Repeated Spectrum Sharing Game with Self-Enforcing Truth-Telling Mechanism

Yongle Wu, Beibei Wang, and K. J. Ray Liu

Department of Electrical and Computer Engineering and Institute for Systems Research University of Maryland, College Park, MD 20742, USA {wuyl, bebewang, kirliu}@umd.edu

Abstract—Dynamic spectrum access has become a promising approach that can coordinate different users' access to adapt to spectrum dynamics to improve spectrum efficiency. However, users competing for an open spectrum may have no incentive to cooperate with each other, and they may even exchange false private information about their channel conditions in order to get more access to the spectrum. Therefore, in this paper, we propose a self-enforcing truth-telling mechanism by modeling the distributed spectrum access as a repeated game. In this game, if any greedy user deviates from cooperation, punishment will be triggered. Through the Bayesian mechanism design, users have no incentive to reveal false channel conditions, and the competing users are enforced to cooperate with each other honestly. The simulation results show that the proposed scheme can greatly improve the spectrum efficiency by alleviating mutual interference; furthermore, the best strategy for each user is demonstrated to be reporting the actual channel condition.

I. INTRODUCTION

With the emergence of new wireless applications and devices, the last decade has witnessed a dramatic increase in the demand for radio spectrum, which has forced the government agencies such as Federal Communications Commission (FCC) to review their policies [1]. The traditional rigid allocation policies by FCC have severely hindered the efficient utilization of the scarce spectrum. Hence, dynamic spectrum access, with the aid of cognitive radio technology [2], has become a promising approach by breaking the paradigm and enabling wireless devices to utilize the spectrum adaptively and efficiently.

Several centralized sharing schemes have been proposed to improve the spectrum efficiency, e.g., [3] and [4]. However, since multiple users compete for the spectrum resources, they may have conflicting interests. Therefore, game theory is a proper and flexible tool to analyze the interactions among the selfish users [5]. A local bargaining mechanism was proposed in [6] to distributively optimize the efficiency of spectrum allocation and maintain fairness. In [7], auction mechanisms were proposed for sharing spectrum among multiple users such that the interference was below a certain level. In [8], the authors proposed a repeated game approach to enlarge the set of achievable rates, in which the spectrum sharing strategy could be enforced by the Nash Equilibrium of dynamic games. In [9], belief-based dynamic pricing approaches were developed to optimize the overall spectrum efficiency based on double auction rules.

Although existing dynamic spectrum access schemes using game theoretical approaches have successfully enhanced spectrum efficiency, in order to achieve more flexible spectrum access in long-run scenarios, some basic questions still remain unanswered. First, the spectrum environment is constantly changing and there is no central authority to coordinate the spectrum access of different users. Thus, the spectrum access scheme should be able to distributively adapt to the spectrum dynamics with only local observations. Moreover, since users compete for an open spectrum, they do not have an incentive to cooperate with each other, and may even exchange false private information about their channel conditions if cheating is profitable. Therefore, novel spectrum sharing schemes should be developed to enhance the efficiency of the spectrum usage.

Motivated by the preceding, in this paper, we propose a selfenforcing truth-telling mechanism for open spectrum sharing by modeling the distributed spectrum access as a repeated game. In this game, punishment will be triggered if any user deviates from cooperation. In this way, users are enforced to access the spectrum cooperatively. Furthermore, through the Bayesian mechanism design [10], users are self-enforced to exchange the private information without any distortion. The proposed sharing scheme with truth-telling mechanism has also been validated by simulation.

The remainder of this paper is organized as follows. In Section II, the model for open spectrum sharing is described. In Section III, we propose a punishment-based scheme that provides players with an incentive to cooperate. We discuss the detection of deviation in Section IV, and develop a truth-telling mechanism in Section V. Simulation results are provided in Section VI, and Section VII concludes the paper.

