "extrinsic" diversity products. The "intrinsic" diversity product depends only on the used signal constellations and the SF code design, while the "extrinsic" diversity product depends only on the applied permutation and the power delay profile of the channel. Based on this decomposition, we propose a permutation strategy and determine the optimum permutation to maximize the "extrinsic" diversity product.

II. Channel Model and SF Code Design Criteria

We consider a SF coded MIMO-OFDM system with M_t transmit antennas, M_r receive antennas and N subcarriers. The MIMO channel is assumed to be constant over each OFDM block period. The frequency selective fading channels between different transmit and receive antenna pairs are assumed to be independent and have the same power delay profile. The channel impulse response from transmit antenna i to receive

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antenna j can be modeled as

$$h_{i,j}(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}(l)\delta(\tau - \tau_l), \qquad (1)$$

where τ_l is the delay of the l-th path, and $\alpha_{i,j}(l)$ is the complex amplitude of the l-th path. The $\alpha_{i,j}(l)$'s are modeled as zero-mean, complex Gaussian random variables with variances $E|\alpha_{i,j}(l)|^2 = \delta_l^2$. The powers of the L paths are normalized such that $\sum_{l=0}^{L-1} \delta_l^2 = 1$. From (1), the frequency response of the channel is given by

$$H_{i,j}(f) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-\mathbf{j}2\pi f \tau_l}, \quad \mathbf{j} = \sqrt{-1}. \qquad (2)$$

The input bit stream is divided into b bit long segments, and each segment is mapped onto a SF codeword. Each SF codeword can be represented as an $N \times M_t$ matrix $C = \{c_i(n)\}_{1 \le i \le M_t, 0 \le n \le N-1}$, in which $c_i(n)$ denotes the channel symbol transmitted over the n-th subcarrier by transmit antenna i. The SF code satisfies the energy constraint $E||C||_F^2 = NM_t$, where $||C||_F$ is the Frobenius norm of C. The transmitter applies an N-point IFFT to each column of the matrix C, and after appending the cyclic prefix, the OFDM symbol corresponding to the i-th column of C is transmitted by transmit antenna i.

After removing the cyclic prefix and applying FFT, the received signal at the n-th subcarrier at receive antenna j is given by

$$y_j(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i(n) H_{i,j}(n) + z_j(n), \qquad (3)$$

where $H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-\mathbf{j}2\pi n \Delta f \tau_l}$ is the channel frequency response at the n-th subcarrier between transmit antenna i and receive antenna j, $\Delta f = 1/T$ is the subcarrier separation, and T is the OFDM symbol period. We assume that $H_{i,j}(n)$ is known at the receiver, but not at the transmitter. In (3), $z_j(n)$ denotes the additive complex Gaussian noise with zero mean and unit variance. The factor $\sqrt{\rho/M_t}$ ensures that ρ is the average signal to noise ratio (SNR) at each receive antenna.

Assuming that the MIMO channel is spatially uncorrelated, the correlation matrix of the channel frequency response between transmit antenna i and receive antenna j can be expressed as [9]

$$R_{i,j} = R = W diag(\delta_0^2, \delta_1^2, \cdots, \delta_{L-1}^2) W^{\mathcal{H}}, \qquad (4)$$

where

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix}_{N \times L},$$

and $w = e^{-\mathbf{j}2\pi\Delta f}$.

For two distinct SF codewords C and $\tilde{C}$, we use the notation $\Delta = (C - \tilde{C})(C - \tilde{C})^{\mathcal{H}}$, where the superscript $\mathcal{H}$ stands for the complex conjugate and transpose. Then, the pairwise error probability between C and $\tilde{C}$ can be upper bounded as [9], [13]

$$P(C \to \tilde{C}) \le \binom{2\nu M_r - 1}{\nu M_r} \left(\prod_{i=1}^{\nu} \lambda_i \right)^{-M_r} \left(\frac{\rho}{M_t} \right)^{-\nu M_r}, \qquad (5)$$

where ν is the rank of $\Delta \circ R$, $\lambda_1, \lambda_2, \cdots, \lambda_\nu$ are the non-zero eigenvalues of $\Delta \circ R$, and $\circ$ denotes the Hadamard product[1]. Based on the upper bound in (5), two design criteria were proposed as follows [9]:

- *Diversity criterion:* The minimum rank of $\Delta \circ R$ over all pairs of distinct C and $\tilde{C}$ should be as large as possible.
- *Product criterion:* The minimum value of the product $\prod_{i=1}^{\nu} \lambda_i$ over all pairs of distinct codewords C and $\tilde{C}$ should also be maximized.

