Collusion-Resistant Multi-Winner Spectrum Auction for Cognitive Radio Networks

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Abstract—In order to fully utilize spectrum, auction-based dynamic spectrum allocation has become a promising approach which allows unlicensed wireless users to lease unused bands from spectrum license holders. Because spectrum resources are reusable by users far apart, in some scenarios, spectrum is more efficiently utilized by awarding one band to multiple secondary users simultaneously, which distinguishes it from traditional auctions where only one user can be the winner. However, the multi-winner auction is a new concept posing new challenges in the traditional auction mechanisms, because such mechanisms may yield low revenue and are not robust to some newly-emerging collusion. Therefore, in this paper, we propose an efficient mechanism for the multi-winner spectrum auction with collusion-resistant pricing strategies, in which the optimal spectrum allocation can be solved by binary linear programming and the pricing is formulated as a convex optimization problem. Furthermore, a greedy algorithm is proposed to reduce complexity for multi-band auctions. Simulation results are presented to evaluate our proposed auction mechanisms.

I. INTRODUCTION

As the demand for wireless spectrum has been growing rapidly with the deployment of new wireless applications and devices in the last decade, the regulatory bodies such as the Federal Communications Commission (FCC) have begun to consider more flexible and comprehensive uses of available spectrum [1]. With the development of cognitive radio technologies [2], dynamic spectrum access becomes a promising approach, which allows unlicensed users (secondary users) to dynamically access the licensed bands from legacy spectrum holders (primary users). There are mainly two kinds of access schemes: secondary users access the licensed spectrum opportunistically when primary users are absent [3]–[5], or lease some channels temporarily from primary users through negotiation [6]–[11].

There are several previous efforts to study the negotiation-based dynamic spectrum access via pricing and auction mechanisms. In [6], the price of anarchy was analyzed for spectrum sharing in WiFi networks. In [7], a demand responsive pricing framework was proposed to maximize the profits of legacy spectrum operators while considering the buyers' response model. An auction-based mechanism was proposed in [8] to efficiently share spectrum among secondary users in interference-limited systems. In [9], the authors considered a multi-unit sealed-bid auction for efficient spectrum allocation. In [10], a real-time spectrum auction framework with interference constraints was proposed to get a conflict-free allocation. In [11], a belief-assisted distributive pricing algorithm was proposed to achieve efficient dynamic spectrum allocation based on double auction mechanisms.

Although existing schemes have enhanced spectrum allocation efficiency through market mechanisms, some critical challenges still remain unanswered. Firstly, in most of the current auctions, one licensed band (or a package of multiple bands) is sold to a unique buyer. However, the spectrum resource is different from other goods since it is interference-limited rather than quantity-limited. In some application scenarios such as a wireless personal area network (WPAN) centered around an individual person's workspace, secondary users transmit with a low power level, and hence users far apart can simultaneously access the same band. In that case, an auction awarding one band to multiple users (multi-winner auction) is a better choice since it improves both spectrum efficiency and the seller's revenue. As traditional mechanisms such as the second-price auction [12] and the Vickery-Clarke-Groves (VCG) mechanism [13] cannot guarantee full efficiency or high revenue, a proper mechanism is needed for the multi-winner auction. Moreover, with the emerging applications of mobile ad hoc networks envisioned in civilian usage, secondary users may be selfish and only aim to maximize their own interests. They may cheat/collude in the spectrum auction if profitable, which will severely undermine the auction, and therefore, the developed mechanism has to be immune to user collusion. In this paper, we propose two pricing strategies as well as full-efficiency allocation for the multi-winner auction. The efficient allocation is determined by a binary linear programming problem, and the pricing strategy can be modeled as a convex optimization problem. It is shown that the proposed strategies not only improve the primary user's revenue, but also resist the possible user collusion.

The rest of this paper is organized as follows. In Section II, the multi-winner spectrum auction model is described. We discuss the limitations of existing auction mechanisms and develop novel collusion-resistant pricing strategies for a single-band auction in Section III, which are further extended to multi-band auctions in Section IV. In Section V, simulation results are presented, and Section VI concludes the paper.