II. SYSTEM MODEL

We consider a situation where K groups of users coexist in the same area, competing for the same unlicensed spectrum band. The users within the same group try to communicate with each other, while their usage of the spectrum will introduce interference to other groups. For simplicity, we assume each group consists of a single transmitter-receiver pair, then the whole system can be modeled as a K-user Gaussian interference channel,

$$y_i = h_{ii}x_i + \sum_{j=1, j \neq i}^{K} h_{ji}x_j + w_i, \quad i = 1, 2, \dots, K,$$
 (1)

where x_i is the transmitted information of the *i*-th pair, y_i is the received signal at the *i*-th receiver, $h_{ji}(j =$ $1, 2, \ldots, K; i = 1, 2, \ldots, K)$ represents the channel gain from the *j*-th transmitter to the *i*-th receiver, and w_i is the white noise at the receiver. We assume the channels are Rayleigh fading, i.e., $h_{ji} \sim C\mathcal{N}(0, \sigma_{ji}^2)$, and distinct h_{ji} 's are independent of each other. The channels are assumed to be invariant during one time slot, whereas they will change independently from slot to slot. The noise is independently identically distributed (i.i.d.) with $w_i \sim C\mathcal{N}(0, N_0)$, where N_0 is the noise power. The transmitter has an average power constraint P_i . For convenience, the bandwidth of the interested spectrum is assumed to be 1, and we define $g_i = |h_{ii}|^2$, which is an exponentially distributed random variable with probability density function $f(g_i) = \frac{1}{\sigma_{ii}^2} \exp(-\frac{g_i}{\sigma_{ii}^2})$. For simplicity, we only discuss the homogenous case in this paper, where

$$P_i = P, h_{ii} \sim \mathcal{CN}(0, 1), h_{ji} \sim \mathcal{CN}(0, \gamma) \text{ for all } i \neq j.$$
 (2)

The results can be easily generalized to a heterogenous scenario.

By treating mutual interference as noise, the long-term averaged transmission rate can be approximated by

$$r_{i}^{S} = E \left[\log \left(1 + \frac{g_{i} p_{i}(g_{i}, P_{i})}{N_{0} + \sum_{j \neq i} p_{j}(g_{j}, P_{j}) |h_{ji}|^{2}} \right) \right], \quad (3)$$

where the expectation is with respect to all channel realizations $\{h_{ji}, j = 1, \ldots, K, i = 1, \ldots, K\}$, and $p_i(g_i, P_i)$ is the power allocation of user *i* according to the power constraint P_i and channel condition g_i . The transmitters can employ waterfilling strategy over the time domain $p_i(g_i, P_i) = (\mu(P_i) - N_0/g_i)_+$, where the notation $(a)_+$ means $\max(a, 0)$, and $\mu(P_i)$ should satisfy the constraint

$$\int_{0}^{+\infty} \left(\mu(P_i) - \frac{N_0}{g_i} \right)_+ f(g_i) \, \mathrm{d}g_i = P_i. \tag{4}$$

Alternatively, the transmitter can simply use uniform power allocation $p_i(g_i, P_i) = P_i$ regardless of the channel conditions.

III. PUNISHMENT BASED SPECTRUM SHARING

When wireless users transmit non-cooperatively, the interference is strong and the resulting capacity is low. On the contrary, if there's a powerful central authority to regulate the access, the spectrum can be shared more efficiently. However, in the unlicensed band, such a powerful authority does not exist. Then, the strategic interaction among selfish users can be well modelled into a game.

The spectrum sharing game consists of K players, where each transmitter-receiver pair is a player. The actions of the players are how much power they will allocate to the time slots, and the payoffs are the obtained throughput. Since the wireless systems coexist over a long period of time, the spectrum sharing game is actually a repeated game, where players care about not only the current payoff but also the rewards in the future. The overall discounted utility for player i is $U_i = (1-\delta) \sum_{n=0}^{+\infty} \delta^n r_i(n)$, where $r_i(n)$ is the player i's payoff at n-th time slot, and $\delta (0 < \delta < 1)$ is the discount factor.

Now we show how cooperation is maintained in the repeated game. If every player accesses the spectrum selfishly, each will receive a very low expected payoff r_i^S in every round because of the strong interference caused by the others; if players make an agreement and share the spectrum orderly, everyone may benefit from the cooperation, achieving a higher average payoff r_i^C . However, one player may probably benefit more by violating the rule when all the others follow the rule, say the reward is r_i^D ($r_i^D > r_i^C$). When each round of the repeated game is played independently, everyone will have the incentive to deviate. On the contrary, if the deviating player will get punished in the future rounds of the game, it is possible to prevent deviation and maintain cooperation.

Therefore, the proposed game consists of the cooperation stage and the punishment stage. In the cooperation stage, players share the spectrum cooperatively, and everyone enjoys the higher payoff r_i^C ; while in the punishment stage, players punish each other by transmitting all the time, and hence the strong interference reduces the payoff to a unfavorable value r_i^S . The "punish-and-forgive" strategy is as follows: if any player deviates in the cooperation stage, punishment will be triggered and the game will jump into the punishment stage for the next T-1 time slots before players "forgive" the deviating behavior and cooperation resumes. Therefore, although deviating increases the current payoff, yet the incurring punishment will reduce the future payoffs. Provided the punishment period is long enough, deviation is not profitable. According to the Folk Theorem [5], as long as $r_i^C > r_i^S$ for all players, and players are patient enough, they will have the incentive to cooperate without any deviations.