If the minimum rank of $\Delta \circ R$ is ν_0, we say that the SF code achieves a diversity order of $\nu_0 M_r$. It has been shown that the maximum achievable diversity is at most $\min\{LM_tM_r, NM_r\}$ [6], [7], [9]. If a SF code achieves full diversity, the *diversity product*, which is the normalized coding advantage, is given by [9], [12]

$$\zeta = \frac{1}{2\sqrt{M_t}} min_{C \ne \tilde{C}} \left| \prod_{i=1}^{\nu_0} \lambda_i \right|^{1/(2\nu_0)}. \qquad (6)$$

III. Full-Rate and Full-Diversity Code Design

In this section, we describe a systematic SF code design method with full rate and full diversity. Specifically, we will design a class of SF codes with diversity order of $\Gamma M_t M_r$ for any integer $\Gamma \, (1 \le \Gamma \le L)$.

A. Code Structure

We propose a coding strategy in which each SF codeword C is a concatenation of some matrices:

$$C = \left[G_1^{\mathcal{T}} \; G_2^{\mathcal{T}} \; \cdots \; G_P^{\mathcal{T}} \; \mathbf{0}_{N-P\Gamma M_t}^{\mathcal{T}} \right]^{\mathcal{T}}, \qquad (7)$$

where $P = \lfloor N/(\Gamma M_t) \rfloor$, and each matrix $G_p \, (1 \le p \le P)$ is of size ΓM_t by M_t. The zero padding in (7) is used if the number of subcarriers N is not an integer multiple of ΓM_t. Each matrix G_p has the same structure given by

$$G = \sqrt{M_t} \, diag \left(X_1, X_2, \cdots, X_{M_t} \right), \qquad (8)$$

where $X_i = [x_{(i-1)\Gamma+1} \; x_{(i-1)\Gamma+2} \cdots x_{i\Gamma}]^{\mathcal{T}}$, $i = 1, 2, \cdots, M_t$, and all x_k, $k = 1, 2, \cdots, \Gamma M_t$, are complex symbols and will be specified later. The energy constraint is $E(\sum_{k=1}^{\Gamma M_t} |x_k|^2) = \Gamma M_t$. The selection of the symbols $\mathbf{X} = [x_1 \; x_2 \cdots x_{\Gamma M_t}]$ is independent for each $G_p \, (1 \le p \le P)$. The symbol rate of the code is $P\Gamma M_t/N$, ignoring the cyclic prefix. If N is a multiple of ΓM_t, the

[1] If $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$ are two matrices of the same size, the *Hadamard product* of A and B is $A \circ B = \{a_{i,j} b_{i,j}\}$.

symbol rate is 1, otherwise it is very close to 1 since usually N is much larger than ΓM_t.

Now, we derive sufficient conditions for the SF codes described above to achieve a diversity order of $\Gamma M_t M_r$. Suppose that C and $\tilde{C}$ are two distinct SF codewords which are constructed from $G_1, G_2, \cdots, G_P$ and $\tilde{G}_1, \tilde{G}_2, \cdots, \tilde{G}_P$, respectively. We would like to determine the rank of $\Delta \circ R$. For two distinct codewords C and $\tilde{C}$, there exists at least one index p_0 $(1 \leq p_0 \leq P)$ such that $G_{p_0} \neq \tilde{G}_{p_0}$. We may further assume that $G_p = \tilde{G}_p$ for any $p \neq p_0$ since the minimum rank of $\Delta \circ R$ can be obtained under this assumption ([21], Corollary 3.1.3, p.149).