II. System Model

We first consider a spectrum auction in which N secondary users (buyers) want to lease a single band from a primary user (seller), and extend it to a multi-band auction in Section IV. We assume there is a spectrum broker (auctioneer) helping coordinate the auction. At the beginning of each leasing period (the length of leasing periods should be decided according to channel dynamics and overhead considerations), the potential buyers simultaneously submit their bids $\boldsymbol{b} = [b_1, b_2, \ldots, b_N]$ to the spectrum broker who will then decide both the allocation $\boldsymbol{x} = [x_1, x_2, \ldots, x_N]$ and the prices $\boldsymbol{p} = [p_1, p_2, \ldots, p_N]$, where $x_i = 1$ (or 0) means secondary user i wins (or loses) the band, and p_i is the price for secondary user i. Alternatively, we define the set of winners as $W \subseteq \{1, 2, \ldots, N\}$, where $i \in W$ if and only if $x_i = 1$. Assume user i gains v_i from transmitting information in the leased band, then his/her reward is

$$r_i = v_i x_i - p_i, \ i = 1, 2, \ldots, N. \tag{1}$$

Given all users' valuations $\boldsymbol{v} = [v_1, v_2, \ldots, v_N]$, the system utility, or the *social welfare*, can be represented by

$$U_{\boldsymbol{v}}(\boldsymbol{x}) = \sum_{i=1}^{N} v_i x_i = \sum_{i \in W} v_i, \tag{2}$$

which measures the total amount of utility realized from the multi-winner auction. An auction is *efficient* if its outcome maximizes the social welfare.

In order to characterize the interference constraints among the secondary users, we adopt an $N \times N$ adjacency matrix $\boldsymbol{C}$, with $C_{ij} = 1$ if user i and user j cannot be assigned the band simultaneously, and $C_{ij} = 0$ otherwise. Collecting reports from secondary users about their neighbors, the spectrum broker keeps the matrix $\boldsymbol{C}$ updated.

Since secondary users want to successfully lease the band with the lowest possible payment, it is reasonable to assume that they are selfish and aim to maximize their own profits. A clique of secondary users may plot collusion before participating in the auction if they believe it is profitable. They may even share a more facilitated way to exchange information and collude, if subscribed to the same service provider. In general, there are several kinds of collusion as follows:

- *Bidding ring collusion.* Some or even all of the secondary users constitute a bidding ring and significantly lower their bids. Then, the revenue of the seller will get reduced.
- *Loser collusion.* Some users, who cannot get the spectrum lease without collusion, may win in the auction if they collude to raise their bids. This will affect the efficiency of the auction as well as the seller's revenue.
- *Sublease collusion.* The secondary users who win in the auction may sublease the spectrum to other users and earn extra profits effortlessly. In other words, the colluding users take away some benefits which should be credited to the primary user.

As the bidding ring collusion can be combated by setting an optimal reserve price as in [11], in this paper, we focus on fighting against the other two kinds of collusion.

III. One-Band Spectrum Auction

In this section, we review the widely used traditional auction mechanisms and analyze their weakness when applied to multi-winner auctions. Then, we propose a proper mechanism consisting of efficient allocation and collusion-resistant pricing strategies.

A. Auctions with the Second Price and the VCG Price

In a second-price auction, the bidder with the highest bid wins the item, and pays the amount of money equal to the second highest bid. It is well-known that submitting bids equal to their true valuations is the dominant strategy [12]. For example, consider Case (a) in Fig. 1 with four secondary users, whose valuations are $v_1 = 15$, $v_2 = 6$, $v_3 = 10$, and $v_4 = 4$, respectively. If the band has to be awarded to only one of them, user 1 will get the spectrum lease by paying $p_1 = 10$, which equals the second highest bid made by user 3.