If δ is given, we can find the proper punishment duration T, which should be large enough to deter players from deviating. For example, the one who deviates at time T_0 will have a higher instantaneous payoff, but will be punished for the next T-1 periods. Assume the most profitable deviation yields a payoff r_i^D , then the highest expected payoff with deviation is

$$U^{D} \stackrel{\triangle}{=} (1-\delta) \left(\delta^{T_0} \overline{r_i^D} + \sum_{n=T_0+1}^{T_0+T-1} \delta^n r_i^S + \sum_{n=T_0+T}^{+\infty} \delta^n r_i^C \right).$$
(5)

Otherwise, cooperation always maintains, yielding the expected payoff

$$U^C \stackrel{\triangle}{=} (1-\delta) \sum_{n=T_0+1}^{+\infty} \delta^n r_i^C.$$
 (6)

From the selfish player's point of view, the one with the higher payoff is the better choice. Therefore, if T is chosen such that $U^C > U^D$, players will be self-enforced to cooperate in spectrum sharing. The necessary condition is $T > \overline{r_i^D} / (r_i^C - r_i^S) + 1$ when δ is close to 1.

Now, the problem becomes how to design a cooperation rule that yields higher payoffs. Out of the many possible choices of cooperation rules, the simplest way is the orthogonal channel allocation, i.e., only one user is allowed to access the spectrum at one time slot according to their channel conditions. The allocation rule $d(g_1, g_2, \ldots, g_K)$ takes channel gains as input parameters, and outputs the index of player that is assigned the channel. Allocating the spectrum to the player with highest instantaneous channel gain at the current time slot, i.e.,

$$d(g_1, g_2, \dots, g_K) = \arg\max g_j,\tag{7}$$

can maximize the total throughput, so we use it as the cooperation rule. Since the interference is avoided and multiuser diversity gain can be reaped, we can expect a higher payoff. As shown by simulation results in later part of this paper, this cooperation rule always benefits players when the interference level among selfish players is medium to high.

Since the system is homogenous as defined in (2), every player can expect a $\frac{1}{K}$ chance to get access to the spectrum. When allocation rule (7) is employed, we can derive the expected cooperative payoff by using order statistics,

$$r_i^C = \int_0^{+\infty} \log\left(1 + \frac{KPg}{N_0}\right) e^{-g} \left(1 - e^{-g}\right)^{K-1} \mathrm{d}g.$$
(8)

IV. OPTIMAL DETECTION TIME

The punishment based spectrum sharing game can provide all players with the incentive to obey the rules, since defection is deterred by the threat of punishment. Detection of the deviating behavior is necessary to ensure the threat to be credible. In our case, the player who is assigned with the spectrum listens to the channel by using detectors such as the energy detector [11]. However, there do exist the possibilities that the detector believes someone else is using the channel although in fact nobody is. This false alarm event will trigger the game into punishment phase by mistake, reducing the system efficiency. In general, the performance of the detector can be improved by increasing the detection time. Nevertheless, the player cannot transmit and detect at the same time because one cannot easily distinguish one's own signal from other players' signal in the same spectrum. As a result, the more time one spends on the detection, the less time one reserves for data transmission. There is a tradeoff between transmission and detection.

Assume that α portion of the time slot is used for detection, while the rest is for transmission. The detector is imperfect with false alarm probability $F(\alpha)$. Now we derive the discounted utility $V_i(\alpha)$, which is made up of two parts: one is the current expected payoff, and the other is the expected rewards in the future. Since α portion of the time has been taken for detection, only the rest $(1 - \alpha)$ part can be used for transmission. Consequently, the expected current transmission throughput reduces to $(1 - \alpha)r_i^C$. Here, we neglect the impact of the parameter α on the power allocation. For the future rewards, the state will remain in the cooperation stage with probability $1 - F(\alpha)$, and it will jump into punishment stage for T - 1 slots with probability $F(\alpha)$. In sum, this modified discount utility should satisfy the following equation

$$V_i(\alpha) = (1-\delta)(1-\alpha)r_i^C + (1-F(\alpha))\delta V_i(\alpha) +F(\alpha)\left((1-\delta)\sum_{n=1}^{T-1}\delta^n r_i^S + \delta^T V_i(\alpha)\right), \quad (9)$$

from which $V_i(\alpha)$ can be solved as

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$$V_i(\alpha) = \frac{(1-\delta)(1-\alpha)r_i^C + (\delta-\delta^T)F(\alpha)r_i^S}{1-\delta + (\delta-\delta^T)F(\alpha)}.$$
 (10)