From (4), we know that the correlation matrix $R = \{r_{i,j}\}_{1 \leq i,j \leq N}$ is a Toeplitz matrix, in which $r_{i,j} = \sum_{l=0}^{L-1} \delta_l^2 w^{(i-j)\tau_l}$, $1 \leq i, j \leq N$. Under the assumption that $G_p = \tilde{G}_p$ for any $p \neq p_0$, we observe that the non-zero eigenvalues of $\Delta \circ R$ are the same as those of $[(G_{p_0} - \tilde{G}_{p_0})(G_{p_0} - \tilde{G}_{p_0})^{\mathcal{H}}] \circ Q$, where $Q = \{q_{i,j}\}_{1 \leq i,j \leq \Gamma M_t}$ is also a Toeplitz matrix whose entries are

$$q_{i,j} = \sum_{l=0}^{L-1} \delta_l^2 w^{(i-j)\tau_l}, \quad 1 \leq i, j \leq \Gamma M_t. \tag{9}$$

Suppose that G_{p_0} and $\tilde{G}_{p_0}$ have symbols $\mathbf{X} = [x_1 \; x_2 \; \cdots \; x_{\Gamma M_t}]$ and $\tilde{\mathbf{X}} = [\tilde{x}_1 \; \tilde{x}_2 \; \cdots \; \tilde{x}_{\Gamma M_t}]$, respectively. Then, the difference matrix between G_{p_0} and $\tilde{G}_{p_0}$ is

$$G_{p_0} - \tilde{G}_{p_0} = \sqrt{M_t}\, diag(\mathbf{X} - \tilde{\mathbf{X}}) \left(I_{M_t} \otimes \mathbf{1}_{\Gamma \times 1} \right), \tag{10}$$

where I_{M_t} is the identity matrix of size $M_t \times M_t$, $\mathbf{1}_{\Gamma \times 1}$ is an all one matrix of size $\Gamma \times 1$, and $\otimes$ stands for the tensor product. Thus, we have

$$\begin{aligned} & \left[(G_{p_0} - \tilde{G}_{p_0})(G_{p_0} - \tilde{G}_{p_0})^{\mathcal{H}} \right] \circ Q \\ = \;& M_t\, diag(\mathbf{X} - \tilde{\mathbf{X}}) \left[(I_{M_t} \otimes \mathbf{1}_{\Gamma \times \Gamma}) \circ Q \right] diag(\mathbf{X} - \tilde{\mathbf{X}})^{\mathcal{H}}. \end{aligned}$$

The determinant of $[(G_{p_0} - \tilde{G}_{p_0})(G_{p_0} - \tilde{G}_{p_0})^{\mathcal{H}}] \circ Q$ is

$$M_t^{\Gamma M_t} \prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k|^2 \cdot (det(Q_0))^{M_t}, \tag{11}$$

where $Q_0 = \{q_{i,j}\}_{1 \leq i,j \leq \Gamma}$ and $q_{i,j}$ is specified in (9). Assuming that $\tau_0 < \tau_1 < \cdots < \tau_{L-1}$, Q_0 is nonsingular. Therefore, if $\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \neq 0$, the determinant of $[(G_{p_0} - \tilde{G}_{p_0})(G_{p_0} - \tilde{G}_{p_0})^{\mathcal{H}}] \circ Q$ is non-zero. This implies that the SF code achieves a diversity order of $\Gamma M_t M_r$. In this case, the diversity product can be calculated as

$$\begin{aligned} \zeta &= \frac{1}{2} min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \left(\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \right)^{\frac{1}{\Gamma M_t}} |det(Q_0)|^{\frac{1}{2\Gamma}} \\ &= \zeta_{in} \cdot |det(Q_0)|^{\frac{1}{2\Gamma}}, \end{aligned} \tag{12}$$

where

$$\zeta_{in} = \frac{1}{2} min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \left(\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \right)^{\frac{1}{\Gamma M_t}} \tag{13}$$

is termed as the "intrinsic" diversity product of the SF code. Thus, we have the following theorem.

Theorem 1: For any SF code described in (7) and (8), if $\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \neq 0$ for any pair of distinct variables $\mathbf{X} = [x_1 \; x_2 \; \cdots \; x_{\Gamma M_t}]$ and $\tilde{\mathbf{X}} = [\tilde{x}_1 \; \tilde{x}_2 \; \cdots \; \tilde{x}_{\Gamma M_t}]$, the SF code achieves a diversity order of $\Gamma M_t M_r$, and the diversity product is $\zeta = \zeta_{in} \; |det(Q_0)|^{\frac{1}{2\Gamma}}$.

We observe from Theorem 1 that $|det(Q_0)|$ depends only on the power delay profile, and the "intrinsic" diversity product ζ_{in} depends only on $min_{\mathbf{X} \neq \tilde{\mathbf{X}}} (\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k|)^{1/(\Gamma M_t)}$, which is called the *minimum product distance* [14]. Therefore, under the code structure in (8), it is desirable to design a set of variables $\mathbf{X}$ such that the minimum product distance is maximized.