Although it is an ideal choice in a single-winner auction, the second-price auction is less efficient in a multi-winner auction. The efficient allocation is determined by the following binary integer programming (BIP) problem

$$\max_{\boldsymbol{x}} U_{\boldsymbol{v}}(\boldsymbol{x}) = \sum_{i=1}^{N} v_i x_i,$$
$$\text{s.t. } x_i + x_j \leq 1, \ \forall i, j \text{ if } C_{ij} = 1, \tag{3}$$
$$x_i = 0 \text{ or } 1, \ i = 1, 2, \ldots, N,$$

with constraints reflecting the interference relationship. Still consider Case (a), where an edge between two users indicates that the two users cannot share the band owing to interference. Then, the efficient allocation is $x_2 = x_3 = x_4 = 1$, yielding a higher system utility ($v_2 + v_3 + v_4 = 20$) than the second-price auction outcome ($v_1 = 15$). This implies the second-price auction may be inefficient in the multi-winner auction.

The VCG mechanism employs the efficient allocation (3). Assume the solution to the optimization problem (3) is $\boldsymbol{x}^*$, and the maximum system utility is $U_{\boldsymbol{v}}^* = U_{\boldsymbol{v}}(\boldsymbol{x}^*)$. We use $\boldsymbol{v}_{-i} = [v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N]$ to denote a new system with only user i excluded. If user i wins the opportunity to access the band, the VCG price will be

$$p_i = v_i + U_{\boldsymbol{v}_{-i}}^* - U_{\boldsymbol{v}}^*, \tag{4}$$

which can be interpreted as the "social opportunity cost" to the system: were user i absent from the system, the maximum system utility would be $U_{\boldsymbol{v}_{-i}}^*$; however, with his/her presence, the total utility of all the other users becomes $U_{\boldsymbol{v}}^* - v_i$, and each winner is asked to compensate the "damage" he/she causes to all the others, i.e., $p_i = U_{\boldsymbol{v}_{-i}}^* - (U_{\boldsymbol{v}}^* - v_i)$. For instance, we can apply the VCG mechanism to Case (a). The efficient allocation (3) results in $U_{\boldsymbol{v}}^* = v_2 + v_3 + v_4 = 20$, and the maximal social welfare, if user 2 were absent, would be achieved by awarding the band to user 1, i.e., $U_{\boldsymbol{v}_{-2}}^* = v_1 = 15$. Hence, user 2 has to pay $p_2 = 1$ according to (4). Prices for other winners are calculated in the same way, which are listed in the tables in Fig. 1.

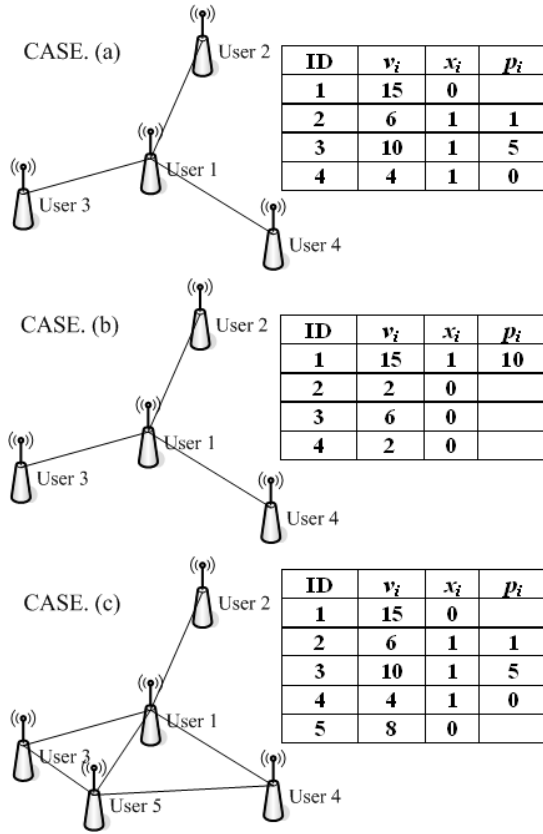However, the VCG mechanism has several drawbacks:

CASE. (a)

ID	v_i	x_i	p_i
1	15	0	
2	6	1	1
3	10	1	5
4	4	1	0

CASE. (b)

ID	v_i	x_i	p_i
1	15	1	10
2	2	0	
3	6	0	
4	2	0	

CASE. (c)

ID	v_i	x_i	p_i
1	15	0	
2	6	1	1
3	10	1	5
4	4	1	0
5	8	0	

Fig. 1. Different auctions with the VCG mechanism employed. v_i is the valuation of user i, x_i and p_i is the allocation and price to user i, respectively.

First, the seller's revenue may be quite low. As in Case (a) with the VCG prices, the total payment collected by the seller is $p_2 + p_3 + p_4 = 6$, which is quite low compared to the system utility. In some unfavorable cases, for example, $v_1 = v_2 = v_3 = v_4 = 10$, the seller's revenue reduces to 0 when the VCG prices are employed.

Second, the losers may take advantage of the VCG pricing by colluding. For example, in Case (b) of Fig. 1, secondary user 1 gets the spectrum lease, and user 2, 3, 4 are losers in the VCG auction. However, if colluding to misrepresent their valuations, they may become winners instead. For instance, they may collude to mimic Case (a) by claiming the same valuations as in Case (a), and gain positive rewards, i.e., $r_2 = 1$, $r_3 = 1$, and $r_4 = 2$ according to (1). The system efficiency is degraded because the spectrum resources are not assigned to the users who value them most.

Third, colluders may extract some profits from the seller by sublease collusion. Consider Case (c) where another secondary user shows up without changing the VCG outcome from Case (a). In this case, user 3 and user 4 may now collude with user 5 by subleasing the band at price $p_5 = 7$, and the income is split between them as 6 and 1. Then, both user 3 and 4 make extra profit by subleasing the band at higher prices than their leasing prices, and user 5 also benefits from subleasing since the reward is $v_5 - p_5 = 1$. Such collusion impairs the spectrum efficiency as well as the primary user's revenue.

B. Collusion-Resistant Auctions

Since the VCG mechanism has severe drawbacks, we need to develop an efficient spectrum auction mechanism with proper pricing strategies to combat user collusion.

We remodel the multi-winner spectrum auction as a single-winner auction by grouping secondary users with negligible interference together as *virtual bidders*, whose valuations equal the sum of the individual valuations. For instance, in Case (a), there are eight virtual bidders with valuations $v(\{1\}) = 15$, $v(\{2\}) = 6$, $v(\{3\}) = 10$, $v(\{4\}) = 4$, $v(\{2,3\}) = 16$, $v(\{2,4\}) = 10$, $v(\{3,4\}) = 14$, and $v(\{2,3,4\}) = 20$. Similar to the second-price strategy, the virtual bidder with the highest bid will be awarded the band (ties are broken randomly if two virtual bidders have the same valuation), and the total payment equals the highest bid made by the virtual bidder consisting of the losers. This can be done by solving two BIP problems in succession without explicitly listing all virtual bidders: We first solve (3) to determine the set of winners W, or the virtual bidder with the highest bid, and after removing all the winners W from the system, we solve the optimization problem again to calculate the maximum utility, denoted by $U_{\boldsymbol{v}_{-W}}^*$. The winners have to pay $U_{\boldsymbol{v}_{-W}}^*$ in total.