Note that the discounted payoff $V_i(\alpha)$ is the convex combination of $(1 - \alpha)r_i^C$ and r_i^S , and thus $r_i^S < V_i(\alpha) < r_i^C$ for all $0 < \alpha < 1 - r_i^S / r_i^C$. Although there is a little loss in the payoff, the players still have the incentive to join in this repeated game and cooperate.

With all other parameters fixed, we can vary the detection time α to obtain the highest discounted payoff, making the impact of non-ideal detection as low as possible. By the first order condition, the optimal α^* is the solution to the equation

 $(1-\delta+(\delta-\delta^T))r_i^C+((1-\alpha)r_i^C-r_i^S)(\delta-\delta^T)\frac{F'(\alpha)}{F(\alpha)}=0,$ (11) where $F'(\alpha)$ is the derivative of $F(\alpha)$. Note that by replacing r_i^C with $V_i(\alpha^*)$, the impact of imperfect detection is incorporated into the game, and needs no further considerations.

V. SELF-ENFORCING TRUTH TELLING

The repeated game discussed so far is based on the assumption of complete information, i.e., all $\{g_i, i = 1, 2, ..., K\}$ are known by all players. In practice, player *i* measures his/her own channel gain g_i , and then broadcasts the information to others through a control channel. However, there is no guarantee that the players will reveal their private information honestly. Furthermore, as our strategy allocates the spectrum to the user with the best channel condition, the selfish players actually tend to exaggerate their channel gains in order to acquire more access to the spectrum. The whole repeated game will be undermined by the distorted information. Therefore, enforcing the players to tell the truth is a crucial problem.

In [8], the truth telling is also ensured by the threat of punishment, but a delicate and complex detection scheme is required to catch the liars. In this paper, a much easier method is proposed. By designing a Bayesian mechanism, all players will get the incentive to tell the truth.

The key point in the Bayesian mechanism is a concept called transfer function. Each player is assigned with a transfer value according to the information they reveal. If the transfer is negative, the player has to pay to others as if they are paying tax; otherwise, the player receives compensation from others. The total payoff is the obtained transmission rate plus the transfer. The player may get the chance to transmit by claiming a higher channel gain, but has to pay more tax; on the contrary, the player will receive the compensation by claiming a lower gain, however, at the cost of less opportunity to occupy the spectrum. By appropriately designing the transfer functions, the total payoff can be maximized when the player claims the exact channel gain, which makes the players self-enforced to tell the truth.

Proposition 1: Assume $\{\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_K\}$ is a realization of independently distributed random variables $\{g_1, g_2, \ldots, g_K\}$ at one time slot, and players claim their channel gains as $\{\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K\}$. The transfer is calculated according to the claimed value, denoted by $t_i(\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K)$ for player *i*. The transfer function $t_i(\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K) = \Phi_i(\hat{g}_i) - \frac{1}{K-1} \sum_{j=1, j\neq i}^K \Phi_j(\hat{g}_j)$, where

$$\Phi_i(\hat{g}_i) = (K-1) \! \int_{\hat{g}_i}^{+\infty} \! \log \left(1 + \frac{KPg}{N_0} \right) e^{-g} \left(1 - e^{-g} \right)^{K-2} \! \mathrm{d}g, \quad (12)$$

will provide the player with the incentive to tell the truth given all the other players report the true values. Furthermore, the



Fig. 1. The payoffs when players share the spectrum cooperatively or selfishly.

designed transfer is balanced, i.e., $\sum_{i=1}^{K} t_i(\hat{g}_1, \hat{g}_2, \dots, \hat{g}_K) = 0$ at any time slot.

Proof: Because the system is homogenous, it suffices to show that player 1 will reveal the true private information given all the others are honest. The expected payoff of player 1 is the sum of the transmission rate and the transfer $t_1(\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K)$. Since player 1 already knows his/her channel condition, the expectation is taken over all realizations of $\{g_2, g_3, \ldots, g_K\}$ throughout this proof.