B. Maximizing the "Intrinsic" Diversity Product ζ_{in}

The problem of maximizing the "intrinsic" diversity product ζ_{in} is related to the problem of constructing signal constellations for Rayleigh fading channels ([15], [16], and the references therein). There are two approaches to design the set of variables $\mathbf{X} = [x_1 \; x_2 \; \cdots \; x_{\Gamma M_t}]$. Denote $K = \Gamma M_t$.

One approach is to apply a transform over a K-dimensional signal set. Specifically, for a signal constellation Ω and any signal vector $S = [s_1 \; s_2 \; \cdots \; s_K] \in \Omega^K$, let

$$\mathbf{X} = S \mathcal{M}_K, \tag{14}$$

where $\mathcal{M}_K$ is a $K \times K$ matrix, and it should be optimized such that the minimum product distance of the set of $\mathbf{X}$ is as large as possible. Both Hadamard transforms and Vandermonde matrices have been proposed for constructing $\mathcal{M}_K$ [15], [16]. The results have been recently used to design space-time block codes [17], [18]. The signal constellations Ω from square lattices, such as QAM, are of practical interest. If $K = 2^s$ $(s \geq 1)$, an optimum transform $\mathcal{M}_K$ for Ω from square lattices has been given by [15], [16]

$$\mathcal{M}_K = \frac{1}{\sqrt{K}} V(\theta_1, \theta_2, \cdots, \theta_K), \tag{15}$$

where $V(\cdot)$ is a Vandermonde matrix, and $\theta_1, \theta_2, \cdots, \theta_K$ are the roots of polynomial $\theta^K - \mathbf{j}$ over field $\mathbf{Q}[\mathbf{j}] \triangleq \{c + d\mathbf{j}$: both c and d are rational numbers$\}$. For more details, we refer the reader to [15], [16].

Suppose that the spectral efficiency of an SF code is r bits/s/Hz. As another approach, one may consider to design a set of $L_0 = 2^{rK}$ variables directly under the energy constraint $E||\mathbf{X}||_F^2 = K$. We can take advantage of cyclic space-time signals [12]:

$$C_l = diag(e^{\mathbf{j} u_1 \theta_l}, \; e^{\mathbf{j} u_2 \theta_l}, \; \cdots, \; e^{\mathbf{j} u_K \theta_l}), \; l = 0, 1, \cdots, L_0 - 1,$$

where $\theta_l = \frac{l}{L_0} 2\pi$, $0 \leq l \leq L_0 - 1$, and the parameters $u_1, u_2, \cdots, u_K \in \{0, 1, \cdots, L_0 - 1\}$ are selected such that $min_{l \neq l'} \prod_{k=1}^{K} \left| e^{\mathbf{j} u_k \theta_l} - e^{\mathbf{j} u_k \theta_{l'}} \right|$ is maximized. Now a set of variables $\mathbf{X} = [x_1 \; x_2 \; \cdots \; x_{\Gamma M_t}]$ can be designed as

$$x_k = e^{\mathbf{j} u_k \theta_l}, \quad k = 1, 2, \cdots, K; \; l = 0, 1, \cdots, L_0 - 1. \tag{16}$$

IV. MAXIMIZING THE CODING ADVANTAGE BY PERMUTATIONS

Assuming that the power delay profile is available at the transmitter, we propose an interleaving/permutation method to maximize the performance of the SF codes.

A. Diversity Product of the SF Codes with Permutations

By permuting the rows of the SF codeword C specified in (7) and (8), we obtain a new codeword $\sigma(C)$. For two distinct codewords C and $\tilde{C}$ constructed from G_p and $\tilde{G}_p$, in order to determine the minimum rank of $[\sigma(C-\tilde{C})\sigma(C-\tilde{C})^{\mathcal{H}}]\circ R$, we assume that $G_{p_0} \neq \tilde{G}_{p_0}$ and $G_p = \tilde{G}_p$ for any $p \neq p_0$. Suppose that G_{p_0} and $\tilde{G}_{p_0}$ have variables $\mathbf{X} = [x_1\ x_2\ \cdots\ x_{\Gamma M_t}]$ and $\tilde{\mathbf{X}} = [\tilde{x}_1\ \tilde{x}_2\ \cdots\ \tilde{x}_{\Gamma M_t}]$, respectively, with $x_k \neq \tilde{x}_k$ for all $1 \leq k \leq \Gamma M_t$. For simplicity, denote $\Delta x_k = x_k - \tilde{x}_k$. After row permutation, we assume that the k-th $(1 \leq k \leq \Gamma M_t)$ row of $G_{p_0} - \tilde{G}_{p_0}$ is located at the n_k-th $(0 \leq n_k \leq N-1)$ row of $\sigma(C - \tilde{C})$. Then, the product of the non-zero eigenvalues of $[\sigma(C-\tilde{C})\sigma(C-\tilde{C})^{\mathcal{H}}]\circ R$ can be determined as [10]