Now, the only unsolved problem is splitting the payment among the secondary users within the winning virtual bidder. This is quite similar to a Nash bargaining game [14] where each selfish player proposes his/her own payment during a bargaining process such that the total payment equals $U_{\boldsymbol{v}_{-W}}^*$, and it is well-known that the Nash bargaining solution (NBS), which maximizes the product of the individual payoffs, is an equilibrium [14]. In our proposed auction, no individual bargaining is necessary; instead, the spectrum broker directly sets the equilibrium prices for each winner by solving

$$\max_{\{p_i \in [0, v_i], i \in W\}} \prod_{i \in W} (v_i - p_i),$$
$$\text{s.t.} \sum_{i \in W} p_i = U_{\boldsymbol{v}_{-W}}^*. \tag{5}$$

By using the fact that $\sum_{i \in W} v_i = U_{\boldsymbol{v}}^*$ and applying Karush-Kuhn-Tucker (KTT) conditions [15], the solution is

$$p_i = \max\{v_i - \rho, 0\}, \text{ for } i \in W, \tag{6}$$

where ρ is chosen such that $\sum_{i \in W} p_i = U_{\boldsymbol{v}_{-W}}^*$. It can be seen that the payment is split in a fair way such that the profits are shared among the winners as equally as possible.

When such a pricing strategy is used, the seller's revenue $U_{\boldsymbol{v}_{-W}}^*$ is often relatively high. Moreover, if some losers collude to beat the winners by raising their bids, they will have to pay more than $U_{\boldsymbol{v}_{-W}}^*$; however, the payment is already beyond what the band is actually worth to them, and as a result, loser collusion is completely eliminated.

In order to completely prevent sublease collusion, a more complicated algorithm is developed by adding more constraints. Notice that sublease collusion happens in this way: a subset of the winners $W_C \subseteq W$ sublease the band to a subset of the losers $L_C \subseteq L$, where $L = \{1, 2, \ldots, N\} - W$ denotes

the set of all losers. The necessary condition for the sublease collusion to happen is $\sum_{i \in W_C} p_i < \sum_{i \in L_C} v_i$, so that they can find a sublease price in between acceptable to both parties. They also have to take interference into consideration: the losers in L_C have to be interference-free with each other, and they will not sublease the band if it turns out to be unusable due to interference with the users in $W - W_C$.

When W is determined by the efficient allocation strategy (3), given any colluding-winner subset $W_C \subseteq W$, the possible colluding losers must come from a subset of the losers whose members are interference-free with those users in $W - W_C$, denoted by $L(W - W_C)$. If the prices are set such that $\sum_{i \in W_C} p_i \geq \max_{L_C \in L(W - W_C)} \sum_{i \in L_C} v_i$, there will be no sublease collusion. Note that $\max_{L_C \in L(W - W_C)} \sum_{i \in L_C} v_i$ is the maximum system utility $U^*_{\boldsymbol{v}_{L(W - W_C)}}$ which can be obtained by solving the BIP problem, then the optimum collusion-resistant pricing strategy is the solution to the following problem,

$$
\begin{aligned}
\max_{\{p_i \in [0, v_i], i \in W\}} & \prod_{i \in W} (v_i - p_i), \\
\text{s.t.} \sum_{i \in W_C} p_i & \geq U^*_{\boldsymbol{v}_{L(W - W_C)}}, \forall W_C \subseteq W.
\end{aligned}
\tag{7}
$$

When $W_C = W$, the constraint reduces to $\sum_{i \in W} p_i \geq U^*_{\boldsymbol{v}_{-W}}$, which incorporates the constraint in (5) as a special case.

It can be shown that (7) is a convex optimization problem with linear inequality constraints, and hence it can be efficiently solved by numerical methods [15]. The major complexity comes from solving $2^{|W|} - 1$ BIP problems in order to get the values $U^*_{\boldsymbol{v}_{L(W - W_C)}}$ for any $W_C \subseteq W$ except $W_C = \Phi$. However, in most cases, the size of $L(W - W_C)$ is relatively small due to the interference constraints, and therefore, the complexity of solving those BIP problems is not a big concern.

In sum, the proposed auction mechanism first determines an efficient allocation according to (3), and then assigns a price to each winner using (7) (or (6) if computational capability is limited), which can completely (or partially) eliminate user collusion.

IV. MULTI-BAND SPECTRUM AUCTION

The proposed mechanism can be easily extended to an M-band spectrum auction. We assume all secondary users are interested in only one band, but they do not care which band they get.