Define the obtained throughput

$$r_i(g_i, d(g_1, \dots, g_K)) = \begin{cases} \log(1 + \frac{KPg_i}{N_0}) & \text{if } d(g_1, \dots, g_K) = i \\ 0 & \text{otherwise,} \end{cases}$$

it can be shown that $\Phi_1(\hat{g}_1)$ given in the proposition is equivalent to $E[\sum_{j=2}^K r_j(g_j, d(\hat{g}_1, g_2, \dots, g_K))]$ by using order statistics. Then, the total payoff of player 1 is $E[r_1(\tilde{g}_1, d(\hat{g}_1, g_2, \dots, g_K)] + t_1(\hat{g}_1, \hat{g}_2, \dots, \hat{g}_K)] = E[r_1(\tilde{g}_1, d(\hat{g}_1, g_2, \dots, g_K) + \sum_{j=2}^K r_j(g_j, d(\hat{g}_1, g_2, \dots, g_K))]] - \frac{1}{K-1} \sum_{j=2}^K \Phi_j(\hat{g}_j)$. Player 1 will claim \hat{g}_1 instead of \tilde{g}_1 if and only if this is profitable, i.e.,

$$E\left[r_{1}(\tilde{g}_{1}, d(\tilde{g}_{1}, g_{2}, \dots, g_{K}) + \sum_{j=2}^{K} r_{j}(g_{j}, d(\tilde{g}_{1}, g_{2}, \dots, g_{K}))\right] < E\left[r_{1}(\tilde{g}_{1}, d(\hat{g}_{1}, g_{2}, \dots, g_{K}) + \sum_{j=2}^{K} r_{j}(g_{j}, d(\hat{g}_{1}, g_{2}, \dots, g_{K}))\right].$$
(13)

Note that the channel allocation strategy in (7) can maximize the total throughput, i.e., for any realization of $\{g_2, g_3, \ldots, g_K\}$, $\sum_{i=1}^{K} r_i(\tilde{g}_i, d(\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_K)) > \sum_{i=1}^{K} r_i(\tilde{g}_i, d')$ for any possible allocation strategy d'. After taking the expectation, we have $E\left[r_1(\tilde{g}_1, d(\tilde{g}_1, g_2, \ldots, g_K)) + \sum_{j=2}^{K} r_j(g_j, d(\tilde{g}_1, g_2, \ldots, g_K))\right] > E\left[r_1(\tilde{g}_1, d') + \sum_{j=2}^{K} r_j(g_j, d')\right]$ for any d', which contradicts (13). As a result, $\hat{g}_1 = \tilde{g}_1$, and player 1 is self-enforced to report the true value.

The proof of the second part of the proposition is quite straightforward. $\sum_{i=1}^{K} t_i(\hat{g}_1, \hat{g}_2, \dots, \hat{g}_K) =$ $\sum_{i=1}^{K} \left(\Phi_i(\hat{g}_i) - \frac{1}{K-1} \sum_{j=1, j \neq i}^{K} \Phi_j(\hat{g}_j) \right) = \sum_{i=1}^{K} \Phi_i(\hat{g}_i) \sum_{j=1}^{K} \Phi_j(\hat{g}_j) = 0$, which concludes the proof. \blacksquare We can see that everyone telling the truth is an equilibrium

We can see that everyone tering the truth is an equilibrium for the game: given all the others are honest, the best choice for an individual is also to report the true private information. Hence, by using the transfer function defined above, nobody will have the incentive to deviate from truth-telling. Since the transfer is balanced, some players pay while others get paid, and the exchange is done among players without surplus. Moreover, since the transfer function only depends on the signal-to-noise ratio (SNR) and the number of players K, the values can be stored in a look-up table, which makes it easier for implementation.

VI. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the proposed mechanism. First, we show under what condition the proposed cooperative sharing can be profitable $(r_i^C > r_i^S)$, and thus players can get the incentive to cooperate. The simplest game consisting of only two players K = 2 is studied. In Fig. 1 (a), the cooperation payoff r_i^C and noncooperation payoff r_i^S are plotted versus the interference level γ (defined in (2)) when the averaged SNR = P/N_0 = 15dB. Both uniform power allocation and waterfilling power allocation are considered. Because for cooperative spectrum usage, only one player gets the transmission opportunity at one time slot, the expected payoff is independent of the strength of the interference, and thus is a horizontal line in the figure. The non-cooperation payoffs drop significantly with increasing interference strength. From the figure, we may see that the cooperative payoff is larger than the non-cooperative counterpart for a wide range of the interference level except when the interference level is too small ($\gamma < 0.1$). Therefore, with medium to high interference that is common in practical situations, the players can benefit from the cooperation, and are self-enforced to join the spectrum sharing game. Similar



Fig. 2. The cooperation gain in a K-player spectrum sharing game.