$$M_t^{\Gamma M_t} \prod_{k=1}^{\Gamma M_t} |\Delta x_k|^2 \prod_{m=1}^{M_t} \left| det(W_m \Lambda W_m^{\mathcal{H}}) \right|. \qquad (17)$$

where $\Lambda = diag(\delta_0^2,\ \delta_1^2,\ \cdots,\ \delta_{L-1}^2)$, and $W_m =$

$$\begin{bmatrix} w^{n_{(m-1)\Gamma+1}\tau_0} & w^{n_{(m-1)\Gamma+1}\tau_1} & \cdots & w^{n_{(m-1)\Gamma+1}\tau_{L-1}} \\ w^{n_{(m-1)\Gamma+2}\tau_0} & w^{n_{(m-1)\Gamma+2}\tau_1} & \cdots & w^{n_{(m-1)\Gamma+2}\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{n_{m\Gamma}\tau_0} & w^{n_{m\Gamma}\tau_1} & \cdots & w^{n_{m\Gamma}\tau_{L-1}} \end{bmatrix}.$$

Therefore, from (6), the diversity product of the permuted SF code can be calculated as: $\zeta = \zeta_{in} \cdot \zeta_{ex}$, where ζ_{in} is the "intrinsic" diversity product (13), and ζ_{ex} denotes the "extrinsic" diversity product, defined as

$$\zeta_{ex} = \left(\prod_{m=1}^{M_t} \left| det(W_m \Lambda W_m^{\mathcal{H}}) \right| \right)^{\frac{1}{2\Gamma M_t}}. \qquad (18)$$

This result is summarized as follows.

Theorem 2: For any permutation, the diversity product of the resulting SF code is

$$\zeta = \zeta_{in} \cdot \zeta_{ex}. \qquad (19)$$

Moreover, ζ_{ex} is upper bounded as follows:

(i) $\zeta_{ex} \leq 1$; and more precisely,

(ii) if we sort the power profile $\delta_0, \delta_1, \cdots, \delta_{L-1}$ in a non-increasing order as: $\delta_{l_1} \geq \delta_{l_2} \geq \cdots \geq \delta_{l_L}$, then

$$\zeta_{ex} \leq \left(\prod_{i=1}^{\Gamma} \delta_{l_i} \right)^{\frac{1}{\Gamma}} \left| \prod_{m=1}^{M_t} det(W_m W_m^{\mathcal{H}}) \right|^{\frac{1}{2\Gamma M_t}}, \qquad (20)$$

where the equality holds when $\Gamma = L$. As a consequence, $\zeta_{ex} \leq \sqrt{L} \left(\prod_{i=1}^{\Gamma} \delta_{l_i} \right)^{1/\Gamma}$.

A proof of Theorem 2 can be found in [10]. We observe that the "extrinsic" diversity product ζ_{ex} depends on the permutation and the power delay profile of the channel. Note that the permutation does not effect the "intrinsic" diversity product ζ_{in}.

B. Maximizing the "Extrinsic" Diversity Product ζ_{ex}

In this subsection, we consider a specific permutation strategy. For a fixed integer $\mu\,(\mu \geq 1)$, we define a one-to-one mapping σ over set $\{0, 1, \cdots, N-1\}$ as follows:

$$\sigma(n) = v_1 \mu \Gamma + e_0 \mu + v_0, \quad n = 0, 1, \cdots, N-1, \qquad (21)$$

where $e_1 = \lfloor \frac{n}{\Gamma} \rfloor$, $e_0 = n - e_1 \Gamma$, $v_1 = \lfloor \frac{e_1}{\mu} \rfloor$, and $v_0 = e_1 - v_1 \mu$. We call the integer μ as a *separation factor*. For any SF codeword C proposed in (7) and (8), we permute the rows of C such that the n-th $(0 \leq n \leq N-1)$ row of C is moved to the $\sigma(n)$-th row. We have the following result.