In the multi-band spectrum auction, the efficient allocation can be similarly determined by the following MN-variable BIP problem,

$$
\begin{aligned}
\max_{\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^M} U_{\boldsymbol{v}}(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^M) &= \sum_{m=1}^{M} \sum_{i=1}^{N} v_i x_i^m, \\
\text{s.t. } x_i^m + x_j^m & \leq 1, \ \forall i, j \text{ if } C_{ij} = 1, \forall m, \\
\sum_{m=1}^{M} x_i^m & \leq 1, \ \forall i, \\
x_i^m = 0 \text{ or } 1, & i = 1, 2, \ldots, N; m = 1, 2, \ldots, M,
\end{aligned}
\tag{8}
$$

where $x_i^m = 1$ implies secondary user i leases a band from primary user m, and $x_i^m = 0$ otherwise. Actually, this is a natural extension of (3) except for an additional constraint requiring that each secondary user can lease at most one band.

However, computational complexity becomes a major concern. To address the problem, we propose a greedy algorithm to reach approximate efficiency by solving M N-variable BIP problems sequentially, reducing the complexity from $O(2^{MN})$ to $M \cdot O(2^N)$. The idea is simply to sell the band one by one, that is, we solve a one-band efficient allocation problem (3) and award the winners band 1, and then, we remove them from the set of potential buyers and solve another one-band problem again to find the winners who will be awarded with band 2, and so on so forth. Denote the set of potential buyers in the m-th iteration as $L^{(m)}$ (initially, $L^{(1)} = \{1, 2, \ldots, N\}$). $W^{(m)}$, the set of winners awarded with band m, can be solved from the following problem ($m = 1, 2, \ldots, M$),

$$
\begin{aligned}
\max_{\boldsymbol{x}} U_{\boldsymbol{v}}(\boldsymbol{x}) &= \sum_{i=1}^{N} v_i x_i, \\
\text{s.t. } x_i + x_j & \leq 1, \ \forall i, j \text{ if } C_{ij} = 1, \\
x_i = 0 \text{ or } 1, & \text{ if } i \in L^{(m)}, \\
x_i = 0, & \text{ if } i \notin L^{(m)}.
\end{aligned}
\tag{9}
$$

After each iteration, $L^{(m+1)} = L^{(m)} - W^{(m)}$ is updated.

We can derive analogous collusion-resistant pricing strategies for the multi-band scenario in a similar way, and moreover, the complexity can be further reduced by a polynomial-time approximation using semi-definite programming (SDP). The details will be presented in our future work.

V. SIMULATION RESULTS

Consider a 1000×1000 m^2 area where N secondary users are uniformly distributed. Assume each secondary user has an R_I-meter coverage radius, that is, two users at least $2R_I$ meters away can share the band without mutual interference. The valuations of different users $\{v_1, v_2, \ldots, v_N\}$ are assumed to be i.i.d. random variables uniformly distributed in $[20, 30]$.

First, we consider the one-band auction, i.e., $M = 1$. Fig. 2 shows the average revenue versus the number of secondary users with $R_I = 150$ or 350. Different from the inefficient second-price auction, the other three mechanisms (we refer to pricing strategies (4), (5), (7) as "VCG price", "Proposed I", and "Proposed II", respectively) guarantee the optimal allocation, but as shown in the figure, the proposed methods can significantly improve the primary user's revenue, e.g., nearly 15% increase compared to the VCG outcome when $R_I = 350$, and 30% increase when $R_I = 150$. This means the proposed algorithms have better performance when more secondary users are admitted to lease the band simultaneously.