Fig. 3. The discounted utility impacted by the length of the detection time.

results can be seen for lower SNR range, for example, Fig. 1 (b) with SNR = 5dB. Note that the waterfilling has little improvement for the cooperative spectrum sharing even when the SNR is low, because it is quite unlikely for a player to get access to the spectrum when in deep fading.

Moreover, since the proposed channel allocation strategy can benefit from the multiuser diversity, we can expect the gain grows with the number of the players in the game. In Fig. 2, the cooperation gain, characterized by the ratio of r_i^C/r_i^S , is illustrated as the number of the players increases, when the interference level is fixed as $\gamma = 1$. As expected, the more players involved in the game, the larger cooperation gain can be obtained, and the gain will saturate when the number of players is large.

Now we show how player's discount utility $V(\alpha)$ depends on the portion of time slot α used for detection. Assume $r_i^S =$ 3 and $r_i^C = 6$, Fig. 3 plots the curve according to (10) when an energy detector with a fixed threshold is used. When the detection time is short, the utility is quite low due to the high false alarm rate; and when the detection time is too long, a significant portion of the transmission opportunity is wasted. The two effects are balanced by choosing the optimal α that maximizes the utility.

Finally, we check the incentive for players to report the true channel parameters through simulations. We assume a 3-



Fig. 4. The total expected payoff with different claimed parameters.

user spectrum sharing game with SNR = 15dB. Given the other two players are honest, the expected payoff (capacity plus transfer) of player 1 is plotted versus the claimed value \hat{g} . In Fig. 4, three curves are provided, with the true channel gain \tilde{g} set to be 0.4, 0.8, and 1.2, respectively. From the figure, we may see player 1 will get a lower payoff if claiming any value other than the true channel gain. Therefore, players are self-enforced to tell the truth with the proposed transfer functions.

VII. CONCLUSIONS

We have proposed a self-enforcing truth-telling mechanism to improve the efficiency of open spectrum sharing. We model the spectrum sharing as a repeated game, where any deviation from cooperation will trigger the punishment. Moreover, we optimize the detection time to alleviate the impact due to imperfect detection of selfish behavior. Finally, we propose a Bayesian mechanism design to enforce the selfish users to report their true channel information.

REFERENCES

- Federal Communications Commission, "Spectrum policy task force report," FCC Document ET Docket No. 02-135, Nov. 2002.
- [2] J. Mitola III, "Cognitive Radio: an integrated agent architecture for software defined radio," Ph.D. Thesis, KTH Royal Institute of Technology, Stockholm, Sweden, 2000.
- [3] S. Keshavamurthy, and K. Chandra, "Multiplexing analysis for spectrum sharing," Proc. of IEEE MILCOMM'06, Washingto D.C., Oct. 2006.
- [4] B. Wang, Z. Ji, and K. J. R. Liu, "Primary-prioritized Markov approach for dynamic spectrum access," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, Dublin, Apr. 2007.
- [5] M. J. Osborne, An Introduction to Game Theory, Oxford University Press, 2004.
- [6] L. Cao, and H. Zheng, "Distributed spectrum allocation via local bargaining," *Proc. of IEEE SECON 2005*, pp. 475–486, Santa Clara, Sept. 2005.
- [7] J. Huang, R. Berry, and M. L. Honig, "Auction-based Spectrum Sharing," ACM/Springer Mobile Networks and Apps., vol. 11, no. 3, pp. 405–418, June 2006.
- [8] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE Journal on selected areas in communications*, vol. 25, no. 3, pp. 517–528, Apr. 2007.
- [9] Z. Ji, and K. J. Ray Liu, "Dynamic Spectrum Sharing: A Game Theoretical Overview," *IEEE Communications Magazine*, vol. 45, no. 5, pp. 88–94, May 2007.
- [10] D. Fudenburg, and J. Tirole, Game Theory, MIT Press, 1993.
- [11] F. F. Digham, M.-S. Alouini and M. K. Simon, "On the energy detection of unknown signals over fading channels," in *Proc. of IEEE International Conference on Communications (ICC'03)*, Anchorage, May 2003.