Theorem 3: For the permutation specified in (21) with a separation factor μ, the "extrinsic" diversity product of the permuted SF code is given by

$$\zeta_{ex} = |det(V_0 \Lambda V_0^{\mathcal{H}})|^{\frac{1}{2\Gamma}}, \qquad (22)$$

where

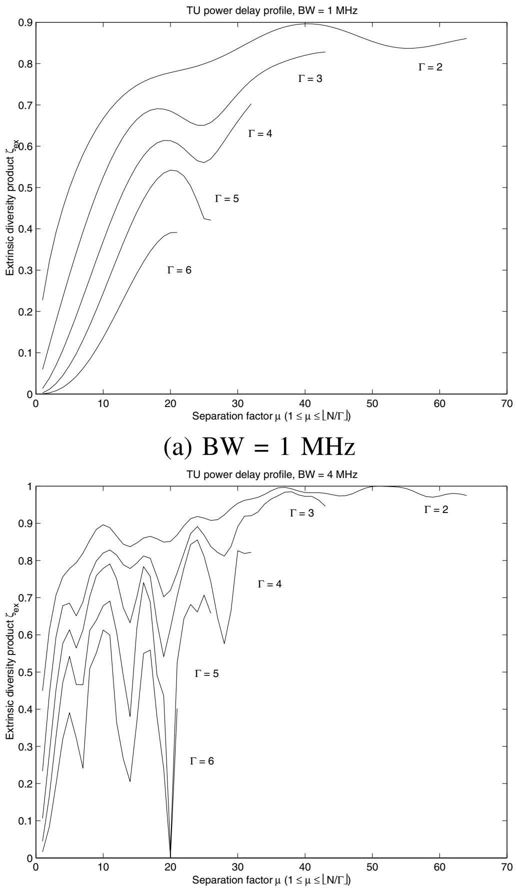
$$V_0 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\mu\tau_0} & w^{\mu\tau_1} & \cdots & w^{\mu\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(\Gamma-1)\mu\tau_0} & w^{(\Gamma-1)\mu\tau_1} & \cdots & w^{(\Gamma-1)\mu\tau_{L-1}} \end{bmatrix}. \qquad (23)$$

The proof of Theorem 3 is described in [10]. The permutation in (21) is determined by the separation factor μ. Our objective is to find a separation factor μ_{op} that maximizes the "extrinsic" diversity product ζ_{ex}:

$$\mu_{op} = arg \max_{1 \leq \mu \leq \lfloor N/\Gamma \rfloor} |det(V_0 \Lambda V_0^{\mathcal{H}})|. \qquad (24)$$

In some cases, closed form solution to (24) can be obtained:

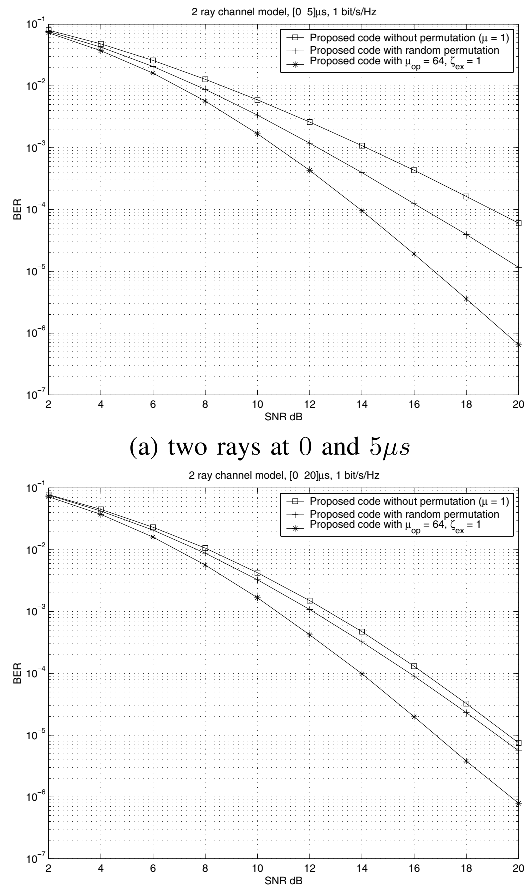
- If $\Gamma = L = 2$, the "extrinsic" diversity product is $\zeta_{ex} = \sqrt{2\delta_0\delta_1}|sin(\mu(\tau_1 - \tau_0)\pi/T)|^{1/2}$. Suppose that the system has $N = 128$ subcarriers, and the total bandwidth is $BW = 1MHz$. Then, the OFDM block duration is $T = 128\mu s$. If $\tau_1 - \tau_0 = 5\mu s$, then $\mu_{op} = 64$. If $\tau_1 - \tau_0 = 20\mu s$, then $\mu_{op} = 16$. In general, if $\tau_1 - \tau_0 = 2^a b\ \mu s$, $a \geq 0$ and b is odd, then $\mu_{op} = 128/2^{a+1}$. In these cases, $\zeta_{ex} = \sqrt{2\delta_0\delta_1}$, which achieves the upper bound in Theorem 2.
- Assume that $\tau_l - \tau_0 = lN_0T/N$, $l = 1, 2, \cdots, L-1$, and N is integer multiple of LN_0. If $\Gamma = L$ or $\delta_0^2 = \delta_1^2 = \cdots = \delta_{L-1}^2 = 1/L$, then the optimum separation factor $\mu_{op} = N/(LN_0)$, and the corresponding "extrinsic" diversity product is $\zeta_{ex} = \sqrt{L}(\prod_{l=0}^{L-1} \delta_l)^{1/L}$ (see [10] for the proof). In particular, in case of $\delta_0^2 = \delta_1^2 = \cdots = \delta_{L-1}^2 = 1/L$, $\zeta_{ex} = 1$. In both cases, the "extrinsic" diversity products achieve the upper bounds in Theorem 2. Note that in case of $\tau_l = lT/N\,(0 \leq l \leq L-1)$, $\Gamma = L$ and N is integer multiple of L, the permutation $\mu_{op} = N/L$ is similar to the optimum subcarrier grouping method proposed in [11], which is not optimal for arbitrary power delay profiles.



(a) BW = 1 MHz

(b) BW = 4 MHz

Fig. 1. Extrinsic diversity product ζ_{ex} vs. separation factor μ for different $\Gamma\,(2 \le \Gamma \le 6)$ in TU channel model. $N = 128$.



(a) two rays at 0 and $5\mu s$

(b) two rays at 0 and $20\mu s$

Fig. 2. Performance of the proposed SF code with different permutations in two ray channel model.

In Figure 1 (a) and (b), we plot the "extrinsic" diversity product ζ_{ex} as the function of the separation factor μ for different $\Gamma\,(2 \le \Gamma \le 6)$ values using the *typical urban* (TU) 6-ray power delay profile [20]. In case of $\Gamma = 2$ and BW = 1 MHz, the maximum "extrinsic" diversity product is $\zeta_{ex} = 0.8963$. The corresponding separation factor is $\mu_{op} = 40$. However, since one-to-one mapping is preferred, we choose $\mu = 64$, which results in $\zeta_{ex} = 0.8606$. In case of $\Gamma = 2$ and BW = 4 MHz, the maximum ζ_{ex} is 0.9998, and the corresponding μ_{op} is 51. Similarly, we choose $\mu = 64$. The resulting ζ_{ex} is 0.9751.

V. Simulation Results

We simulated a MIMO-OFDM system with two transmit and one receive antennas, and $N = 128$ subcarriers. The full-rate full-diversity SF codes were constructed according to (7) and (8) with the G matrix

$$G = \sqrt{2}\begin{bmatrix} x_1 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & x_4 \end{bmatrix}^{\mathcal{T}}, \qquad (25)$$

where the x_i's were obtained by (14) and (15) as follows:

$$[x_1\, x_2\, x_3\, x_4] = [s_1\, s_2\, s_3\, s_4] \cdot V(\theta, -\theta, \mathbf{j}\theta, -\mathbf{j}\theta)/2, \qquad (26)$$

in which s_i's are chosen from BPSK or QPSK, and $\theta = e^{\mathbf{j}\pi/8}$.