Moreover, the proposed auction mechanisms can effectively combat user collusion. We use the percentage of the system utility taken away by colluders to represent the vulnerability to sublease colluding attacks. Fig. 3 demonstrates the results from 100 independent runs, with a line segment representing the range of the results, and a marker representing their mean.
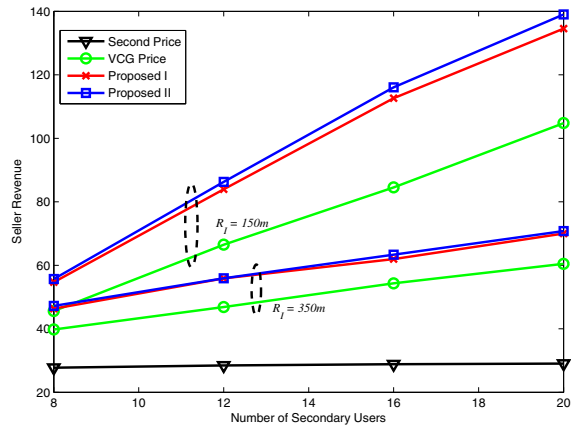
Fig. 2. Seller's revenue when different auction mechanisms are employed.
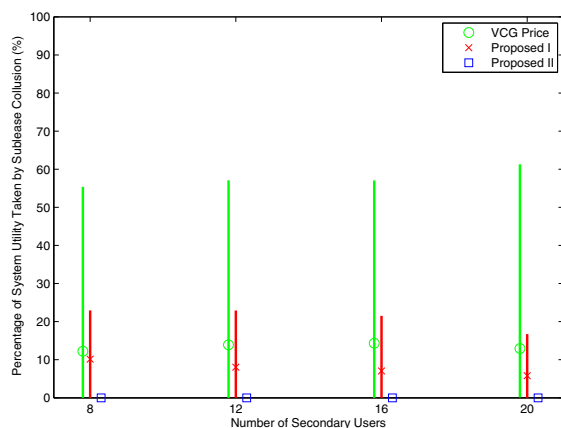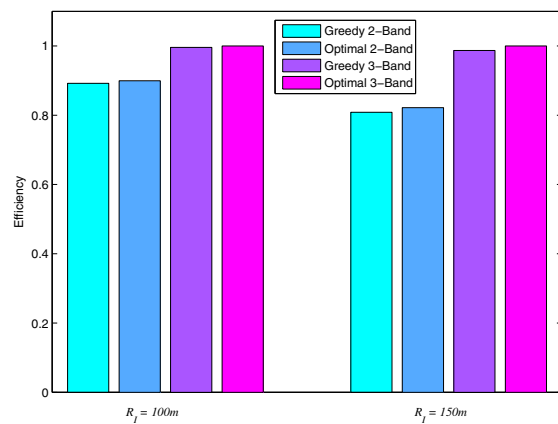

Fig. 4. Approximate efficiency of the greedy algorithm compared to the optimal solution.

trum efficiency and a higher seller's revenue. We further extend the one-band auction to a multi-band case, and propose a greedy algorithm achieving almost the same efficiency as the optimal solution with complexity greatly reduced.

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Fig. 3. Normalized collusion gains when different auction mechanisms are employed.

With the VCG pricing strategy, colluders could steal away more than 10% of the social welfare on average, whereas in the worst case, they may even grasp up to half of the system utility. If using pricing strategy (5) instead, the system will be more robust against colluder attacks, as colluding gains drop considerably. Furthermore, the proposed strategy (7) can completely prevent user collusion, as shown in the figure.

Finally, we show that for the multi-band auction ($M > 1$) the proposed greedy algorithm can approximately achieve the efficient allocation. As illustrated in Fig. 4, the normalized system utilities are evaluated for both the greedy and optimal algorithms. We can see that for both two-band and three-band auction cases, the proposed greedy algorithm (9) can achieve a comparable outcome with the optimal solution to (8).

VI. CONCLUSIONS

In this paper, we investigate the pricing mechanism in a multi-winner spectrum auction, in which secondary users can lease some unused bands from primary users. As the existing schemes, such as the second-price auction and the VCG mechanism, have several drawbacks, we propose collusion-resistant one-band auction mechanisms which yield full spec-