First, we compare the performance of the proposed SF codes with three permutation schemes: no permutation, random permutation, and the proposed optimum permutation. The random permutation was generated by the Takeshita-Constello method [19]

$$\sigma(n) = mod(n(n+1)/2,\ N), \quad n = 0, 1, \cdots, N-1. \qquad (27)$$

We simulated the proposed code with the structure (25) and (26), and s_1, s_2, s_3, s_4 were chosen from BPSK. We assumed a simple 2-ray, equal-power delay profile, with a delay $\tau\ \mu s$ between the two rays. The total bandwidth was BW = 1 MHz. From Figures 2 (a) and (b), we observe that the performance of the proposed SF code with the random permutation is better than that without permutation. With the optimum permutations, the performance is further improved. In case of $\tau = 5\mu s$, there is a 3 dB gain between the optimum permutation ($\mu_{op} = 64$) and the random permutation at a BER of 10^{-5}. In case of $\tau = 20\mu s$, the performance improvement of the optimum permutation ($\mu_{op} = 16$) over the random permutation is 2 dB at a BER of 10^{-5}.

Then, we compare the performance of the proposed full-rate full-diversity SF codes with that of the full-diversity SF codes described in [9]. We simulated the proposed code with the structure (25) and (26), in which s_1, s_2, s_3, s_4 were

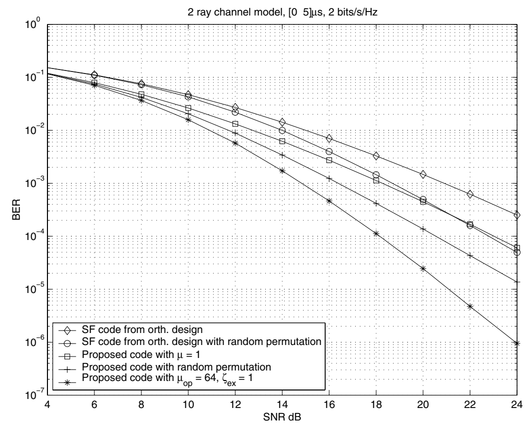


Fig. 3. Comparison of the proposed SF code and the code from orthogonal design in two ray channel model with $\tau = 5\mu s$.

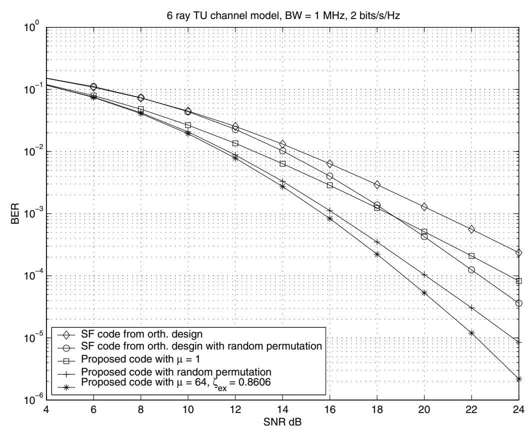


Fig. 4. Comparison of the proposed SF code and the code from orthogonal design in TU channel model with BW = 1 MHz.

chosen from QPSK, so the spectral efficiency was 2 bits/s/Hz, ignoring the cyclic prefix. The full-diversity SF code of [9] is a repetition of the orthogonal design two times, in which all symbols were chosen from 16-QAM in order to maintain the same spectral efficiency.

We considered the 2-ray, equal-power delay profile with $\tau = 5\mu s$. From Figure 3, we observe that without permutation, the proposed SF code outperforms the SF code from orthogonal design by about 3 dB at a BER of 10^{-4}. With the random permutation, the proposed code outperforms the orthogonal code by about 2 dB at a BER of 10^{-4}. With the optimum permutation ($\mu_{op} = 64$), the proposed code has an additional gain of 3 dB at a BER of 10^{-4}. Compared to the orthogonal code with the random permutation, the proposed code with the optimum permutation has a total gain of 5 dB at a BER of 10^{-4}.

We also compare the two SF codes using the TU channel model with BW = 1 MHz in Figure 4. We can see that without permutation, the proposed SF code outperforms the SF code from orthogonal design by about 2 dB at a BER of 10^{-4}. With the random permutation, the performance of the proposed code is better than that of the orthogonal code by about 2.5 dB at a BER of 10^{-4}. With the proposed permutation ($\mu = 64$), an additional improvement of 1 dB at a BER of 10^{-4} is achieved by the proposed SF code.

VI. Conclusion

In this paper, we proposed a general SF code structure that can guarantee full-rate and full-diversity transmission in MIMO-OFDM systems. In addition, assuming that the power delay profile of the channel is available at the transmitter, we proposed an optimum interleaving scheme to further improve the performance. The simulation results showed that the proposed SF codes outperformed previously existing approaches and demonstrated the importance of interleaving in SF coded MIMO-OFDM systems.

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